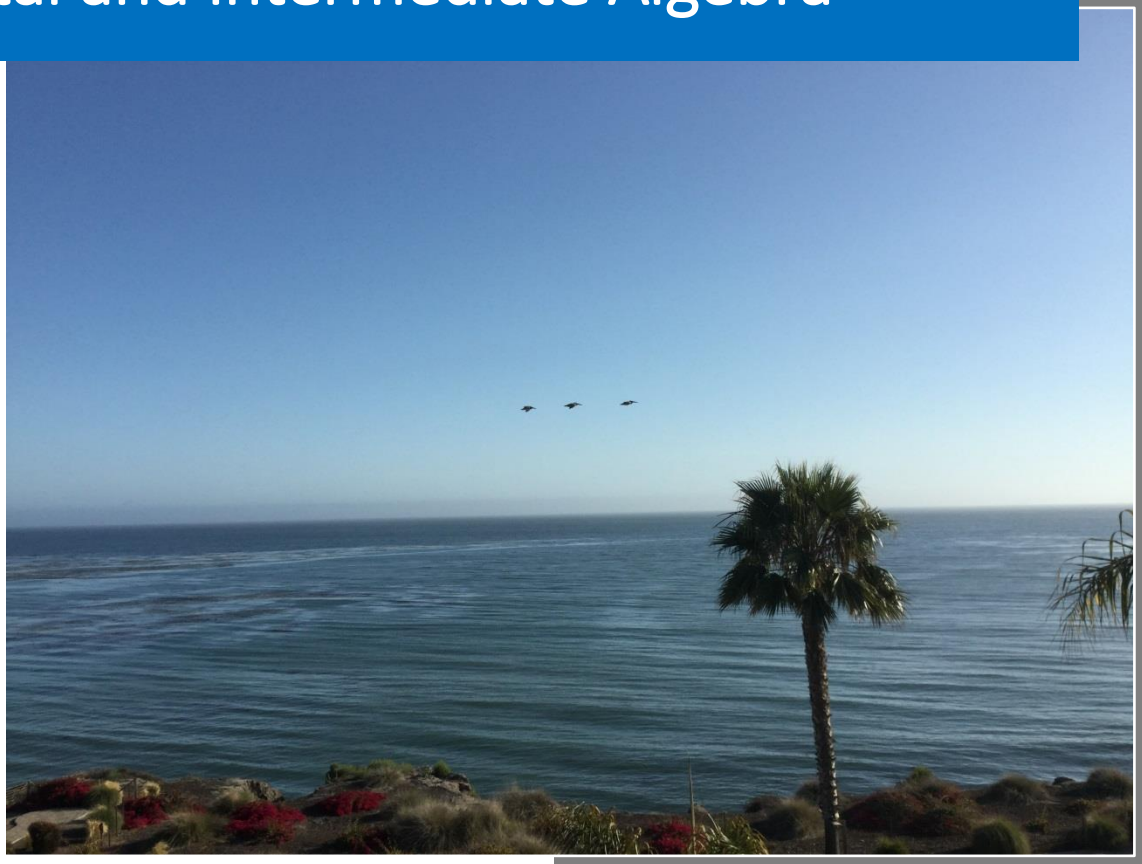
Worksheets

Developmental and Intermediate Algebra



Stalder & Martin

Supplement to Developmental and Intermediate Algebra

Developmental and Intermediate Algebra Supplemental Worksheets

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About the authors

Dr. Shubhangi Stalder is a full professor of mathematics at the University of Wisconsin Waukesha. She has received her doctoral degree in mathematics from the University of Wisconsin Milwaukee in 1993. She has decades of teaching experience and her focus has always been to reach out to those who are struggling in mathematics. Her main belief is that everyone can learn basic mathematics if they tried. The key is to understand the "Why" and the "How, and to be able to see the patterns across different mathematical processes. She believes that in the long run rote memorization does not work to learning mathematics. She uses yoga and meditation techniques with her students who experience math and test anxiety and continues to include mindfulness practice in her teaching of mathematics. She has received the UW System Board of Regents Teaching in Excellence Award (the state of Wisconsin's highest teaching award), the UW Colleges Chancellor's Excellence in Teaching Award, and the UW Colleges Kaplan Teacher Award.

Dr. Paul Martin is a full professor of mathematics at the University of Wisconsin Marathon. He received his doctoral degree in mathematics from the University of Wisconsin Madison in 1994. He has decades of teaching experience and his focus has always been to help his students see how mathematics connects to the real world. He does this through building 3-dimensional models, modeling physical processes such as heat loss through the attic of a house, to connecting mathematics to his students' other classes from chemistry to music. As a teacher, Martin stresses the importance of reasoning over memorization. He has received several teaching awards over his career including the prestigious UW Colleges Chancellor's Excellence in Teaching Award.

Acknowledgements

The worksheets accompany the second edition of the Developmental and Intermediate Algebra text by the same authors. The material on which the worksheets are based on initially were developed based on a two year Developmental and Intermediate Algebra curricular redesign project funded by the UW-System Committee on Baccalaureate Expansion (COBE) grant. The material in this book is inspired by Sybilla Beckmann's work in her *Mathematics for Elementary Teachers* textbook, brain plasticity research, and Jo Boaler's growth mindset research.

The authors would like to thank their families who gave their professional opinions, their constant support and put up with long hours on this project (without their support it would have been difficult to put this book together in a such a short time).

We would like to thank Mark Fuller, Scott Budzien, Sunita Mehrotra, and Leah Rineck for their valuable feedback that lead to the development of the this supplemental worksheets.. In addition, we want to thank all the countless brave souls (students and teachers) who were willing to give us their time in trying this material, and giving us their feedback to make this project better. We hope their work and ours has made it possible to get a product that we hope will be useful to other students and teachers.

Contents

Pre-Class Worksheet	Section 0.1	**Date:**_____ **Name:**_____

E-text pages 1-18

- **Introduction to Content (3min)** **http://www.youtube.com/watch?v=SiXM29eUw2k**
- **Introduction to the class and study skills (11 min)** **http://www.youtube.com/watch?v=th4cI8ugE-I**

1. What is the basic premise of the course?

2. Who can learn mathematics barring any brain dysfunction?

3. What is brain plasticity?

4. Give the modular structure of the content of the course.

5. What is “Mathematics”?

6. What is a “Problem”?

7. What is a “Mistake”?

8. Why do you think so many students when faced with a difficult problem immediately jump to the key phrase of “I don’t know”?

9. What is “Habit Energy”?

Difficulties encountered :

Pre-Class Worksheet	Section 0.1	**Date:_____ Name:_____**

E-text pages 1-18

- **Introduction to the class and study skills (8 min) http://www.youtube.com/watch?v=I5OktRxH79c**
- **Mindfulness of thoughts https://www.youtube.com/watch?v=5xrKjyLRnAw (3 min)**
- **Body Scan https://www.youtube.com/watch?v=vdm06q3AmwY (17:29 min)**
- **Exam Part 1 https://www.youtube.com/watch?v=NllnoZeJ2X4 (5:13min)**
- **Exam Part 2 https://www.youtube.com/watch?v=14g0h3AA9_o (4:54min)**

10. List at least one problem solving strategy.

11. What is mindfulness?

12. List one mindfulness practice you might use when stressed out.

13. List the importance of getting help and the role ego might play.

14. List one organizational tip you learned.

15. List one study skills tip you learned.

16. What is the role of proper nutrition, sleep, and exercise in learning of mathematics?

Difficulties encountered :

Pre-Class Worksheet	Section 1.1	**Date:**_____ **Name:**_____

E-text pages 19-27

Decimal Number System **https://www.youtube.com/watch?v=B6GA-o6YoLw** **(12 min)**

1. Write briefly about steps you followed in attempting the counting project.

2. Write the base of the different number systems below.

Number System	**Tally Mark**	**Babylonian**	**Aztec or Mayan**	**Hindu Arabic**	**Binary**
Base					

3. Which of these number systems are still currently being used today?

4. What is a decimal number?

5. Explain in your own words the terminology below

Terminology	Your own words to describe it
Decimal Number in expanded form	
Terminating Decimal	
Non-terminating Decimal with repeating pattern	
Non-terminating Decimal without repeating pattern	

6. What is the notation to write a Non-terminating Decimal with a repeating pattern? _____

7. Give an example of the similarity of the expanded form of a natural number and of a third degree polynomial.

Difficulties encountered :

Pre-Class Worksheet	Section 1.2	**Date:**_____ **Name:**_____

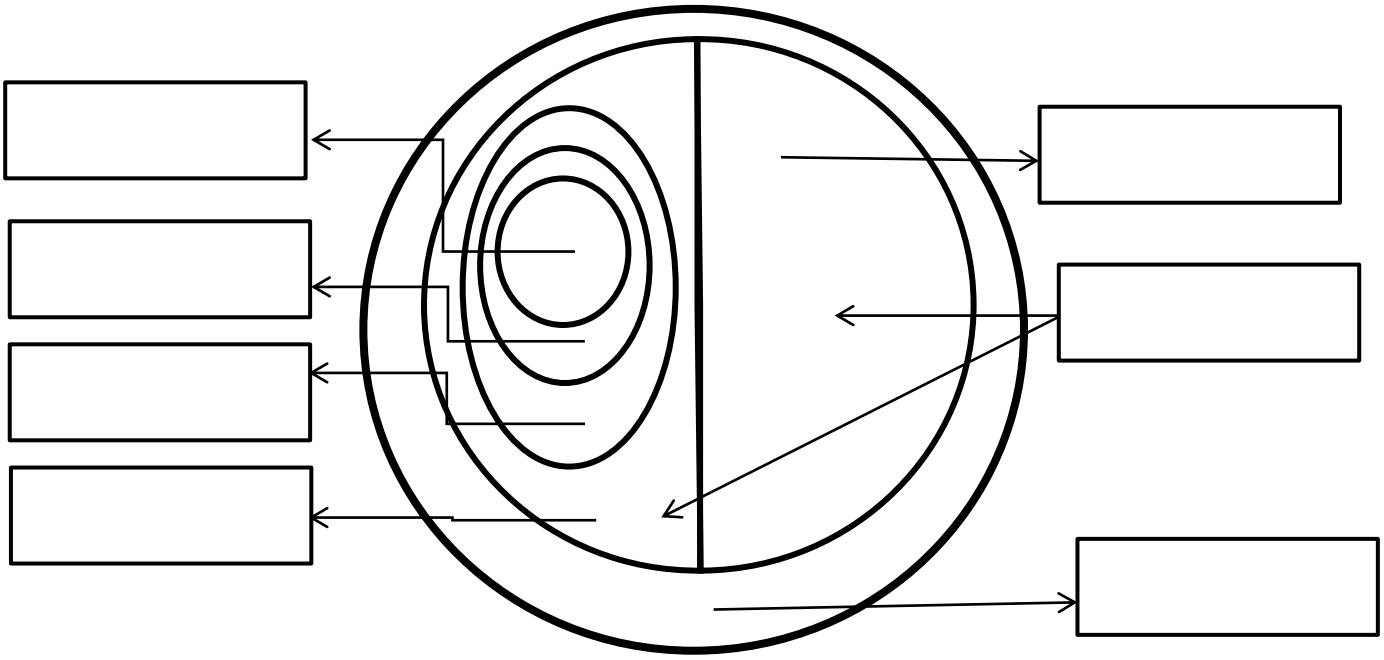
E-text pages 28-31

Natural through Complex Numbers http://www.youtube.com/watch?v=MH946PzUGIg **(13 min)**

1. Explain in your own words the terminology below and provide examples asked for

Terminology	Your own words to describe it	Examples
Counting Numbers		_____ is a counting number.
Whole Numbers		_____ is a whole number that is not a counting number.
Integers		_____ is an integer that is not a whole number.
Rational Numbers		_____ is a rational number that is not an Integer.
Irrational Numbers		_____ is an irrational number.
Real Numbers		_____ is a real number.
Imaginary Numbers		_____ is an imaginary number.
Complex Numbers		_____ is a complex number that is not a real number.

2. Fill the Venn diagram or the parts in the pictorial representation of the hierarchy of all the number sets as shown in the video or e-text.



Difficulties encountered :

Pre-Class Worksheet	Section 1.3	**Date:_____ Name:_____**

E-text pages 35-40

Geometry http://www.youtube.com/watch?v=X4v0CZzC9ec (10 min)

1. Explain in your own words the terminology below and draw an example showing how to visualize each term.

Terminology	**In your own words**	**Draw a picture to visualize each term**
Point		
Line Segment		
Line		
Ray		
Angle		
Acute Angle		
Obtuse Angle		
Right Angle		
Parallel Lines		
Perpendicular Lines		
Transversal		
Alternate Interior Angles		
Alternate Exterior Angles		
Corresponding Angles		
Complementary Angles		
Supplementary Angles		
Polygons		
Triangle		
Rectangle		

Continuation	Section 1.3	**Date:_____ Name:_____**

Terminology	**In your own words**	**Draw a picture to visualize each term**
Square		
Quadrilateral		
Hexagon		
Circle		

Difficulties encountered :

Pre-Class Worksheet	Section 1.4	**Date:**_____ **Name:**_____

E-text pages 43-47

One Visualization of Rational numbers http://www.youtube.com/watch?v=79ZjO2MTiOc **(11 min)**

1. Describe briefly in your own words what a rational number is.

2. For a rational number that looks like $\frac{a}{b}$, a is called _____, and b is called _____.

3. Show three different visual representations of different rational numbers below.

a.	b.	c.

4. List at least one other mathematical object or concept that arises as an extension of a rational number and explain the similarities between that concept and a rational number.

5. What is a difference between an improper fraction and a proper fraction? Give two examples.

6. What is the restriction on the denominator to have a valid rational number or a rational mathematical object?

Difficulties encountered :

Pre-Class Worksheet	**Section 1.4**	**Date:_____ Name:_____**

E-text pages 47-51

Equivalent Fractions **http://www.youtube.com/watch?v=xruSTzZcpns** **(11 min)**

7. What are equivalent fractions?

8. Sketch a visual representation of two equivalent fractions.

9. Why does multiplying or dividing both the numerator and denominator by the same factor create an equivalent fraction?

10. What is a fraction in simplest or lowest terms?

11. What kind of rational numbers can be written as a terminating decimal?

12. Show how to represent the fractions below as decimal numbers.
    a. $\frac{3}{40}$

    b. $\frac{1}{3}$

Difficulties encountered :

Pre-Class Worksheet	**Section 1.4**	**Date:_____ Name:_____**

E-text pages 51-54

Ratios and Percents http://www.youtube.com/watch?v=Z5JYj_FQx7M (15 min)

13. Describe in your own words what a ratio is and give a real life example of a ratio.

14. Give an example that involves the number 15% and draw a diagram to represent the example.

15. For the percentages below, create equivalent fractions, representing them visually using percent diagrams, and write a real life scenario where you might encounter these.

Percentage	Fraction	Visual Representation	Real Life Scenario
75%			
200%			
60%			

16. Show how to convert the percentages below into a decimal number and vice-versa.

Percentage	Decimal
230%	
30%	
0.25%	

Percentage	Decimal
	30.4
	0.07
	0.005

17. Show how to compute the percentages below visually.

a. 5% sales tax on a TV that costs $1240.	**b. A 90$ table is on 60% off sale. Find the sale price of the table.**

Difficulties encountered :

Pre-Class Worksheet	**Section 1.4**	**Date:_____ Name:_____**

E-text pages 57-59

Ordering Numbers http://www.youtube.com/watch?v=Wjcel8TB4mg (8min)

18. What is the Trichotomy property of real numbers?

19. How would you order the numbers below in descending order?

Numbers	**Descending Order**	**Explain in words the strategies used**
$\mathbf{45.69,\ 3.40,\ 23.40,\ 3,49}$		
$\frac{2}{3}, \frac{4}{5}, \frac{3}{5}, -\frac{1}{3}$		

20. How would you order the numbers below in ascending order?

Numbers	**Descending Order**	**Explain in words the strategies used**
$0.34, 20, -43.45, 20.34$		

21. How can ordering be used to determine which pile has more of the amounts represented below?

Quantities		**Which one has more ants?**
a. Ants that come into a home for food. 	b. Ant Colony 	

Difficulties encountered :

Pre-Class Worksheet	Section 1.5	**Date:**_____ **Name:**_____

E-text pages 60-64

Plotting Numbers on a Number Line http://www.youtube.com/watch?v=BohmHn8NgOA **(14 min)**

1. What is a number line?

2. What is a scale or increment on a number line?

3. Give an example of where you might use a horizontal number line in real life.

4. Give an example of where you might use a vertical number line in real life.

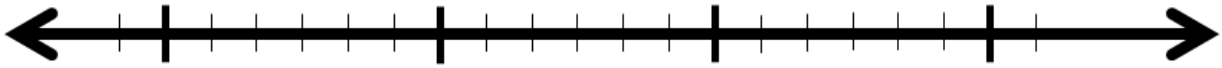
5. When you plot decimal numbers on the number line what kind of increments can we use?

6. In plotting decimal numbers we can use powers of ten as our increments. Give an example of a tool in real life where the increments are not powers of ten?

7. Plot a number on each number line below and show what the increment between each tick mark is.

   a.

   b.



8. Show how to plot complex number $-\mathbf{5}+\mathbf{3}\boldsymbol{i}$.

Difficulties encountered :

Pre-Class Worksheet	**Section 1.5**	**Date:_____ Name:_____**

E-text pages 68-71

Scientific Notation http://www.youtube.com/watch?v=4lM8zwi0kWY (10 min)

9. What is scientific notation and why is it used?

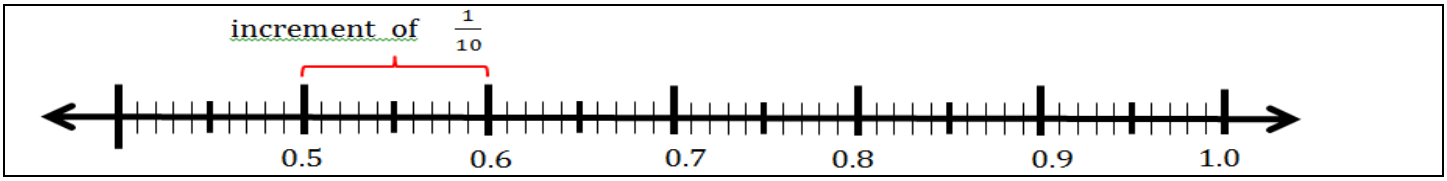
10. Give two examples in real life where you see Scientific Notation being used.

11. Fill in the table below.

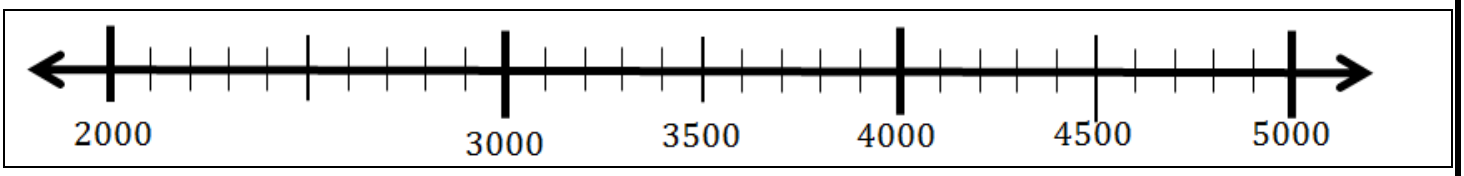
Decimal Number	Scientific Notation
345000	
0.00049	

Decimal Number	Scientific Notation
	5.13×10^{6}
	7.8×10^{-12}

12. Use the number line below to show how you would round $\mathbf{0.752}$ to the nearest tenth's place?



13. Use the number line below to show how you would round $\mathbf{3435}$ to the nearest thousand's place?



14. List two examples of rounding in real life and explain what that means.

Difficulties encountered :

Pre-Class Worksheet	Section 1.6	**Date:**_____ **Name:**_____

E-text pages 73-77

Exponents http://www.youtube.com/watch?v=QlnQTDKNH_Q **(9 min)**

1. For any counting number $\boldsymbol{n}$, $\boldsymbol{a^n}$ =_____

2. In the mathematical expression $\boldsymbol{a^n}$,

a is called the _____	n is called the _____

3. We read $\boldsymbol{a^n}$ as (Please write the English words as if you were speaking it out loud below)

4. Please fill in the table below.

Example	Base	Exponent	English words as spoken out loud	Expanded form	Evaluate
2^3					
$(-5)^3$					
-4^2					
a^2					
$-(f(x))^3$					
$(ab)^5$					
$(a^2c^3)^4$					

Difficulties encountered :

Pre-Class Worksheet	**Section 1.6**	**Date:_____ Name:_____**

E-text pages 77-81

Product Rule of Exponents http://www.youtube.com/watch?v=qS2yuBEXcxk **(9 min)**

5. What does a mathematician do to make sense of something not encountered before?

6. Explain how to make sense of the following using the definition of exponents. Explain as if you are talking to someone not in your class.

Question	**Making sense by writing in expanded notation**	**Explanation in words so someone not in your class understands**
$\mathbf{2^3 \times 2^4}$		
$\mathbf{3^2 \times 3^5}$		
$\mathbf{4^{22} \times 4^{100}}$		
$\mathbf{(10^2)^4}$		
$(5x+7)^3(5x+7)^8$		
$((2x+4)^4)^3$		
$-(c^2c^3)^4$		

7. State the Product Rule.

8. State the Power Rule.

9. Which number is bigger and why?

a. 2^{10}	b. 3^{10}

Difficulties encountered :

Pre-Class Worksheet	**Section 1.6**	**Date:_____ Name:_____**

E-text pages 82-84

Quotient Rule of Exponents http://www.youtube.com/watch?v=SgEyb7s1Vcw **(5 min)**

10. Explain how to make sense of the following using the definition of exponents. Explain as if you are talking to someone not in your class.

Question	**Making sense by writing in expanded notation**	**Explanation in words so someone not in your class understands**
$\frac{10^6}{10^2}$		
$\frac{x^8}{x^3}$		
$\left(\frac{10^6}{10^2}\right)^3$		
$\frac{(2a-b)^5}{(2a-b)^2}$		
$\left(\frac{d^{13}}{d^2}\right)^3$		
$\frac{(-b)^{12}}{(-b)^3}$		

11. State the Quotient Rule.

Difficulties encountered :

Pre-Class Worksheet	Section 1.7	**Date:**_____ **Name:**_____

E-text pages 86-97

Zero and Negative Exponents http://www.youtube.com/watch?v=3_pnpRr93hA **(14 min)**

1. Why should we define $\boldsymbol{a^0 = 1}$ for all non-zero real numbers $\boldsymbol{a}$? Explain in words to someone not in your class with examples.

2. What is the meaning of $\boldsymbol{a^{-n}}$ for a non-zero real number $\boldsymbol{a}$, and counting number $\boldsymbol{n}$? Explain in words to someone not in your class using examples.

3. What is the meaning of $\frac{\boldsymbol{1}}{\boldsymbol{a^{-n}}}$ for a non-zero real number $\boldsymbol{a}$, and counting number $\boldsymbol{n}$? Explain in words to someone not in your class using examples.

4. What is the difference between $\boldsymbol{-3^2}$, and $\boldsymbol{3^{-2}}$? Explain your answer.

5. What is the difference between $\boldsymbol{(-3)^2}$, and $\boldsymbol{(-3)^{-2}}$? Explain your answer.

6. Write in English words how you would read $\boldsymbol{-3^2}$ as opposed to $\boldsymbol{(-3)^2}$?

Difficulties encountered :

Pre-Class Worksheet	**Section 1.7**	**Date:_____ Name:_____**

7. Fill in the blanks below.

	Base	**Exponent**	**English words to read it**	**Evaluate**
a. -3^2				
b. -3^{-2}				
c. $(-3)^2$				
d. $(-3)^{-2}$				
e. $(-2)^3$				
f. $(-2)^{-3}$				
g. $-\frac{1}{3^2}$				
h. $-\frac{1}{3^{-2}}$				
i. $\frac{1}{(-3)^2}$				
j. $\frac{1}{(-3)^{-2}}$				
k. $\frac{1}{(-2)^3}$				
l. $\frac{1}{(-2)^{-3}}$				

8. Fill in the blanks below for a summary of all the rules learnt so far regarding exponents.

➢ $\boldsymbol{a^n a^m =}$	➢ $\boldsymbol{\frac{a^n}{a^m} =}$	➢ $\boldsymbol{(a^m)^n =}$	➢ $\boldsymbol{a^0 =}$	➢ $\boldsymbol{a^{-n} =}$	➢ $\boldsymbol{\frac{1}{a^{-n}} =}$

Difficulties encountered :

Pre-Class Worksheet	Section 1.8	**Date:**_____ **Name:**_____

E-text pages 100-103

Rational Exponents http://www.youtube.com/watch?v=GJVtvQ2bm8M **(13 min)**

1. We know that $\mathbf{2^5 = 32}$. What is the meaning of $\mathbf{32^{\frac{1}{5}}}$?

2. In general then for any counting number $\boldsymbol{n}$ and a real number $\boldsymbol{a}$, what does $\boldsymbol{a^{\frac{1}{n}}}$ mean? Explain in your own words to someone not in your class.

3. For any counting number $\boldsymbol{n}, \boldsymbol{m}$ and a real number $\boldsymbol{a}$ What does $\boldsymbol{a^{\frac{m}{n}}}$ mean? Explain in your own words to someone not in your class.

4. What is the radical notation for $\boldsymbol{a^{\frac{1}{n}}}$?

5. In the notation $\sqrt[n]{\boldsymbol{a}}$, $\boldsymbol{a}$ is called _____, and $\boldsymbol{n}$ is called _____.

6. Evaluate the following.

Evaluate	English Words to Read it	Evaluate	English Words to Read it
a. $\sqrt{25} =$		b. $\sqrt[3]{-8} =$	
c. $\sqrt[3]{64} =$		d. $\sqrt{-4} =$	

Difficulties encountered :

Pre-Class Worksheet	**Section 1.8**	**Date:_____ Name:_____**

E-text pages 103-110

Radical Notation http://www.youtube.com/watch?v=B0zdWX3CzFE **(8 min)**

7. Fill in the empty columns below.

Exponential Notation	Radical Notation
$a^{\frac{1}{2}}$	
	$\sqrt[3]{a}$
$a^{\frac{1}{5}}$	
$a^{-\frac{1}{3}}$	
	$\sqrt{\frac{1}{a}}$
	$\sqrt[5]{a^2} = \left(\sqrt[5]{a}\right)^2$

Exponential Notation	Radical Notation
$a^{\frac{7}{2}}$	
$49^{\frac{1}{2}}$	
	$\sqrt[6]{3}$
$8^{-\frac{1}{5}}$	
	$\sqrt{x+y}$
	$-\left(\sqrt[3]{5}\right)^2$

8. Evaluate the following. Your final answer should be without a radical. Assume all variables are nonzero positive real numbers.

a. $\sqrt[5]{3^5}$	**b.** $\frac{1}{\sqrt[7]{a^7}}$
c. $\sqrt{49}$	**d.** $\sqrt[3]{27}$
e. $-\sqrt{81}$	**f.** $\sqrt{-81}$

Difficulties encountered :

Pre-Class Worksheet	**Section 1.8**	**Date:_____ Name:_____**

E-text pages 111-113

Estimating Radicals http://www.youtube.com/watch?v=hMWQUtQuTKI (5 min)

9. Using estimations fill in the blanks with an integer less than, and an integer greater than the given radical. Explain your answer.

a.	b.	c.
$\square < \sqrt{6} < \square$	$\square < \sqrt{29} < \square$	$\square < \sqrt[3]{10} < \square$

10. For each number in the left column, state whether it is less than, equal to, or greater than the number at the top of each column. Follow the example in the first row.

	$2^{\frac{1}{2}}$	2^3	-2^2	$\frac{1}{\sqrt{2}}$	$4\sqrt{2}$	$-\frac{1}{2^{-\frac{1}{2}}}$
$\sqrt{2}$	$=$	$<$	$>$	$>$	$<$	$>$
$\frac{1}{2^2}$						
-4						
4						
$\sqrt{32}$						
$2^{-\frac{1}{2}}$						
$\frac{1}{2^{-\frac{1}{2}}}$						
$-\sqrt{2}$						

For My Eyes Only column

Difficulties encountered :

Pre-Class Worksheet	Section 1.9	**Date:_____ Name:_____**

E-text pages 118-121

Polynomials https://www.youtube.com/watch?v=GjpAlev8o8E (14 min)

1. What happens to the expanded form of a decimal number when we replace the base ten with a variable like x? Give an example to demonstrate this concept.

2. What is a polynomial?

3. Give the definitions below along with concrete examples.

Definition	**Example**
➢ **Degree of a polynomial:**	
➢ **Leading Coefficient:**	
➢ **Constant Term:**	
➢ **Monomial:**	
➢ **Binomial:**	
➢ **Trinomial:**	

4. What is a rational expression?

Difficulties encountered :

Pre-Class Worksheet	Section 1.9	Date:_____ Name:_____

E-text pages 121-124

- **Translating Words Part 1** http://www.youtube.com/watch?v=Ff-bOPs5iz4 **(13 min)**
- **Translating Words Part 2** http://www.youtube.com/watch?v=xVKV_9OsNeQ **(6 min)**

5. List at least two places where a polynomial is used in writing a formula for some quantity.

6. Write a polynomial for the following situations.

Words	**Polynomial**	**Words**	**Polynomial**
Perimeter of a rectangle		"Twice as much as 3 more than a quantity"	
Area of a rectangle		half of a quantity"	
Perimeter of a square		"37% of a quantity"	
Area of square		"x times as much as y"	
Circumference of a circle		"x more than y"	
Area of a circle		"x less than y"	
Surface area of a rectangular prism		"r % of y"	
Volume of a rectangular prism		"r percent more than y"	
"Twice" a quantity		"r percent less than y"	
"Two more than a quantity"		"x **is** at least as much as y"	
"30 more than twice a quantity"		"x is at most as much as y"	

Difficulties encountered :

Pre-Class Worksheet	Section 1.10	Date:_____ Name:_____

E-text pages 121-124

Introduction to Functions http://www.youtube.com/watch?v=GHR4QiPoBi8 **(11 min)**

1. Fill in the chart below.

Concept	In your words describe what these concepts mean to you
➢ Relation	
➢ Function	
➢ Domain of a function	
➢ Range of a function	
➢ Four different ways to represent a function (give examples)	

2. What is the function notation? Given an example below.

Difficulties encountered :

Pre-Class Worksheet	Section 1.10	Date:_____ Name:_____

3. Describe what the functions below are and then give an example of each type.

Functions	Examples
a. Constant Function:	
b. Square root function:	
c. Exponential Function:	
d. Polynomial Function:	
e. Rational Function:	

4. Evaluate the following
   a. $\boldsymbol{f(x) = \sqrt{x} + 1}$, find $\boldsymbol{f(3)}$
   b. $\boldsymbol{C(x) = 100}$, find $\boldsymbol{C(50)}$
   c. $\boldsymbol{Absolute(t) = |t|}$, $\boldsymbol{Absolute(-200)}$

Difficulties encountered :

Pre-Class Worksheet	Section 2.1	**Date:**_____ **Name:**_____

1. **Identifying Like Units** **http://www.youtube.com/watch?v=Zqzb5VpogNs** **(6 min)**
2. Fill in the blanks to complete the definitions or sentences as you see fit.
   a. Addition is a _____which means it acts on two objects with specific rules.

   b. The objects themselves are called _____or_____.

   c. The mathematical notation or symbol for addition is _____.

   d. You can only add quantities or mathematical objects that have _____.

3. Not using the correct units can have huge consequences. Please write briefly about one disaster that can be attributed to not using appropriate units.

4. Write two quantities below that you consider to have the same units and can be added together.

5. **Properties of Addition and Introduction to Adding Decimal Numbers**
   http://www.youtube.com/watch?v=b12XsziOpJA **(10 min)**
6. Briefly describe the properties and/or terminology related to addition in your own words.

Name of Property	
a. Commutative property of addition	
b. Associative property of addition	
c. Additive Identity	

Difficulties encountered :

Pre-Class Worksheet	**Section 2.1**	**Date:_____ Name:_____**

Addition of Decimal Numbers, Polynomials, Radical Expressions, and Functions
http://www.youtube.com/watch?v=Xwwy9_-NQ0M (14 min)

1. In your own words describe the basic principle behind any addition algorithm?

2. What are similarities and dissimilarities between adding decimal numbers and polynomials?

3. Show how these similarities and dissimilarities play out in the examples a & b, c & d, and e & f.

a. $3489 + 896$	**b.** $(3x^3 + 4x^2 + 8x + 9) + (8x^2 + 9x + 6)$
c. $(3 + 4i) + (5 + 6i)$	d. $\left(3 + 4\sqrt{x}\right) + \left(5 + 6\sqrt{x}\right)$
e. $6(3x + 4) + 5(3x + 4)$	f. $6(3x + 4) + 5x(3x + 4)$

Difficulties encountered :

Pre-Class Worksheet	**Section 2.1**	**Date:_____ Name:_____**

Introduction to Addition of Fractions and Rational Expressions http://www.youtube.com/watch?v=y_LvHKSC10E (9 min)

4. Draw a strip diagram to show how to add $\frac{3}{4}+\frac{5}{6}$. Used colored pens/pencils to show all the relevant parts of the addition in your picture and how it relates to the final answer.

5. What basic principle becomes visible from drawing the strip diagram above regarding adding of fractions?

6. Show how this basic principle carries over to addition of fractions below.

a. $2\frac{3}{4}+4\frac{1}{3}$

b. $\frac{3}{x}+\frac{2}{5}$

Difficulties encountered :

Pre-Class Worksheet	Section 2.2	**Date:**_____ **Name:**_____

Properties of Multiplication http://www.youtube.com/watch?v=5tt0WWHEJm4 **(10 min)**

1. Fill in the blanks to complete the definitions or sentences as you see fit.
   - **a.** Multiplication is a _____which means it acts on two objects with specific rules.
   - **b.** Mathematical notation or symbol used for multiplication is _____.
   - **c.** Multiplication of whole numbers can be thought of as repeated _____.

2. Briefly describe the properties and/or terminology related to multiplication in your own words.

Name of Property	
a. Commutative property of multiplication	
b. Associative property of multiplication	
c. Multiplicative Identity	
d. Distributive property of multiplication over addition	

3. Answer true or false and briefly explain your answer.
   a. $\mathbf{3} \times (\mathbf{2} \times \mathbf{5}) = (\mathbf{3} \times \mathbf{2}) \times (\mathbf{3} \times \mathbf{5})$

   b. $\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c}) = (\boldsymbol{a} \times \boldsymbol{b}) \times (\boldsymbol{a} \times \boldsymbol{c})$

Difficulties encountered :

Pre-Class Worksheet	**Section 2.2**	**Date:_____ Name:_____**

Visualizing Multiplication **http://www.youtube.com/watch?v=0ofeTiqGSFs** **(13 min)**

4. Show how to visualize the multiplications below. Used colored pens/pencils to show all the relevant parts of the multiplication in your picture and how it relates to the final answer.

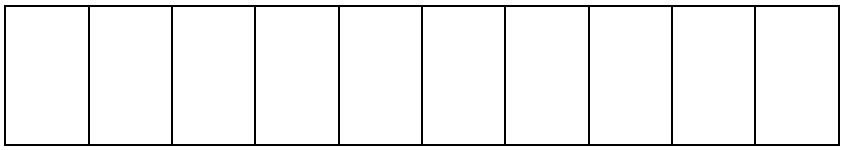
a. $\mathbf{23 \times 35}$

b. $\mathbf{(2x + 3)(3x + 5)}$

5. Describe the similarities and dissimilarities between the two multiplication problems above.

6. Explain why when multiplying decimal numbers like $\mathbf{4.59 \times 64.3}$, we can multiply $\mathbf{459 \times 643}$ and then adjust the decimal point.

7. Suppose that 15% of a school budget is to be used for school supplies and the school budget totals $360,000. Then what is the amount that could be spent on school supplies? Show the solution visually and then explain how multiplication plays a role in the final answer. Use colored pens/pencils to show your explanation.



Difficulties encountered :

Pre-Class Worksheet	**Section 2.2**	**Date:_____ Name:_____**

Adding Rational Expressions Using Multiplication https://www.youtube.com/watch?v=J9qz5xSlIOA **(6 min)**

8. Show all your steps in adding the following rational expressions and how multiplication plays a role in getting the final answer.

a. $\frac{2}{3+4x}+\frac{4}{x}$

b. $\frac{1}{3+a}+\frac{2}{1+a}$

c. $\frac{5x}{2+x}+\frac{x}{2x+3}$

d. $\frac{3x+1}{x+3}+\frac{5x+12}{2x+1}$

Difficulties encountered :

Pre-Class Worksheet	Section 2.3	**Date:**_____ **Name:**_____

Properties of Subtraction http://www.youtube.com/watch?v=W9PEgpFyAYg **(15 min)**

1. Fill in the blanks to complete the definitions or sentences.
   a. Subtraction is _____ process of addition.

   b. An object "$\boldsymbol{a}$"multiplied by $-\mathbf{1}$ yeilds its additive _____.

   c. Another way to think of – $\boldsymbol{a}$ is _____.

   d. $\boldsymbol{a}(-\boldsymbol{b}) = -\boldsymbol{a}(\boldsymbol{b}) =$ _____.

   e. $-\boldsymbol{a}(-\boldsymbol{b}) =$ _____.

   f. $\boldsymbol{a}(\mathbf{0}) = (\mathbf{0})(\boldsymbol{a}) =$ ____, and is called the _____ property of real numbers.

   g. If $(\boldsymbol{a})(\boldsymbol{b}) = \mathbf{1}$, then $\boldsymbol{a}$ is called the _____ of $\boldsymbol{b}$ and vice versa.

   h. Exponential notation for multiplicative inverse of $\boldsymbol{a}$ is _____.

   i. $\boldsymbol{a}$ and $\boldsymbol{a}^{-\mathbf{1}}$ have the same _____.

   j. $\boldsymbol{a} \times \boldsymbol{a}^{-\mathbf{1}} =$_____.

2. Explain the difference between the terms $-\boldsymbol{a}$ and $\boldsymbol{a}^{-\mathbf{1}}$.

3. Explain briefly the distributive property of multiplication over subtraction.

Difficulties encountered :

Pre-Class Worksheet	**Section 2.3**	**Date:_____ Name:_____**

Subtraction Algorithm http://www.youtube.com/watch?v=azaR-4ySSwQ **(9 min)**

4. In subtraction what is the process of unbundling? Explain using the examples below.

a. $53 - 26$	***b.*** $\mathbf{5ft\ 3in - 1ft\ 10in}$ ***(remember*** $\mathbf{1\ ft = 12\ inches)}$
c. $679.43 - 287.56$	
d. $(5\ weeks + 3\ days + 5\ hours + 23\ minutes) - (4\ weeks + 6\ days + 19\ hours + 53\ minutes)$	

5. Explain what role units of an object play in subtraction compared to addition?

Difficulties encountered :

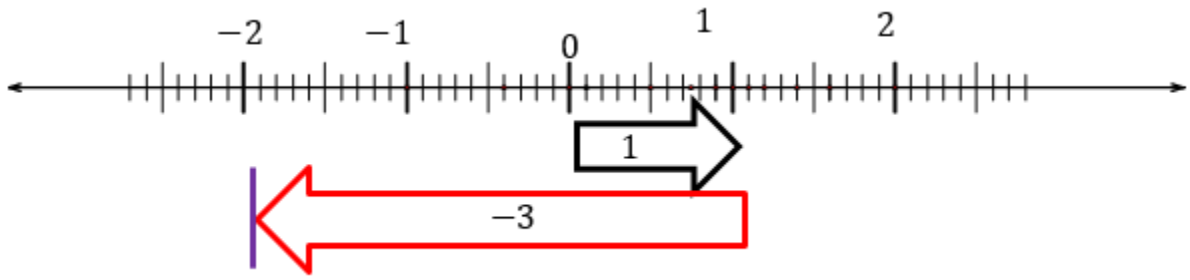
Pre-Class Worksheet	**Section 2.3**	**Date:**_____ **Name:**_____

Visualizing Subtraction http://www.youtube.com/watch?v=PwQGc_1p0jQ **(8 min)**

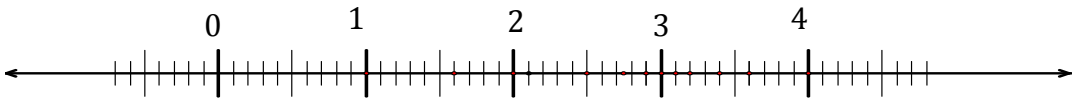
6. Explain to someone not in your class how visualize subtraction of real numbers on a number line.

7. Explain how to make sense of the following subtraction problems using real life examples.
   a. $\mathbf{-6-3}$
   b. $\mathbf{-100-(-25)}$

8. Explain the diagram below both as an addition problem and as a subtraction problem.



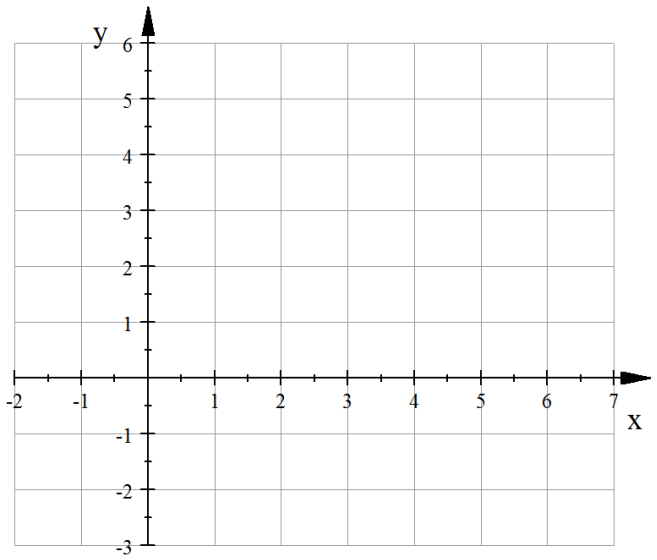
9. Use arrow diagrams like above to show the result of $\mathbf{4.5+(-3.5)-(-2)}$



10. Define absolute of a real number?

Difficulties encountered :

11. Show how to visualize the subtraction of complex numbers in the complex plane to the right $(2 + 4i) - (4 - 3i)$.



12. **Define absolute value of a complex number.**

13. **Find**
$|2 + 4i| =$ _____ **, and** $|4 - 3i| =$ _____

Subtraction http://www.youtube.com/watch?v=E7Cj8QnEmNo **(12 min)**

14. Describe the subtraction algorithm including the borrowing or unbundling principles for the examples below.

$$\begin{array}{r} 92356 \\ -\ 59417 \\ \hline \end{array} \qquad \begin{array}{r} 9x^4 + 2x^3 + 3x^2 + 5x + 6 \\ -\ 5x^4 + 9x^3 + 4x^2 + x + 7 \\ \hline \end{array}$$

15. Describe what the similarities and differences are between the two problems.

16. Draw a strip diagram to show how to subtract $\frac{2}{5} - \frac{1}{3}$. Used colored pens/pencils to show all the relevant parts of the subtraction in your picture and how it relates to the final answer.

17. What basic principle becomes visible from drawing the strip diagram above regarding subtraction of fractions?

18. Show how these basic principle carry over to addition of fractions below.

a. $\mathbf{5\frac{3}{4} - 2\frac{2}{3}}$

b. $\mathbf{\frac{2}{x} - \frac{3}{5}}$

c. $\mathbf{\frac{2+x}{x+4} - \frac{5x-2}{x+4}}$ (What is the like unit here?

19. Explain why we need a common denominator to subtract rational expressions.

20. Identify the "like terms" in each and complete the subtraction

a. $9\ cm + 6\ cm^2 - 4\ cm - 2\ cm^2$

b. $-3(2x + 1) - 7(2x + 1)$

c. $9\sqrt{x} + 15\sqrt[3]{x} - 3\sqrt{x} - 7\sqrt[3]{x}$

d. $(3 + 4i) - (5 + 2i)$

21. With $\boldsymbol{f(x) = 3x^2 - 2x + 4}$ and $\boldsymbol{g(x) = 5x^2 + 3x + 7}$, compute:

a. $(f - g)(x) =$

b. $(f - g)(-2) =$

22. Use subtraction with a 24-hour clock time to compute the number of days, hours and minutes that passed between Monday May 4th at 3:45PM and later that week on Saturday at 8:47 AM. Express this vertically and show the unbundling that is required at the three unit levels of day, hour, and minute.

Difficulties encountered :

Pre-Class Worksheet	Section 2.3	Date:_____ Name:_____

Subtraction of Rational Expressions http://www.youtube.com/watch?v=Vuvmrq54b4w **(8 min)**

23. Perform the following arithmetic operations and simplify your answers.

a. $-2-(-45)$

b. $(3x-4)(-5x^2)$

c. $-2(5x^3-4x-6)$

d. $(3x-4)(5-2x)$

e. $\frac{2}{3-4x}-\frac{4}{x}$

f. $\frac{1}{3-a}-\frac{2}{1-a}$

g. $\frac{5x}{2-x}-\frac{x}{2x+3}$

h. $\frac{3x}{3-4x}-\frac{2x-1}{x+2}$

Difficulties encountered :

Pre-Class Worksheet	Section 2.4	**Date:**_____ **Name:**_____

Prime factors and multiples of expressions http://www.youtube.com/watch?v=wy7pm8wjm_8 **(8 min)**

Multiples http://www.youtube.com/watch?v=f3ZdozzChjQ **(9 min)**

1. Please fill in the blanks below.

   a. In a product $\boldsymbol{ab}$, where $\boldsymbol{a}$ and $\boldsymbol{b}$ are any two mathematical objects we have studied so far, the individual objects $\boldsymbol{a}$ and $\boldsymbol{b}$ are referred to as _____.

   b. In a product $\boldsymbol{ab}$, where $\boldsymbol{a}$ and $\boldsymbol{b}$ are any two mathematical objects we have studied so far, the quantity $\boldsymbol{ab}$ can be referred to as a_____ of $\boldsymbol{a}$, and also as a _____ of $\boldsymbol{b}$.

2. Give at least two multiples of each expression below.

   a. 24

   b. $3x^2y^3$

3. Write each number as a product of prime factors.

   a. 84

   b. 3528

4. Give your definition of what it means for expression A to be a multiple of expression B.

5. Which of the following, if any, is a multiple of $\boldsymbol{(x-1)^2 \cdot x^6 \cdot (3x+1)^2}$**:**

   a. $(x-1)^3 \cdot x^{12} \cdot (3x+1)^3$

   b. $(x-1)^2 \cdot x^5$

Difficulties encountered :

Pre-Class Worksheet	**Section 2.4**	**Date:_____ Name:_____**

Least Common Multiples http://www.youtube.com/watch?v=wJCWNcytyXE **(15 min)**

6. Describe what the concept of the least common multiple is in your own words.

7. Find the least common multiple of the following.

a. 24, 15

b. $3a^2b^{10},\ b^3c^{12}d^5$

c. $x,\ x+1$

d. $x(x+1),\ x(x-1),$

Adding Rational Expressions Using LCM http://www.youtube.com/watch?v=O0V6hbTE-2s **(12 min)**

8. Perform the following operations and simplify your answers.

a. $\frac{5}{12}-\frac{7}{18}$

b. $\frac{1}{2^2\times3^2\times5}+\frac{1}{2^3\times3^2\times7}$

c. $\frac{c^3}{a^5b^2}-\frac{a^2}{7b^3c^2}$

d. $\frac{(5x-1)}{(x+1)(x-2)}-\frac{3x}{(x-2)(x-1)}$

e. $\frac{(2x+1)}{(x-1)(3x+2)}-\frac{(5x-2)}{(3x-2)(x-1)}$

Difficulties encountered :

Pre-Class Worksheet	Section 2.4	Date:_____ Name:_____

- Factoring Whole Numbers http://www.youtube.com/watch?v=snMzQARfX_M (8 min)
- Introduction to Factoring Polynomials http://www.youtube.com/watch?v=JR4rMAd0Mhg (13 min)

9. Give your definition of what it means for a factor to be prime.

10. When given the prime factorization of two numbers or expressions A and B, how can you tell if expression A is a multiple of expression B?

11. Explain in your own words what the concept of a greatest common factor of two or more expressions is?

11. Find the greatest common factors of the algebraic expressions given below.

a. $360,\ 2100$

b. $2^4 \times 3^{10} \times 7^3,\quad 2^{12} \times 3^5 \times 5^2$

c. $(x+1) \times (2x-3)^{10} \times x^3,\ x^{12} \times (x+1)^4 \times (x-3)^2$

d. $(x+1) \times (2x-3)^{10} \times x^3,\ (x-1)^4 \times (x-3)^2$

12. Rewrite the polynomials below as a product of factors, one of which must be the greatest common factor of all the terms in the polynomial.

a. $-3a^3b + 12a^2b$

b. $24a^6b^3 - 15a^2b^5$

c. $p(q-p) + q^2(p-q)$

d. $(2x-3)^4(5x+7) + (3x-1)(2x-3)^3$

Difficulties encountered :

Pre-Class Worksheet	**Section 2.4**	**Date:_____ Name:_____**

Adding and Subtracting Rational Expressions http://www.youtube.com/watch?v=p8tMoTPFyPI **(5 min)**

13. Perform the operations and simplify as much as you can.

For my eyes only:

a. $\dfrac{1}{x^2+4x} - \dfrac{x}{x+4}$

b. $\dfrac{(2-4x)}{x(x-1)+5(x-1)} - \dfrac{(x+1)}{2(x+5)-x(x+5)}$

Difficulties encountered :

Pre-Class Worksheet	Section 2.5	**Date:**_____ **Name:**_____

Factoring by Grouping Geometrically as Rectangles http://www.youtube.com/watch?v=JPWGp83_DUE **(6 min)**

1. What does factoring a polynomial mean?

2. Show how to visualize the factor by grouping below using rectangles.
   $\boldsymbol{3x + 3 + ax + a = 3(x + 1) + a(x + 1) = (3 + a)(x + 1)}$

Factoring by Grouping http://www.youtube.com/watch?v=yyMzSSw8KLQ **(5 min)**

3. Factor the following $\boldsymbol{3x + ay + ax + 3y}$

Factoring Trinomials using Algebra Tiles http://www.youtube.com/watch?v=-Xy0zEGIb54 **(11 min)**

Factoring Trinomials Algebraically http://www.youtube.com/watch?v=Ib9eeHyxwm4 **(10 min)**

4. Show how to use algebra tiles to factor $\boldsymbol{3x^2 + 8x + 4}$**.**

Difficulties encountered :

Pre-Class Worksheet	**Section 2.5**	**Date:_____ Name:_____**

Factoring Trinomials by Grouping http://www.youtube.com/watch?v=hvuH6eXbXWQ **(14 min)**

5. Factor the following algebraically. Show all your steps.

a. $x^2 + 5x + 6$

b. $x^2 - 12x + 20$

c. $\mathbf{6x^2 + 11x + 4}$

d. $\mathbf{x^2t + 5xt + 6t}$

e. $\mathbf{24a^2b - 28ab^2 - 20b^3}$

f. $\mathbf{-2x^2 + 5x - 3}$

Difficulties encountered :

Pre-Class Worksheet	Section 2.5	Date:_____ Name:_____

Application to Adding and Subtracting Rational Expressions http://www.youtube.com/watch?v=Ja2ul4TGuH0 **(7 min)**

6. Show how factoring is used to simplify the subtraction $\dfrac{x+1}{2x^2-5x+3} - \dfrac{x-1}{2x^2+x-6}$

Difficulties encountered :

Factoring the Difference of Two Perfect Squares http://www.youtube.com/watch?v=cy_n_YfFQlQ **(9 min)**

7. Explain the difference of squares formula below using rectangles.
   $\boldsymbol{a^2 - b^2 = (a - b)(a + b)}$

8. Show how to use difference of squares formulas to factor the following.
   a. $25x^2 - 16y^2$
   b. $18x^3y - 8xy^3$

Factoring the Sum or Difference of Perfect Cube Terms http://www.youtube.com/watch?v=2XvIb_JtvQQ **(12 min)**

9. Show how to use the sum and difference of perfect cubes formulas below to factor the following.
   $\boldsymbol{a^3 - b^3 = (a - b)(a^2 + ab + b^2)}$ and $\boldsymbol{a^3 + b^3 = (a + b)(a^2 - ab + b^2)}$
   a. $8x^3 - 27y^3$
   b. $64a^3 + 125b^3$

10. List a summary of all the factoring facts you have learned so far.
   - Factoring trinomial by grouping
   - Factoring difference of cubes
   - Factoring difference of squares
   - Factoring sum of cubes

11. Can the sum of squares $\boldsymbol{a^2 + b^2}$ be factored? Explain your answer.

Difficulties encountered :

Pre-Class Worksheet	Section 2.5	Date:_____ Name:_____

Application of Factoring http://www.youtube.com/watch?v=fm0NEqFloMA (7 min)

12. Combine the rational expressions into a single rational expression in lowest terms. Show all your steps.

a. $\dfrac{3x-2}{x^2-16} - \dfrac{2x+3}{x^2-5x+4}$

For my eyes only:

b. $\dfrac{-2x+1}{x^2+4x+16} + \dfrac{5x^2+3x+4}{x^3-64} - \dfrac{2}{x-4}$

Difficulties encountered :

Pre-Class Worksheet	Section 2.6	Date:_____ Name:_____

Multiplication of Rational Numbers and Expressions http://www.youtube.com/watch?v=e-F4CpSXzJ4 **(10 min)**

1. Show how to visualize the following multiplications.

   a. 2×3

   b. $5 \times \frac{2}{3}$

   c. $\frac{4}{5} \times \frac{2}{3}$

   d. $\frac{3}{4} \times \frac{2}{5}$

2. When adding two fractions we need to make a common denominator. Explain if making common denominator is necessary when multiplying two fractions.

Difficulties encountered :

Pre-Class Worksheet	Section 2.6	Date:_____ Name:_____

Multiplying Rational Numbers and Expressions http://www.youtube.com/watch?v=czoI6D3NNeg **(12 min)**

3. Write each product of rational numbers or expressions as a single rational expression in reduced or lowest form.

a. $\frac{42}{25} \times \frac{65}{56}$

b. $\frac{24}{-45} \times -\frac{30}{9}$

c. $-\frac{200}{15} \times \frac{18}{60} \times \frac{-12}{25}$

d. $\frac{a^4 b^{-3}}{b^2 c} \times \frac{-a^5 c^7}{b^3 c^5}$

e. $\frac{x^2 - 4}{x^2 - 5x} \times \frac{x^2 - 4x - 5}{x^2 - 3x + 2}$

For My Eyes Only

Difficulties encountered :

Pre-Class Worksheet	Section 2.6	Date:_____ Name:_____

Multiplication and Exponents http://www.youtube.com/watch?v=ExausXVXu_E **(15 min)**

4. Please fill in the blanks below to review all the rules of exponents.

For all integers n, m, and all nonzero real numbers a:

Product Rule: $a^m a^n =$_____

Power Rule 1: $(a^m)^n =$_____

Quotient Rule: $\frac{a^m}{a^n} =$_____

Negative Exponent:

$a^{-n} =$_____ and $\frac{1}{a^{-n}} =$_____

Zero Exponent: $a^0 =$_____

Radicals or Fractional Exponents:

$\sqrt[n]{a} =$_____ or $a^{\frac{1}{n}} =$_____

5. Simplify the following and do not leave any negative exponents in the answer.

a. $(2 \times 5)^3$

b. $\left(\frac{2}{5}\right)^3$

c. $a^{-3}a^5$

d. $(2x^2y^{-3})^{-2}$

e. $\left(\frac{3x^{-4}}{b^3}\right)^{-2}$

f. $\left(\frac{q^{-1}xy^{-2}}{x^{-4}y^3q^{-3}}\right)^{-2}$

g. $\left(\frac{3x^{-3}}{y^2}\right)^2\left(-\frac{3x^2}{5y^3}\right)^{-3}$

Difficulties encountered :

Review of radicals and fractional powers and simplifying radicals http://www.youtube.com/watch?v=Ab-epECGrl4 **(9 min)**

6. For all positive real numbers $\boldsymbol{a}$, fill the blanks below.

a. $\sqrt{a}\sqrt{a} =$_____

b. $\sqrt[3]{a}\sqrt[3]{a}\sqrt[3]{a} =$ _____

c. $\left(\sqrt[n]{a}\right)^n =$_____

d. $\sqrt[n]{a^n} =$_____

e. $\sqrt[n]{a}\sqrt[n]{b} =$_____

f. $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} =$_____

7. Multiply and simplify the following.

a. $\sqrt{3}\sqrt{3}$

b. $\dfrac{x^{\frac{5}{3}}}{x^{\frac{1}{2}}}$

c. $3\sqrt{y} \times 2\sqrt{y}$

d. $(2\sqrt{x} - 3)(5 + 3\sqrt{x})$

e. $\sqrt[3]{125b^3a^5}$

f. $(2 + \sqrt{3})(2 - \sqrt{3})$

Difficulties encountered :

Rationalizing Denominators of Radical Expressions http://www.youtube.com/watRch?v=BM7KwGKZBbs **(8 min)**

8. Rationalize the denominator so that your final answer does not have radical terms in the denominator.

a. $\sqrt{\frac{1}{2}}$

b. $\frac{3+2\sqrt{x}}{2-3\sqrt{x}}$

c. $\frac{3x}{\sqrt{12x^3}}$

d. $\frac{3-2i}{4+5i}$

Difficulties encountered :

Pre-Class Worksheet	Section 2.7	Date:_____ Name:_____

Introduction to Division http://www.youtube.com/watch?v=7gZ4yW1nr9Y **(13 min)**

1. Fill in the blanks below.
   a. Division is a binary operation denoted by the symbol _____.

   b. In the notation $\boldsymbol{a \div b = c}$, $\boldsymbol{a}$ is called the _____, $\boldsymbol{b}$ is called the _____and the result of the division $\boldsymbol{c}$ is called the _____.

   c. $\frac{\boldsymbol{a}}{\boldsymbol{b}} \div \frac{\boldsymbol{c}}{\boldsymbol{d}} =$_____

2. What are the two different interpretations of the division $\boldsymbol{a \div b}$, with of two real numbers $\boldsymbol{a, b}$, with $\boldsymbol{b \neq 0}$.

Introduction to Division of Rational Numbers http://www.youtube.com/watch?v=9LTICGxqwKE **(10 min)**

3. Draw a strip diagram to explain visual interpretation of the divisions below.

a. $4 \div \frac{1}{2}$

b. $\frac{4}{5} \div \frac{2}{3}$

Difficulties encountered :

Pre-Class Worksheet	**Section 2.7**	**Date:_____ Name:_____**

Division of Decimal Numbers and Rational Expressions http://www.youtube.com/watch?v=BGReDOGObbk **(7 min)**

4. Perform the following divisions.

a. $288 \div 12$

b. $\frac{32.5}{0.25}$

c. $\sqrt{3} \div \sqrt{2}$

d. $\dfrac{x^2-9}{x^2+2x+4} \div \dfrac{x^2-5x+6}{x^3-8}$

e. $\dfrac{8x^2-2y^2}{6x^2+13xy+5y^2} \div \dfrac{x^3y-xy^3}{3x^2+2xy-5y^2}$

Difficulties encountered :

Pre-Class Worksheet	**Section 2.7**	**Date:_____ Name:_____**

Division Algorithm for Decimal Numbers and Polynomials http://www.youtube.com/watch?v=XXr0ixy8PfA **(8 min)**

Division Algorithm for Decimal Polynomials http://www.youtube.com/watch?v=PQrIt8PhFAE **(11 min)**

5. Fill in the blanks below.
   a. Let $\boldsymbol{a}$ and $\boldsymbol{b}$ be any two mathematical objects where $\boldsymbol{b}$ ***is*** the divisor and $\boldsymbol{b} \neq \boldsymbol{0}$. If we set $\boldsymbol{c}$ to be the quotient of $\boldsymbol{a} \div \boldsymbol{b}$, and let $\boldsymbol{r}$ to be the remainder, then we have $\boldsymbol{a} \div \boldsymbol{b} =$_____.

   b. $\boldsymbol{0} \div \boldsymbol{a}$=_____

6. Perform the long division below and find the quotient and remainder.
   a. $676 \div 32$

   b. $0.324 \div 12$

   c. $\frac{23}{47}$

   d. $(6x^2 + 7x + 6) \div (3x + 2)$

   e. $(2x^3 - 5x^2 + x - 10) \div (x^2 - 4x + 1)$

Difficulties encountered :

Pre-Class Worksheet	**Section 2.8**	**Date:_____** **Name:_____**

Order of Operations http://www.youtube.com/watch?v=iHvTbraDV38 **(11 min)**

1. Explain the order of operations?

2. Use order of operations to find the value of the following.

a. $(2-5)\times 4+3$

b. $\sqrt{2^2+3^2}$

c. $\dfrac{\frac{2}{3}+5}{5^2-4}$

d. $3|5-8|-12$

e. $3|2\times 3^2-8\times 3|-5\times 2^2$

f. $\dfrac{3^2+2\times 5-10}{\sqrt{25}-3\times 2}$

g. $\dfrac{-7+(3\times 2-4)^2}{3^2-4+2\times 5}$

Difficulties encountered :

Pre-Class Worksheet	Section 2.8	Date:_____ Name:_____

Complex Fractions http://www.youtube.com/watch?v=_epR6si0ncc **(4 min)**

3. Evaluate the following.

a. $\dfrac{\frac{1}{2}-3}{\frac{1}{3}-2}$

b. $\dfrac{\frac{3}{x}-2}{\frac{2}{x^2}-1}$

c. $\dfrac{\frac{4}{x+1}-3}{\frac{1}{x-1}-2}$

d. $\dfrac{\frac{1}{a^2}-\frac{1}{b^2}}{\frac{a}{b}-\frac{b}{a}}$

Difficulties encountered :

Pre-Class Worksheet	Section 3.1	Date:_____ Name:_____

Introduction to Equations and Inequalities (11 min) http://www.youtube.com/watch?v=vZ2mjSUvneQ

1. Explain what an equation is.

2. Explain what an inequality is.

3. What are the different symbols that are used to write an inequality? Explain the meaning of all of them.

4. What is the difference between solving an equation and an inequality?

5. Determine which operations will undo the operations listed below.

Operation	Undone by
a. Addition	a.
b. Subtraction	b.
c. Multiplication	c.
d. Division	d.
e. Odd Power	e.
f. Even Power	f.

Difficulties encountered :

6. What is a solution to an equation?

7. What is a solution to an inequality?

8. What is an identity?

9. What is an extraneous solution?

Interval Notation (14 min) **http://www.youtube.com/watch?v=P1IIz3XtJLs**

10. Describe the difference between using a round bracket verses rectangular bracket when writing solutions to an inequality in interval notation.

Difficulties encountered :

Pre-Class Worksheet	Section 3.2	Date:_____ Name:_____

- **Additive Property of Equalities and Inequalities (10 min) http://www.youtube.com/watch?v=Emlxj6Xj4w0**
- **Multiplicative Property of Equalities and Inequalities (8 min) http://www.youtube.com/watch?v=IUaQxG8Vn-8**
- **Solving Equations and Inequalities (12 min) http://www.youtube.com/watch?v=9Ky4kZA1unE**
- **Solving Equations and Inequalities (8 min) http://www.youtube.com/watch?v=kRbOrSNxKy0**
- **Solving Equations (11 min) http://www.youtube.com/watch?v=6oGn22clCwA**

1. Solve the following and explain why you can do the steps you did. For inequalities, write your answer algebraically, in interval notation, and represent it graphically on a number line. For equations, if there are extraneous solutions, please state so.

Equations	Inequalities
a. $x + 5 = 3$	a. $x + 5 < 3$
b. $x - \frac{1}{2} = \frac{7}{3}$	b. $x - \frac{1}{2} \geq \frac{7}{3}$
c. $-\frac{2}{3}t = \frac{5}{2}$	c. $-\frac{2}{3}t < \frac{5}{2}$
d. $2.5x + 4.5 = -5.6$	d. $2.5x + 4.5 > -5.6$

Difficulties encountered :

Pre-Class Worksheet	**Section 3.2**	**Date:**_____ **Name:**_____

Equations	**Inequalities**
e. $-2(3x-5)+5x=4(x-3)$	e. $-2(3x-5)+5x<4(x-3)$
f. $2-\frac{1}{x+1}=\frac{7}{3}$	f. $2-\frac{1}{x+1}\geq\frac{7}{3}$
g. $\lvert 3x-4\rvert=\frac{5}{3}$	**g.** $\lvert 3x-4\rvert>\frac{5}{3}$
h. $\sqrt{x+1}=5$	
i. $(x+2)^5=7$	
j. $\sqrt[3]{2x-1}=4$	

Difficulties encountered :

Percentage, Ratio and Proportions Problems (11 min) http://www.youtube.com/watch?v=oLoRCRXTYv4

2. Explain the difference between a percentage, a ratio, and a proportion.

3. Solve the following.

a. The gender wage gap varies by occupation. Female waitresses on average earn 82% as much as male waiters. If a typical female waitress earns $12/hr in New York City, what does a typical male waiter earn in New York City? In other words what wage is $12/hr 82% of?

b. A car dealer marked the price of a new car down by 8% and the sale price was $18,000. What was the original price?

Algebraic Solution	**Visual Solution**

c. A tall building casts a shadow of length 40 paces (100 feet) on a level city street. Also, the shadow of a six foot person is 2.5 feet long. Determine how tall the building is using the fact that the shadows and heights of the person and building form similar triangles where the ratio of corresponding sides is constant.

d. A cookie recipe calls for 1.5 cups of flour and 7 oz. of almond paste and makes 2 dozen cookies. How much flour would be needed to make a huge batch of these cookies that utilizes a 90 oz. can of almond paste? Also how many cookies would this make?

Difficulties encountered :

Pre-Class Worksheet	Section 3.2	Date:_____ Name:_____

Direct and Inverse Variation (5 min)

http://www.youtube.com/watch?v=sezsOC5fggo

4. Explain what the difference is between direct, inverse variation, and a joint variation?

5. Solve the following

   a. The number of miles, $\boldsymbol{M}$, that Joe drives each week is inversely proportional to the price of gas, $\boldsymbol{g}$. When the price was at \$3/gal, Joe drove 240 miles each week. Find the relationship between $\boldsymbol{M}$ and $\boldsymbol{g}$. Predict how many miles Joe drives if the price of gas is \$5/gal.

   b. The length of a shadow varies directly with the height of an object. A six foot person has a 10 foot shadow. Find the relationship between the shadow length l and the height h of an object. Also determine the height of an object that has a shadow that is 58 feet long.

Difficulties encountered :

Pre-Class Worksheet	Section 3.3	**Date:**_____ **Name:**_____

Zero Product Property (10 min) http://www.youtube.com/watch?v=5zKug2bfT48

Examples (7 min) http://www.youtube.com/watch?v=0FFGzy5Bw4s

1. Solve the following equations.

a. $x(x+5)=0$

b. $(3x-5)(2x+3)=0$

c. $2x^2+3x-5=0$

d. $4x^2+15x+9=28$

e. $\frac{3-x}{7-2x}=\frac{3x^2+7}{2x^2-9x+7}+\frac{x+1}{x-1}$

Difficulties encountered :

Pre-Class Worksheet	Section 3.4	**Date:**_____ **Name:**_____

Radical equations (9 min) http://www.youtube.com/watch?v=qibBpu5vixk

1. What is the difference between solving radical and power equations?

Radical Equations	Power Equations
2. $\sqrt{x} = 4$	3. $x^2 = 4$
4. $\sqrt{x+3} = 4$	5. $(x+3)^2 = 4$
6. $\sqrt{5-3x} = 4$	7. $(1-2x)^2 = 3$

Difficulties encountered :

8. $\sqrt{x+2} - x = -4$

9. $\sqrt{x+4} - \sqrt{2x+1} = -1$

Difficulties encountered :

Pre-Class Worksheet	Section 3.5	**Date:**_____ **Name:**_____

Quadratic Equations (15 min) https://www.youtube.com/watch?v=29_SBzxChMw

1. Solve the following equations.

Equations With Real Zeros	**Equations with Complex Zeros**
a. $x^2 = 4$	b. $x^2 = -4$
c. $x^2 = 5$	d. $x^2 = -5$
e. $(x+3)^2 = 5$	f. $(x+3)^2 = -5$
g. $4(x+3)^2 = 5$	h. $4(x+3)^2 = -5$

Difficulties encountered :

2. Explain what completing the square is algebraically and visually for the example $\boldsymbol{x^2 + 6x}$

3. Explain how to use completing the squares to solve the following examples.

Equations With Real Zeros	**Equations with Complex Zeros**
$x^2 + 6x - 14 = 0$	$x^2 + 6x + 14 = 0$

4. What is the quadratic formula?

5. Find all the solutions to the equation using the quadratic formula.
   $\boldsymbol{3x^2 - x + 3 = 0.}$

Difficulties encountered :

Pre-Class Worksheet	Section 4.1	**Date:**_____ **Name:**_____

- **Cartesian Coordinate System** http://www.youtube.com/watch?v=QdqdISLovuM **(11 min)**
- **Midpoint Formula** http://www.youtube.com/watch?v=kRivyxLD_IM **(3 min)**
- **Pythagorean Theorem and Distance Formula**
  http://www.youtube.com/watch?v=KrLZuOhus4U **(10min)**

1. Explain what the rectangular or Cartesian coordinate system is.

2. Give at least one example where a rectangular coordinate representation is useful.

3. Explain what the distance formula is. Give an example to illustrate your answer.

4. Explain what the midpoint formula is. Give an example to illustrate your answer.

Difficulties encountered :

Pre-Class Worksheet	Section 4.2	**Date:**_____ **Name:**_____

Plotting solutions to equations in x and y. http://www.youtube.com/watch?v=MEs1zAr--bc (11 min)

1. Explain what determines if a coordinate (a, b) is a solution to an equation in two variables. Give an example to illustrate your answer.

2. Explain what determines if a coordinate (a, b) is a solution to an inequality in two variables. Give an example to illustrate your answer.

Graphing Equations and Inequalities in Two Variables http://www.youtube.com/watch?v=lHCGlPoewJc (9 mins)

3. Sketch the graphs of the equations and inequalities below. For inequalities please make use of colored pencils.

Equation	Inequality
a. $y = x + 2$	**b.** $y > x + 2$

Difficulties encountered :

Pre-Class Worksheet	**Section 4.2**	**Date:_____ Name:_____**

Equations of Circles **http://www.youtube.com/watch?v=fzNXmoCHRCk** **(10 min)**

4. Explain what a circle is, and how to find its equation given the center and radius. Give an example to illustrate your answer.

5. Find the equation of the circle with center at $(\mathbf{3}, \mathbf{5})$ with radius 2. Then sketch the graph of the circle.

Difficulties encountered :

Pre-Class Worksheet	Section 4.3	**Date:**_____ **Name:**_____

Horizontal and Vertical lines and their equations **http://www.youtube.com/watch?v=IamhB_5youg** **(6 min)**

1. How many points determine a unique line? Explain why that might be.

2. Explain one difference regarding points that belong to a vertical line verses those that belong to a horizontal line. Give examples to illustrate your answer. You may want to use colored pencils to demonstrate the differences.

Slopes of lines **http://www.youtube.com/watch?v=hbrLS3ifskQ** **(13 min)**

3. Explain the concept of slope of a line. Give an example to illustrate your answer.

4. How can you determine if two lines were perpendicular or not?

Difficulties encountered :

Pre-Class Worksheet	**Section 4.3**	**Date:_____** **Name:_____**

Lines **http://www.youtube.com/watch?v=l2TPmIzfkLo** **(6 min)**

5. Explain what happens to the graph of the line $\boldsymbol{y = 2x}$ when you add or subtract a number from the $\boldsymbol{2x}$, e.g. $y = 2x + 5$ *or* $y = 2x - 7$.

6. Sketch the graphs of the following lines.

a. $\boldsymbol{y = 2x + 8}$

b. $\boldsymbol{y = 2x - 16}$

7. Write the formula to find slope of a line passing through the points $(\boldsymbol{x_1, y_1})$ and $(\boldsymbol{x_2, y_2})$.

8. Explain the meaning of slope using a real life example.

9. What determines if two lines are parallel, perpendicular or neither?

Difficulties encountered :

Pre-Class Worksheet	**Section 4.3**	**Date:**_____ **Name:**_____

Slope-intercept and point-slope forms of equations of lines **http://www.youtube.com/watch?v=Mqh-1mGnuU0** **(15 min)**

Fill in the blanks below
10. A point (x_1, y_1) belongs to the line $y = mx + b$ if and only if _____is a true statement.
11. _____points determine a unique line.
12. A _____on the line and the _____ of the line also determine a unique line.
13. To find points on the line $y = mx + b$ we have to _____
14. Equation of a line in the slope intercept form is given by _____.
15. Equation of a line in the point slope form is given by _____.
16. Equation of a line in the standard form is given by _____.
17. Slope of a line passing through points (x_1, y_1), (x_2, y_2) is given by _____.
18. Slope represents the _____ of the y-value per one unit increase in x.
19. Equation of a vertical line passing through the point (a, b) is given by _____.
20. Equation of a horizontal line passing through the point (a, b) is given by _____.

Difficulties encountered :

Pre-Class Worksheet	**Section 4.3**	**Date:_____ Name:_____**

Solving Systems of Equations and Inequalities http://www.youtube.com/watch?v=Ek8oBqJ2E_4 (14 min)

21. 999List three different ways to solve a system of equations. Give examples of each to illustrate. Please use colored pencils where necessary to highlight your answer.

22. What is the main difference between solving a system of equations verses solving a system of inequalities?

23. When solving a system of equations we can have system that is either _____, or _____, or _____.

Difficulties encountered :

Pre-Class Worksheet	Section 4.4	**Date:**_____ **Name:**_____

Applications of lines and graphs Reading Graphs (7 min) **http://www.youtube.com/watch?v=plhuVnNFYxc**

1. Explain what modeling a real life problem is in mathematics.

2. What kind of information is expressed in graphical representations of an event? Use an example to illustrate your answer.

Linear Models (11 min) **http://www.youtube.com/watch?v=8DXMehKa6_w**

3. Explain how a linear model can help with estimating population growth.

Applications (8 min) **http://www.youtube.com/watch?v=idPmgnUD-X0**

4. Explain how software like Excel can help us create models using a set of data points.

Difficulties encountered :

Pre-Class Worksheet	Section 4.5	**Date:**_____ **Name:**_____

Applications (14 min) **http://www.youtube.com/watch?v=WXLZKrZUnKQ**

1. List some basic steps one can use to convert a word problem into a mathematical equation.

2. Find a visual and an algebraic solution to the problem below.
   The tank on Karl's truck went from $\frac{1}{3}$ full to ½ full when he added 4 gallons of gas. Use this information to determine how many gallons the tank holds when full.

Visual	**Algebraic**

Applications (7 min) **http://www.youtube.com/watch?v=dsbrQ54So38**

3. Solve the following.
   A school bus can hold maximum of 72 kids. If there are 353 kids that are to be taken on field trip to Chicago's Science and Industry Museum, how many buses will be needed?

Difficulties encountered :

Pre-Class Worksheet	**Section 4.5**	**Date:_____ Name:_____**

Applications (11 min) **http://www.youtube.com/watch?v=CZQu8Q3maEk**

4. Show how organizing information in a chart can help create your mathematical model to solve word problems below.
   a. Anita works at two jobs for 50 hours a week, one in the math lab at a pay rate of $8/hr and the other at a nursing home at a rate of $9.50/hr. In a week where she earned a total of $460, how many hours did she work at each job?

   b. A theater sold 550 tickets. Some were discounted and sold for $6 each with the remaining tickets sold for full price of $8 a ticket. If the total revenue from ticket sales was $4050, how many of each type of ticket was sold?

   c. A financial planner needs to allocate $500,000 into two funds so that her client will receive $35,000 of earnings per year. Fund A has a return of 9% per year and Fund B earns 6% per year. How many dollars should be invested in each account to obtain the total of $35,000 earnings in a year?

5. Explain how to solve a word problem using a visual representation.

Difficulties encountered :

Pre-Class Worksheet	Section 4.6	**Date:**_____ **Name:**_____

Rate Problems (11 min) **http://www.youtube.com/watch?v=prwMJFjTD24**

1. Solve the following. Make a chart to organize your information.
   a. A brick that is dropped from the top of a 15 story building is traveling 99 mph just before it hits the ground. Determine the speed of the brick at this time in feet per second. Note that 1 mile = 5280 feet.

   b. A plane travels $\mathbf{1110}$ miles from Milwaukee, WI, to Boston, MA, in 2 hours with the wind. The plane then returns to Milwaukee against the wind in 2 hours and 25 minutes. Assume that the speed of the wind stays constant. Also assume that the plane speed going to Boston is the speed of the plane in still air plus the wind speed and that the plane speed returning to Milwaukee is the speed of the plane in still air minus the wind speed. Use a system of equations to describe this event, and find the wind speed and the speed of the plane in still air.

   c. It takes an old copier 15 minutes to print 50 copies of a weekly report, while a new copier can print the 50 copies in 8 minutes. How long will it take the copiers to print 50 copies of the weekly report if they are both used?

Difficulties encountered :

Pre-Class Worksheet	Section 4.7	**Date:**_____ **Name:**_____

Applications (10 min) **http://www.youtube.com/watch?v=ys0RRfBJ7HU**

1. List at least three different fields of study where mathematical modeling can be used. Which one was your favorite application?

2. Show how you would use the knowledge gained so far to solve
   A rectangular garden was enclosed using a 50 foot roll of rabbit proof wire fence. The width of the garden was 10 feet. Determine the area of the garden in square feet.

Difficulties encountered :