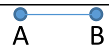
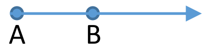
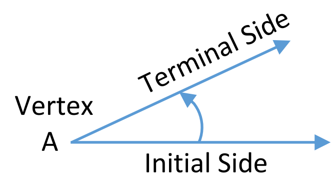
Section 1.1:

<u>Some Basic Terminology:</u>

Line AB	
Line Segment AB	
Ray AB	



<u>Types of Angles:</u>

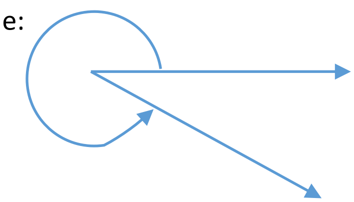
Acute angle $0° < \theta < 90°$

Right angle $\theta = 90°$

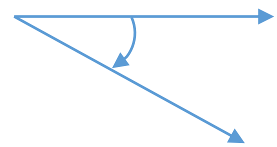
Obtuse angle $90° < \theta < 180°$

Straight angle $\theta = 180°$

Positive Angle:



Negative Angle:



<u>Two angles A and B are:</u>

complementary angles if $A + B = 90°$

supplementary angles if $A + B = 180°$

<u>#4:</u> Find the complement and supplement of an 18° angle.

a) $90° - 18° = 72°$
b) $180° - 18° = 162°$

<u>Measuring angles in **degrees, minutes, seconds**:</u>

Every degree is 60 minutes ($1° = 60'$)

Every minute is 60 seconds ($1' = 60''$)

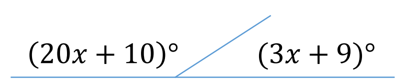
<u>#12:</u> Find the complement and supplement of a $50°\,40'\,50''$ angle.

a) $90° - 50°\,40'\,50'' = 89°\,59'\,60'' - 50°\,40'\,50'' = 39°\,19'\,10''$
b) $180° - 50°\,40'\,50'' = 179°\,59'\,60'' - 50°\,40'\,50'' = 129°\,19'\,10''$

<u>Note:</u> An angle is in standard position if its vertex is at the origin.

Two angles with the same initial and terminal sides that their measures differ by a multiple of 360° are called coterminal angles.

<u>#14:</u> Find the measure of each unknown angle.



$(20x + 10)° + (3x + 9)° = 180° \quad \rightarrow \quad 20x + 10 + 3x + 9 = 180 \quad \rightarrow \quad 23x + 19 = 180$

$23x = 161 \quad \rightarrow \quad x = \dfrac{161}{23} = 7 \quad \rightarrow \quad (20x + 10)° = (20(7) + 10)° = 150°$

$(3x + 9)° = (3(7) + 9)° = 30°$

#22: Find the measure of each unknown complementary angles with measures $3x-5$ and $6x-40$ degrees.

$$3x-5+6x-40=90 \quad \rightarrow \quad 9x-45=90 \quad \rightarrow \quad 9x=135 \quad \rightarrow \quad x=15$$

#40: Perform each calculation

$$47^\circ\, 23' - 73^\circ\, 48' = -26^\circ\, 25'$$

#58: Convert each angle measure to decimal degrees. If applicable, round to the nearest thousandth of a degree.

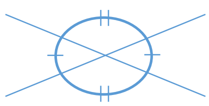
$$34^\circ\, 51'\, 35'' = 34^\circ + \left(\frac{51}{60}\right)^\circ + \left(\frac{35}{3600}\right)^\circ = 34^\circ + 0.85^\circ + 0.0097^\circ = 34.8597^\circ$$

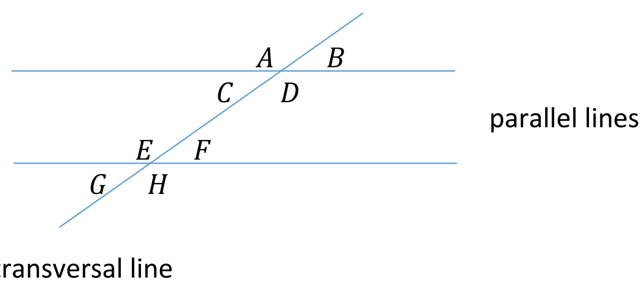
#64: Convert each angle measure to degrees, minutes, and seconds. Round answers to the nearest second, if applicable.

$$174.255^\circ = 174^\circ\,(0.255\times 60)' = 174^\circ\, 15.3' = 174^\circ\, 15'\,(0.3\times 60)'' = 174^\circ\, 15'\, 18''$$

Section 1.2:

Vertical Angles have equal measures:

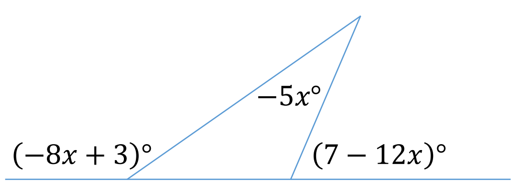




$A=D=E=H$ And $B=C=F=G$

Note: The sum of measures of the angles of any triangle is 180°.

#10: Find the measure of each marked angle.



$$A = 180^\circ - (-8x+3)^\circ = (177+8x)^\circ \quad \& \quad B = 180^\circ - (7-12x)^\circ = (173+12x)^\circ$$

$$A + B + (-5x)^\circ = 180^\circ \quad \rightarrow \quad (177+8x)^\circ + (173+12x)^\circ + (-5x)^\circ = 180^\circ$$

$$350+15x=180 \quad \rightarrow \quad 15x=-170 \quad \rightarrow \quad x=-\frac{34}{3}$$

$$A = \left(177+8\left(-\frac{34}{3}\right)\right)^\circ = 86.\overline{3}^\circ = 86^\circ\, 20'$$

<u>Types of Triangles:</u>

Acute Triangle – all of the angles of the triangle are acute angles

Right Triangle – one angle of the triangle is a right angle

Obtuse Triangle – one angle of the triangle is an obtuse angle

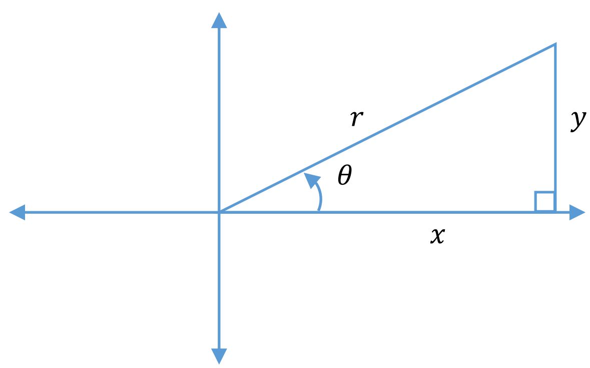
Equilateral Triangle – all of the angles of the triangle are equal (therefore all of the sides of the triangle are equal)

Isosceles Triangle – two of the angles of the triangle are equal (therefore two of the sides of the triangle are equal)

Scalene Triangle – all of the angles of the triangle are unequal (therefore all of the sides of the triangle are unequal)

Section 1.3:

<u>Trigonometric Functions:</u>



$\sin\theta = \frac{y}{r}$ $\qquad$ $\csc\theta = \frac{r}{y} \quad (y \neq 0)$

$\cos\theta = \frac{x}{r}$ $\qquad$ $\sec\theta = \frac{r}{x} \quad (x \neq 0)$

$\tan\theta = \frac{y}{x} \quad (x \neq 0)$ $\qquad$ $\cot\theta = \frac{x}{y} \quad (y \neq 0)$

$r = \sqrt{x^2 + y^2}$

<u>#4:</u> Sketch an angle θ in standard position such that θ has the least positive measure, and the given point is on the terminal side of θ. Then find the values of the six trigonometric functions for each angle. Rationalize denominators when applicable. $(-4, -3)$

$$r = \sqrt{(-4)^2 + (-3)^2} = \sqrt{16 + 9} = \sqrt{25} = 5$$

$$\sin\theta = -\frac{3}{5} \;,\quad \cos\theta = -\frac{4}{5} \;,\quad \tan\theta = \frac{3}{4} \;,\quad \csc\theta = -\frac{5}{3} \;,\quad \sec\theta = -\frac{5}{4} \;,\quad \cot\theta = \frac{4}{3}$$

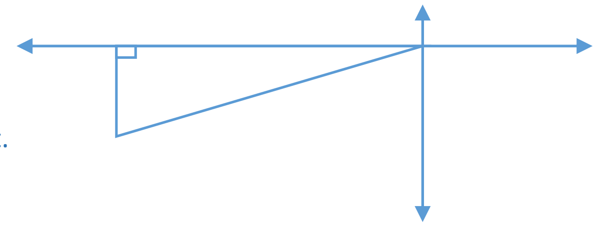
<u>#6:</u> Sketch an angle θ in standard position such that θ has the least positive measure, and the given point is on the terminal side of θ. Then find the values of the six trigonometric functions for each angle. Rationalize denominators when applicable. $(15, -8)$

$$r = \sqrt{(15)^2 + (-8)^2} = \sqrt{289} = 17$$

$$\sin\theta = -\frac{8}{17} \;,\quad \cos\theta = \frac{15}{17} \;,\quad \tan\theta = -\frac{8}{15} \;,\quad \csc\theta = -\frac{17}{8} \;,\quad \sec\theta = \frac{17}{15} \;,\quad \cot\theta = -\frac{15}{8}$$

#26: Suppose that the point (x, y) is in the indicated quadrant. Decide whether the given ratio is positive or negative.

III, $\frac{y}{r}$



It is negative in the third quadrant.

#48: An equation of the terminal side of an angle θ in standard position is given with a restriction on x. Sketch the least positive such an angle θ, and find the values of the six trigonometric functions of θ.

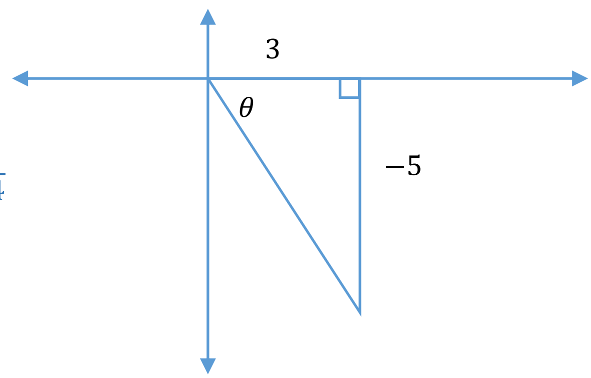
$-5x - 3y = 0 \ , \ \ x \leq 0$

$-3y = 5x \quad \rightarrow \quad y = -\frac{5}{3}x$

$r = \sqrt{(3)^2 + (-5)^2} = \sqrt{9 + 25} = \sqrt{34}$

$\sin\theta = -\frac{5}{\sqrt{34}} = -\frac{5\sqrt{34}}{34}$

$\cos\theta = \frac{3}{\sqrt{34}} = \frac{3\sqrt{34}}{34}$



$\tan\theta = -\frac{5}{3} \ , \quad \csc\theta = -\frac{\sqrt{34}}{5} \ , \quad \sec\theta = \frac{\sqrt{34}}{3} \ , \quad \cot\theta = -\frac{3}{5}$

#60: For each quadrantal angle, identify appropriate values of x, y, and r to find the indicated function value. $\cot 90°$

Coordinate is $(0,1)$ and $r = 1$. So, $\cot 90° = \frac{0}{1} = 0$

#64: For each quadrantal angle, identify appropriate values of x, y, and r to find the indicated function value. $\cos(-90°)$

Coordinate is $(0,-1)$ and $r = 1$. So, $\cos(-90°) = \frac{0}{1} = 0$

Section 1.4:

Reciprocal Identities:

For all angles θ for which both functions are defined,

$\sin\theta = \frac{1}{\csc\theta} \quad \text{and} \quad \csc\theta = \frac{1}{\sin\theta}$

$\cos\theta = \frac{1}{\sec\theta} \quad \text{and} \quad \sec\theta = \frac{1}{\cos\theta}$

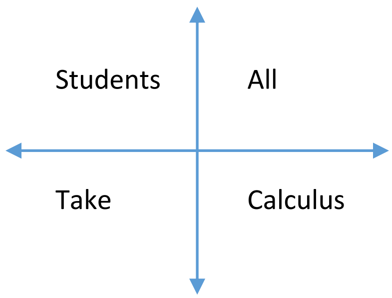
$\tan\theta = \frac{1}{\cot\theta} \quad \text{and} \quad \cot\theta = \frac{1}{\tan\theta}$

#2: Use the appropriate reciprocal identity to find each function value. Rationalize denominator when applicable.

$$\cos\theta = \frac{5}{8} \quad \rightarrow \quad \sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{5}{8}} = \frac{8}{5}$$

#10: Use the appropriate reciprocal identity to find each function value. Rationalize denominator when applicable.

$$\csc\theta = \frac{\sqrt{24}}{3} \quad \rightarrow \quad \sin\theta = \frac{3}{\sqrt{24}} = \frac{3\sqrt{24}}{24} = \frac{\sqrt{24}}{8} = \frac{2\sqrt{6}}{8} = \frac{\sqrt{6}}{4}$$



#26: Determine the signs of the trigonometric functions of an angle in standard position with the given measure. $-15°$

Only $\cos\theta$ and $\sec\theta$ are positive.

Range of Trigonometric Functions:

$\sin\theta$, $\cos\theta$: $[-1,1]$

$\tan\theta$, $\cot\theta$: $(-\infty,\infty)$

$\sec\theta$, $\csc\theta$: $(-\infty,-1] \cup [1,\infty)$

#38: Identify the quadrant of an angle θ that satisfies the given conditions. $\csc\theta > 0$, $\cot\theta > 0$

$\csc\theta$ is positive in quadrants 1 and 2, and $\cot\theta$ is positive in quadrants 1 and 3. Thus, θ is in quadrant 1.

#46: Decide whether each statement is possible or impossible for some angle θ. $\sin\theta = 3$

It is impossible since the range of sine is $[-1,1]$.

#48: Decide whether each statement is possible or impossible for some angle θ. $\cos\theta = -0.56$

It is possible since -0.56 is within the range.

Pythagorean Identities:

$$\sin^2\theta + \cos^2\theta = 1 \ , \quad \tan^2\theta + 1 = \sec^2\theta \ , \quad \cot^2\theta + 1 = \csc^2\theta$$

Quotient Identities:

$$\tan\theta = \frac{\sin\theta}{\cos\theta}, \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

#62: Find $\sin\theta$, given that $\cos\theta = 4/5$ and θ is in quadrant IV.

$$\sin^2\theta + \cos^2\theta = 1 \quad \to \quad \sin^2\theta + \left(\frac{4}{5}\right)^2 = 1 \quad \to \quad \sin^2\theta + \frac{16}{25} = 1$$

$$\sin^2\theta = 1 - \frac{16}{25} = \frac{25}{25} - \frac{16}{25} = \frac{9}{25} \quad \to \quad \sin\theta = \pm\sqrt{\frac{9}{25}} = \pm\frac{3}{5}$$

Since θ is in quadrant 4, $\sin\theta = -\frac{3}{5}$

#70 Find the five remaining trigonometric function values for each angle θ.

$$\cos\theta = -\frac{3}{5}, \text{ and } \theta \text{ is in quadrant III}$$

#76 Find the five remaining trigonometric function values for each angle θ.

$$\cos\theta = \frac{\sqrt{5}}{8}, \text{ and } \tan\theta < 0$$

Extra Credit: #81

Section 2.1:

Right-Triangle-Based Definition of Trigonometric Functions:

$$\sin\theta = \frac{y}{r} = \frac{opposite}{hypotnuse} \qquad \csc\theta = \frac{r}{y} = \frac{hypotnuse}{opposite}$$

$$\cos\theta = \frac{x}{r} = \frac{adjacent}{hypotnuse} \qquad \sec\theta = \frac{x}{r} = \frac{hypotnuse}{adjacent}$$

$$\tan\theta = \frac{y}{x} = \frac{opposite}{adjacent} \qquad \cot\theta = \frac{x}{y} = \frac{adjacent}{opposite}$$

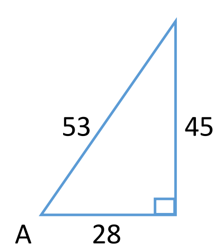
Cofunction Identities:

$$\sin\theta = \cos(90° - \theta) \qquad \cot\theta = \tan(90° - \theta)$$

$$\cos\theta = \sin(90° - \theta) \qquad \csc\theta = \sec(90° - \theta)$$

$$\tan\theta = \cot(90° - \theta) \qquad \sec\theta = \csc(90° - \theta)$$

#2 Find exact values or expressions for $\sin A$, $\cos A$, and $\tan A$.



$$\sin A = \frac{45}{53}, \quad \cos A = \frac{28}{53}, \quad \tan A = \frac{45}{28}$$

$$\csc A = \frac{53}{45}, \quad \sec A = \frac{53}{28}, \quad \cot A = \frac{28}{45}$$

#14: Suppose ABC is a right triangle with sides of lengths a, b, and c, and right angle at C. Find the unknown side length using the Pythagorean theorem, and then find the values of the six trigonometric functions for angle B. Rationalize denominators when applicable.

$b = 7$, $c = 12$

$$a = \sqrt{c^2 - b^2} = \sqrt{12^2 - 7^2} = \sqrt{144 - 49} = \sqrt{95}$$

$$\sin B = \frac{7}{12} \quad , \quad \cos B = \frac{\sqrt{95}}{12} \quad , \quad \tan B = \frac{7}{\sqrt{95}} = \frac{7\sqrt{95}}{95}$$

$$\csc B = \frac{12}{7} \quad , \quad \sec B = \frac{12}{\sqrt{95}} = \frac{12\sqrt{95}}{95} \quad , \quad \cot B = \frac{\sqrt{95}}{7}$$

#18 Suppose ABC is a right triangle with sides of lengths a, b, and c, and right angle at C. Find the unknown side length using the Pythagorean theorem, and then find the values of the six trigonometric functions for angle B. Rationalize denominators when applicable.

$a = \sqrt{2}$, $c = 2$

$$b = \sqrt{c^2 - a^2} = \sqrt{2^2 - \left(\sqrt{2}\right)^2} = \sqrt{4 - 2} = \sqrt{2}$$

$$\sin B = \frac{\sqrt{2}}{2} \quad , \quad \cos B = \frac{\sqrt{2}}{2} \quad , \quad \tan B = \frac{\sqrt{2}}{\sqrt{2}} = 1$$

$$\csc B = \frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2} \quad , \quad \sec B = \frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2} \quad , \quad \cot B = \frac{\sqrt{2}}{\sqrt{2}} = 1$$

#26: Write each function in terms of its cofunction. Assume that all angles in which an unknown appears are acute angles.

$$\tan 25.4° = \cot(90° - 25.4°) = \cot 64.6°$$

#28: Write each function in terms of its cofunction. Assume that all angles in which an unknown appears are acute angles.

$$\cos(\theta + 20°) = \sin\left(90° - (\theta + 20°)\right) = \sin(90° - \theta - 20°) = \sin(70° - \theta)$$

#34: Find one solution for each equation. Assume that all angles in which an unknown appears are acute angles.

$$\sec(\beta + 10°) = \csc(2\beta + 20°)$$

$$\sec(\beta + 10°) = \csc\left(90° - (\beta + 10°)\right) = \csc(80° - \beta)$$

$$\csc(2\beta + 20°) = \csc(80° - \beta) \quad \rightarrow \quad 2\beta + 20° = 80° - \beta \quad \rightarrow \quad 3\beta = 60° \quad \rightarrow \quad \beta = 20°$$

#54: For each expression, give the exact value.

$$\csc 30° = \frac{1}{\sin 30°} = \frac{1}{\frac{1}{2}} = 2$$

Section 2.2:

#24: Find exact values of the six trigonometric functions for $480°$. Rationalize denominators when applicable.

Coterminal of $480°$ is $120°$, so

$$\sin 480° = \sin 120° = \sin 60° = \frac{\sqrt{3}}{2} \quad , \quad \csc 480° = \frac{1}{\sin 480°} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\cos 480° = \cos 120° = -\cos 60° = -\frac{1}{2} \quad , \quad \sec 480° = \frac{1}{\cos 480°} = -2$$

$$\tan 480° = \frac{\sin 480°}{\cos 480°} = \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -\sqrt{3} \quad , \quad \cot 480° = \frac{1}{\tan 480°} = \frac{1}{-\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

#32: Find exact values of the six trigonometric functions for $-1020°$. Rationalize denominators when applicable.

Coterminal of $-1020°$ is $60°$, so

$$\sin(-1020°) = \sin 60° = \frac{\sqrt{3}}{2} \quad , \quad \csc(-1020°) = \csc 60° = \frac{1}{\sin 60°} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\cos(-1020°) = \cos 60° = \frac{1}{2} \quad , \quad \sec(-1020°) = \sec 60° = \frac{1}{\cos 60°} = 2$$

$$\tan(-1020°) = \frac{\sin(-1020°)}{\cos(-1020°)} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3} \quad , \quad \cot(-1020°) = \frac{1}{\tan(-1020°)} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

#38: Find the exact value of each expression.

$$\sin 1500° = \sin 1140° = \sin 780° = \sin 420° = \sin 60° = \frac{\sqrt{3}}{2}$$

#48: Evaluate each of the following.

$$\cot^2 135° - \sin 30° + 4\tan 45° = (\cot 135°)^2 - \sin 30° + 4(\tan 45°) = (-1)^2 - \frac{1}{2} + 4(1)$$

$$= 1 - \frac{1}{2} + 4 = 5 - \frac{1}{2} = \frac{10}{2} - \frac{1}{2} = \frac{9}{2}$$

#50: Evaluate each of the following.

$$\cot^2 90° - \sec^2 180° + \csc^2 135° = \frac{\cos^2 90°}{\sin^2 90°} - \frac{1}{\cos^2 180°} + \frac{1}{\sin^2 135°} = \frac{0}{1} - \frac{1}{(-1)^2} + \frac{1}{\left(\frac{\sqrt{2}}{2}\right)^2}$$

$$= 0 - 1 + \frac{1}{\frac{1}{2}} = -1 + 2 = 1$$

#56: Determine whether each statement is true or false. If false, tell why.

$\cos 60° = 2\cos^2 30° - 1$

$\text{True, because } \cos 60° = \frac{1}{2}$

$\text{and } 2\cos^2 30° - 1 = 2\left(\frac{\sqrt{3}}{2}\right)^2 - 1 = 2\left(\frac{3}{4}\right) - 1 = \frac{3}{2} - 1 = \frac{3}{2} - \frac{2}{2} = \frac{1}{2}$

Section 2.3:

#6: Use a calculator to find a decimal approximation for each value. Give as many digits as your calculator displays.

$\cos 41°\ 24' = \cos 41.4° = 0.75011107$

#12: Use a calculator to find a decimal approximation for each value. Give as many digits as your calculator displays.

$\tan(-80°\ 06') = \tan(-80.1°) = -5.75011107$

Inverse of Trigonometric Functions:

$$\sin\theta = \frac{y}{r} \quad \leftrightarrow \quad \sin^{-1}\left(\frac{y}{r}\right) = \theta \quad \text{or} \quad \arcsin\left(\frac{y}{r}\right) = \theta$$

$$\cos\theta = \frac{x}{r} \quad \leftrightarrow \quad \cos^{-1}\left(\frac{x}{r}\right) = \theta \quad \text{or} \quad \arccos\left(\frac{x}{r}\right) = \theta$$

$$\tan\theta = \frac{y}{x} \quad \leftrightarrow \quad \tan^{-1}\left(\frac{y}{x}\right) = \theta \quad \text{or} \quad \arctan\left(\frac{y}{x}\right) = \theta$$

$$\csc\theta = \frac{r}{y} \quad \leftrightarrow \quad \csc^{-1}\left(\frac{r}{y}\right) = \theta \quad \text{or} \quad \text{arccsc}\left(\frac{r}{y}\right) = \theta$$

$$\sec\theta = \frac{r}{x} \quad \leftrightarrow \quad \sec^{-1}\left(\frac{r}{x}\right) = \theta \quad \text{or} \quad \text{arcsec}\left(\frac{r}{x}\right) = \theta$$

$$\cot\theta = \frac{x}{y} \quad \leftrightarrow \quad \cot^{-1}\left(\frac{x}{y}\right) = \theta \quad \text{or} \quad \text{arccot}\left(\frac{x}{y}\right) = \theta$$

#24: Find a value of θ in the interval $[0°, 90°]$ that satisfies each statement. Write each answer in decimal degrees to six decimal places as needed.

$\tan\theta = 6.4358841 \quad \rightarrow \quad \theta = \tan^{-1}(6.4358841) = 81.168073°$

#30: Find a value of θ in the interval $[0°, 90°]$ that satisfies each statement. Write each answer in decimal degrees to six decimal places as needed.

$\sec\theta = 1.1606249 \quad \rightarrow \quad \frac{1}{\cos\theta} = 1.1606249 \quad \rightarrow \quad \cos\theta = \frac{1}{1.1606249}$

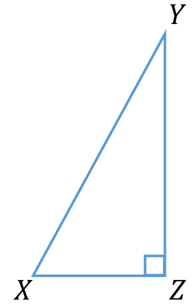
$\theta = \cos^{-1}\left(\frac{1}{1.1606249}\right) = 30.502748°$

#40: Use a calculator to evaluate each expression.

$$\cos 100° \cos 80° - \sin 100° \sin 80° = (-0.17365)(0.17365) - (0.98481)(0.98481) = -1$$

Section 2.4:

#10: Solve each right triangle. When two sides are given, give angles in degrees and minutes.



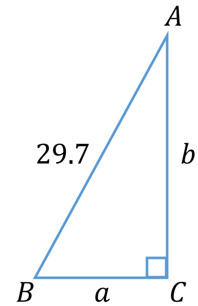
$X = 47.8°$, $z = 89.6\ Cm$

$Y = 90° - 47.8° = 42.2°$

$$\sin 47.8° = \frac{x}{89.6} \quad \rightarrow \quad x = 89.6 \sin 47.8° = 66.4\ Cm$$

$$\cos 47.8° = \frac{y}{89.6} \quad \rightarrow \quad y = 89.6 \cos 47.8° = 60.2\ Cm$$

#22: Solve each right triangle. In each case, $C = 90°$.



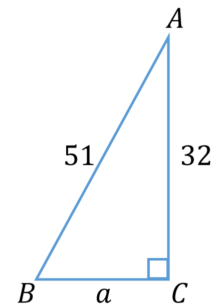
$B = 46.0°$, $c = 29.7\ m$

$A = 90° - 46.0° = 44.0°$

$$\sin 46.0° = \frac{b}{29.7} \quad \rightarrow \quad b = 29.7 \sin 46.0° = 21.36\ m$$

$$\cos 46.0° = \frac{a}{29.7} \quad \rightarrow \quad a = 29.7 \cos 46.0° = 20.63\ m$$

#28: Solve each right triangle. In each case, $C = 90°$.



$b = 32\ ft$, $c = 51\ ft$

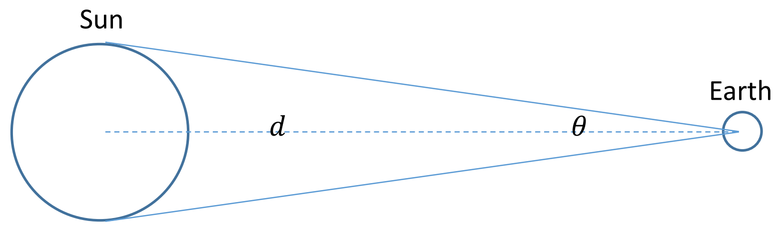
$$\sin B = \frac{32}{51} \quad \rightarrow \quad B = \sin^{-1}\left(\frac{32}{51}\right) = 38.86°$$

$A = 90° - 38.86° = 51.14°$

$$a = \sqrt{(51)^2 - (32)^2} = \sqrt{2601 - 1024} = \sqrt{1577} \approx 39.71\ ft$$

#44: To determine the diameter of the sun, an astronomer might sight with a **transit** (a device used by surveyors for measuring angles) first to one edge of the sun and then to the other, estimating that the included angle equals $32'$. Assuming that the distance d from Earth to the sun is 92,919,800 mi, approximate the diameter of the sun.

$d = 92919800\ mi$ & $\theta = 32'$



$$\theta = 32' = \frac{8}{15}° \quad \rightarrow \quad \frac{\theta}{2} = \frac{4}{15}°$$

$$\sin\frac{\theta}{2} = \frac{r}{92919800}$$

$$r = 92919800 \sin\frac{\theta}{2} = 92919800 \sin\frac{4}{15}° = 432466.8\ mi$$

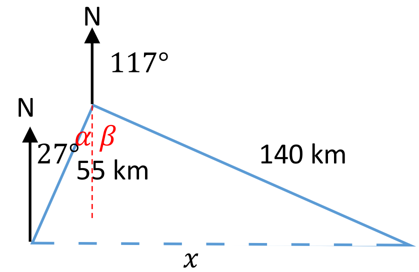
$$d = 2(432466.8) = 864933.6\ mi$$

Section 2.5:

#16: A ship travels 55 km on a bearing of 27° and then travels on a bearing of 117° for 140 km. Find the distance from the starting point to the ending point.

By Vertical Angles, $\alpha = 27°$ and $\beta = 117°$. So, we have a right triangle with adjacent sides of 55 km and 140 km.

So, $x = \sqrt{55^2 + 140^2} = 150.42$ km



#32: Find h as indicated in the figure.

$$\tan 41.2° = \frac{h}{168 + x} \quad \& \quad \tan 52.5° = \frac{h}{x} \quad \rightarrow \quad h = (168 + x)\tan 41.2° \quad \& \quad h = x \tan 52.5°$$

$$x \tan 52.5° = (168 + x)\tan 41.2°$$

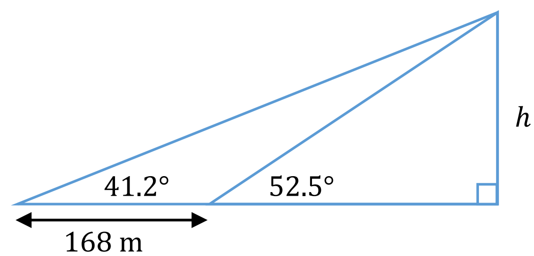
$$x \tan 52.5° = 168 \tan 41.2° + x \tan 41.2°$$

$$x \tan 52.5° - x \tan 41.2° = 168 \tan 41.2°$$

$$x(\tan 52.5° - \tan 41.2°) = 168 \tan 41.2°$$

$$x = \frac{168 \tan 41.2°}{\tan 52.5° - \tan 41.2°} = 343.8\, m$$

$$h = x \tan 52.5 = 353.8 \tan 52.5° = 448\, m$$



Section 3.1:

Converting between degrees and radians:

$$\frac{Angle\ in\ degrees}{180°} = \frac{Angle\ in\ radians}{\pi}$$

#8: Convert each degree measure to radians. Leave answers as multiple of π.

$$\frac{30°}{180°} = \frac{x}{\pi} \quad \rightarrow \quad x = \frac{30° \times \pi}{180°} = \frac{\pi}{6}$$

#14: Convert each degree measure to radians. Leave answers as multiple of π.

$$\frac{-315°}{180°} = \frac{x}{\pi} \quad \rightarrow \quad x = \frac{-315° \times \pi}{180°} = -\frac{7\pi}{4}$$

#30: Convert each radian measure to degrees.

$$\frac{x}{180°} = \frac{\frac{8\pi}{3}}{\pi} \quad \rightarrow \quad x = \frac{\frac{8\pi}{3} \times 180°}{\pi} = \frac{8\pi \times 60°}{\pi} = 480°$$

#40: Convert each radian measure to degrees.

$$\frac{x}{180°} = \frac{-\frac{7\pi}{20}}{\pi} \quad \rightarrow \quad x = \frac{-\frac{7\pi}{20} \times 180°}{\pi} = \frac{-7\pi \times 9°}{\pi} = -63°$$

#48: Convert each degree measure to radians.

$$\frac{264.9°}{180°} = \frac{x}{\pi} \quad \rightarrow \quad x = \frac{264.9° \times \pi}{180°} = 4.623 \ radians$$

#50: Convert each degree measure to radians.

$$\frac{174° \ 50'}{180°} = \frac{x}{\pi} \quad \rightarrow \quad x = \frac{174° \ 50' \times \pi}{180°} = \frac{174.83° \times \pi}{180°} = 3.05 \ radians$$

#60: Convert each radian measure to degrees. Write answers to the nearest minutes.

$$\frac{x}{180°} = \frac{3.06}{\pi} \quad \rightarrow \quad x = \frac{3.06 \times 180°}{\pi} = 175.325° = 175° \ (0.325 \times 60)' = 175° \ 20'$$

#70: Find the exact value of each expression without using a calculator.

$$\csc\frac{\pi}{4} = \csc 45° = \frac{1}{\sin 45°} = \frac{1}{\frac{\sqrt{2}}{2}} = \frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

Section 3.2:

Arc Length: $s = r\theta$ when θ is measured in radians and r is the radius

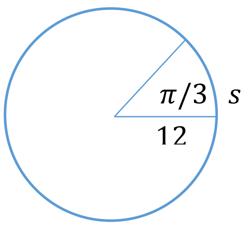
Area of a Sector:

$$A = \frac{\theta}{2\pi}(\pi r^2) = \frac{1}{2}\theta r^2$$

when θ is in radians.

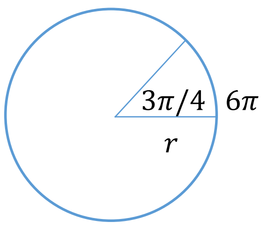
#2: Find the exact length of each arc intercepted by the given central angle.

$$s = 12\left(\frac{\pi}{3}\right) = 4\pi$$



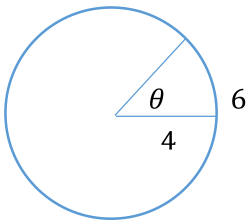
#4: Find the radius of each circle.

$$6\pi = r\left(\frac{3\pi}{4}\right) \quad \rightarrow \quad r = 6\pi\left(\frac{4}{3\pi}\right) = 8$$



#8: Find the measure of each central angle (in radians).

$$6 = 4\theta \quad \rightarrow \quad \theta = \frac{6}{4} = 1.5 \ radians$$



#14: Find the length to three significant digits of each arc intercepted by a central angle θ in a circle of radius r.

$$r = 3.24\ mi\ ,\ \ \theta = \frac{7\pi}{6}$$

$$s = 3.24 \times \frac{7\pi}{6} = 11.875\ mi$$

#16: Find the length to three significant digits of each arc intercepted by a central angle θ in a circle of radius r.

$$r = 71.9\ cm\ ,\ \ \theta = 135°$$

$$\theta = 135° = \frac{3\pi}{4} \quad \rightarrow \quad s = 71.9 \times \frac{3\pi}{4} = 169.410\ cm$$

#22: Find the distance in kilometers between each pair of cities, assuming they lie on the same north-south line. Use $r = 6400\ km$ for the radius of earth.

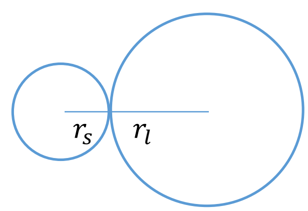
$$r = 6400\ km\ ,\ \ Farmersville, California, 36°N \quad and \quad Penticton, British\ Colombia, 49°N$$

$$\theta = 49° - 36° = 13° = \frac{13\pi}{180}\ radians$$

$$s = 6400 \times \frac{13\pi}{180} = 1452\ km$$

#32: Find the radius of the larger wheel if the smaller wheel rotates 120° when the larger wheel rotates 60°.

$$r_s = 14.6\ in\ \ \&\ \ \theta_s = 120°\ \ \&\ \ \theta_l = 60°$$



$$\theta_s = 120° = \frac{120° \times \pi}{180°} = \frac{2\pi}{3}\ \ \&\ \ \theta_l = 60° = \frac{60° \times \pi}{180°} = \frac{\pi}{3}$$

$$s = r_s\theta_s\ \ \&\ \ s = r_l\theta_l \quad \rightarrow \quad r_s\theta_s = r_l\theta_l$$

$$14.6 \times \frac{2\pi}{3} = r_l \times \frac{\pi}{3} \quad \rightarrow \quad r_l = 14.6 \times \frac{2\pi}{3} \times \frac{3}{\pi} = 29.2\ in$$

#40: Suppose the tip of the minute hand of a clock is 3 in from the center of the clock. For each duration, determine the distance travelled by the tip of the minute hand.

$$r = 3\ in\ ,\ \ find\ arc - length\ at\ 6\frac{1}{2}\ hour$$

$$\theta = 6\frac{1}{2} \times 2\pi = \frac{13}{2} \times 2\pi = 13\pi$$

$$s = 3 \times 13\pi = 39\pi$$

Section 3.3:

Unit Circle: A circle with radius of 1 unit.

$$s = r\theta = 1\theta = \theta$$

Thus, we have:

$\sin s = y$ $\quad\quad$ $\csc s = \frac{1}{y} \quad (y \neq 0)$

$\cos s = x$ $\quad\quad$ $\sec s = \frac{1}{x} \quad (x \neq 0)$

$\tan s = \frac{y}{x} \quad (x \neq 0)$ $\quad\quad$ $\cot s = \frac{x}{y} \quad (y \neq 0)$

Domain of Circular Functions:

Domain of sine and cosine functions: $(-\infty, \infty)$

Domain of tangent and secant functions: $\left\{s \mid s \neq (2n+1)\frac{\pi}{2} \text{ , where } n \text{ is an integer}\right\}$

Domain cotangent and cosecant functions: $\{s \mid s \neq n\pi \text{ , where } n \text{ is an integer}\}$

#6: Find the exact circular function value for each of the following.

$$s = -\frac{3\pi}{2}$$

$$\sin s = \sin\left(-\frac{3\pi}{2}\right) = \sin\frac{\pi}{2} = 1$$

$$\cos s = \cos\left(-\frac{3\pi}{2}\right) = \cos\frac{\pi}{2} = 0$$

$$\tan s = \frac{\sin s}{\cos s} = \frac{1}{0} \quad \rightarrow \quad undefined$$

#16: Find the exact circular function value for each of the following.

$$\sec\frac{5\pi}{4} = \frac{1}{\cos\frac{5\pi}{4}} = -\frac{1}{\cos\frac{\pi}{4}} = -\frac{1}{\frac{\sqrt{2}}{2}} = -\frac{2}{\sqrt{2}} = -\frac{2\sqrt{2}}{2} = -\sqrt{2}$$

#24: Find a calculator approximation for each circular function value.

$\sin 0.8203 = 0.7314$

#48: Without using a calculator, decide whether each function value is positive or negative.

$\cos 6$ is positive since 6 is in the 4th quadrant

#52: Evaluate the six circular function value of θ. $\left(-\frac{15}{17}, \frac{8}{17}\right)$

$$\sin\theta = \frac{8}{17} \qquad \csc\theta = \frac{1}{\sin\theta} = \frac{17}{8}$$

$$\cos\theta = -\frac{15}{17} \qquad \sec\theta = \frac{1}{\cos\theta} = -\frac{17}{15}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{8}{17}}{-\frac{15}{17}} = -\frac{8}{15} \qquad \cot\theta = \frac{1}{\tan\theta} = -\frac{15}{8}$$

#60: Find the value of s in the interval $\left[0, \frac{\pi}{2}\right]$ that makes each statement true.

$\csc s = 1.0219$

$$\frac{1}{\sin s} = 1.0219 \quad \rightarrow \quad \sin s = \frac{1}{1.0219} \quad \rightarrow \quad s = \sin^{-1}\left(\frac{1}{1.0219}\right) = 1.3634$$

#66: Find the exact value of s in the given interval that has the given circular function value.

$$\left[\frac{3\pi}{2}\ ,\ 2\pi\right]\ , \qquad \cos s = \frac{\sqrt{3}}{2}$$

$$s = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6} \text{ or } \frac{11\pi}{6}$$

since s is in the 4th quadrant, $s = 11\pi/6$

Section 3.4:

Linear Speed: $v = \frac{s}{t}$

Angular Speed: $\omega = \frac{\theta}{t}$

Linear to Angular **or** Angular to Linear Speed:

$$v = \frac{s}{t} = \frac{r\theta}{t} = r\ \frac{\theta}{t} = r\omega$$

So, $v = r\omega$

#6: Use the formula $\omega = \frac{\theta}{t}$ to find the value of the missing variable.

$$\omega = \frac{\pi}{4}\ radians/min\ ,\ t = 5\ min$$

$$\frac{\pi}{4} = \frac{\theta}{5} \quad \rightarrow \quad \theta = \frac{\pi}{4} \times 5 = \frac{5\pi}{4}$$

#12: Use the formula $\omega = \frac{\theta}{t}$ to find the value of the missing variable.

$$\theta = 5.225\ radians\ ,\ t = 2.515\ seconds$$

$$\omega = \frac{5.225}{2.515} = 2.078\ radians/seconds$$

#16: Use the formula $v = r\omega$ to find the value of the missing variable.

$$r = 8\ cm\ ,\ \omega = \frac{9\pi}{5}\ radians/seconds$$

$$v = 8 \times \frac{9\pi}{5} = \frac{72\pi}{5}\ cm/seconds$$

#18: Use the formula $v = r\omega$ to find the value of the missing variable.

$$v = 18\ ft/seconds\ ,\ r = 3\,ft$$

$$18 = 3\omega \quad \rightarrow \quad \omega = \frac{18}{3} = 6\ radians/seconds$$

#22: Use the formula $s = r\omega t$ to find the value of the missing variable.

$$r = 9\ yards\ \ \&\ \ \omega = \frac{2\pi}{5}\ radians/seconds\ \ \&\ \ t = 12\ seconds$$

$$\frac{s}{t} = r\omega \quad \rightarrow \quad s = r\omega t = 9 \times \frac{2\pi}{5} \times 12 = \frac{216\pi}{5}\ yards$$

#26: Use the formula $s = r\omega t$ to find the value of the missing variable.

$$s = \frac{8\pi}{9}\ m\ \ \&\ \ r = \frac{4}{3}\ m\ \ \&\ \ t = 12\ seconds$$

$$s = r\omega t \quad \rightarrow \quad \omega = \frac{s}{rt} = \frac{\frac{8\pi}{9}}{\frac{4}{3} \times 12} = \frac{\frac{8\pi}{9}}{16} = \frac{8\pi}{9} \times \frac{1}{16} = \frac{\pi}{18}\ radians/seconds$$

#38: Mars rotates on its axis at the rate of about 0.2552 radian per hour. Approximate how many hours are in a Martian day?

$$\omega = 0.2552\ radians/hour$$

$$0.2552 = \frac{2\pi}{t} \quad \rightarrow \quad t = \frac{2\pi}{0.2552} \approx 24.62\ hours$$

#42: The two pulleys have radii of 15 cm and 8 cm, respectively. The larger pulley rotates 25 times in 36 seconds. Find the angular speed of each pulley in radians per seconds.

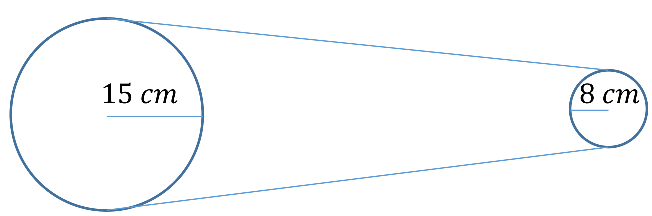
distance travelled by the larger pully: $s_l = r_l\theta_l = 15 \times (25 \times 2\pi) = 750\pi$

distance travelled by the smaller pully is equal to the distance travelled by the larger pully.

So, $s_s = s_l \quad \rightarrow \quad r_s\theta_s = 750\pi \quad \rightarrow \quad 8\theta_s = 750\pi \quad \rightarrow \quad \theta_s = \frac{750\pi}{8} = \frac{375\pi}{4}$

$$\omega_l = \frac{25 \times 2\pi}{36} = \frac{25\pi}{18}\,\frac{radians}{seconds}$$

$$\omega_s = \frac{\frac{375\pi}{4}}{36} = \frac{375\pi}{144} = \frac{125\pi}{48}\ radians/seconds$$



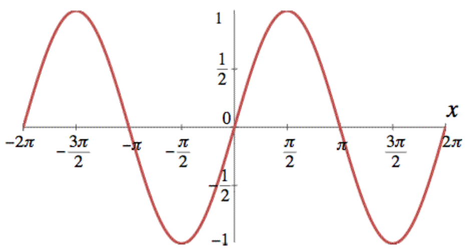
Section 4.1:

<u>Period:</u> the interval in which the function has a full cycle.

<u>Amplitude:</u> half of the distance between the maximum and minimum value.

<u>Graph of since and cosine funcions:</u>

$f(x) = \sin x$



Domain: $(-\infty, \infty)$

Range: $[-1,1]$

Continuous over $(-\infty, \infty)$

x-int: $n\pi$, where n is an integer

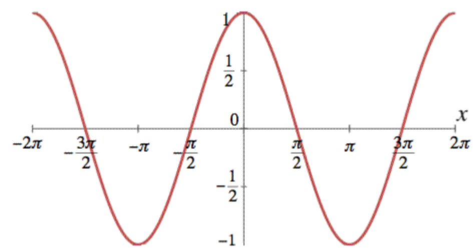
Period: 2π

Symmetric with respect to the origin

An odd function

$\sin(-x) = -\sin x$

$g(x) = \cos x$



Domain: $(-\infty, \infty)$

Range: $[-1,1]$

Continuous over $(-\infty, \infty)$

x-int: $n\pi + \frac{\pi}{2}$, where n is an integer

Period: 2π

Symmetric with respect to the y-axis

An even function

$\cos(-x) = \cos x$

<u>Example:</u>

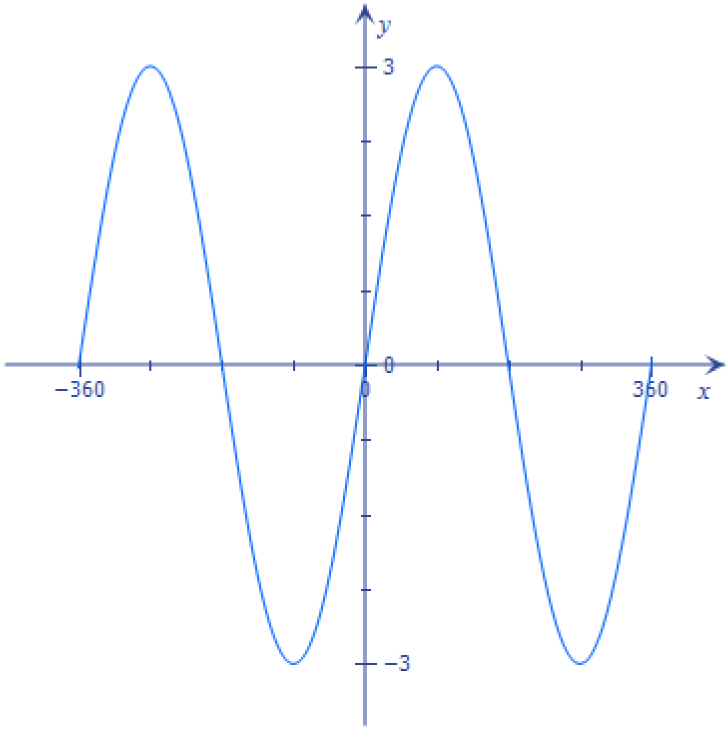
$f(x) = a\sin(bx)$

Period: $\dfrac{2\pi}{b}$

Amplitude: $|a|$

<u>#14:</u> Graph each function over the interval $[-2\pi, 2\pi]$. Give the amplitude.

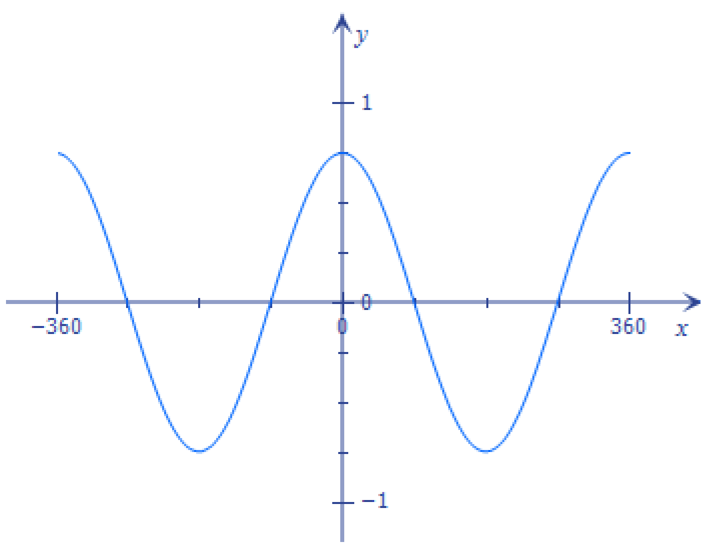
$y = 3\sin x$



Period: 2π , Amplitude: 3

<u>#16:</u> Graph each function over the interval $[-2\pi, 2\pi]$. Give the amplitude.

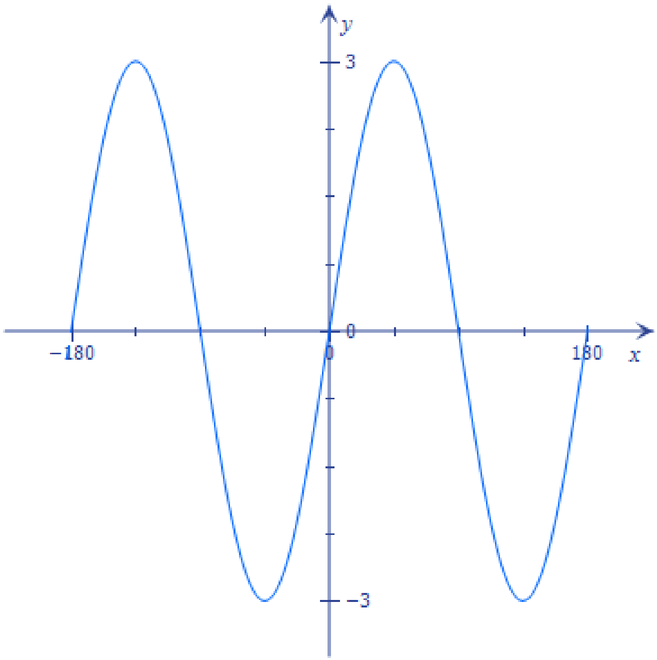
$y = \frac{3}{4}\cos x$



Period: 2π , Amplitude: $\frac{3}{4}$

#30: Graph each function over a two-period interval. Give the period and amplitude.

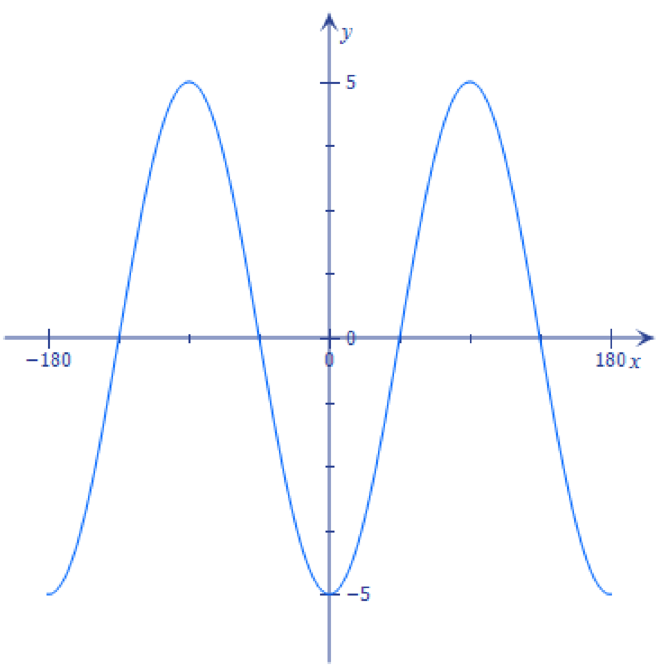
$y = 3\sin 2x$



Period: $\frac{2\pi}{2} = \pi$, Amplitude: 3

#32: Graph each function over a two-period interval. Give the period and amplitude.

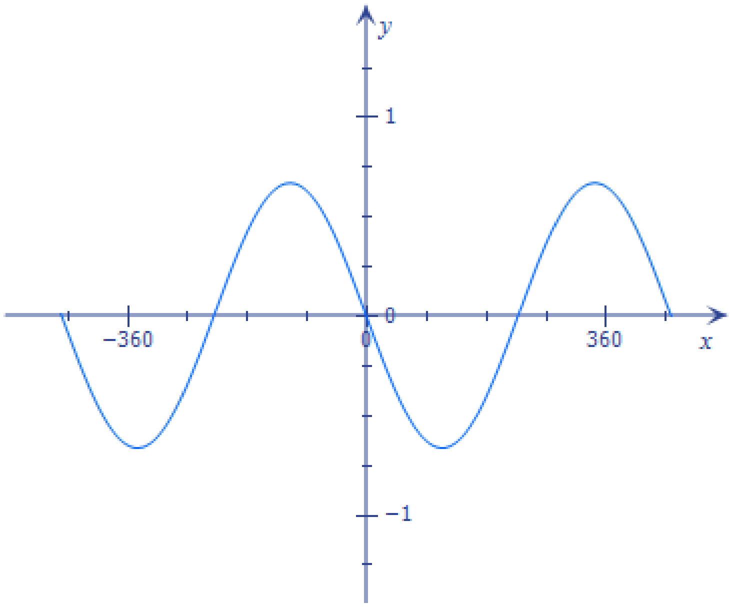
$y = -5\cos 2x$



Period: $\frac{2\pi}{2} = \pi$, Amplitude: 5

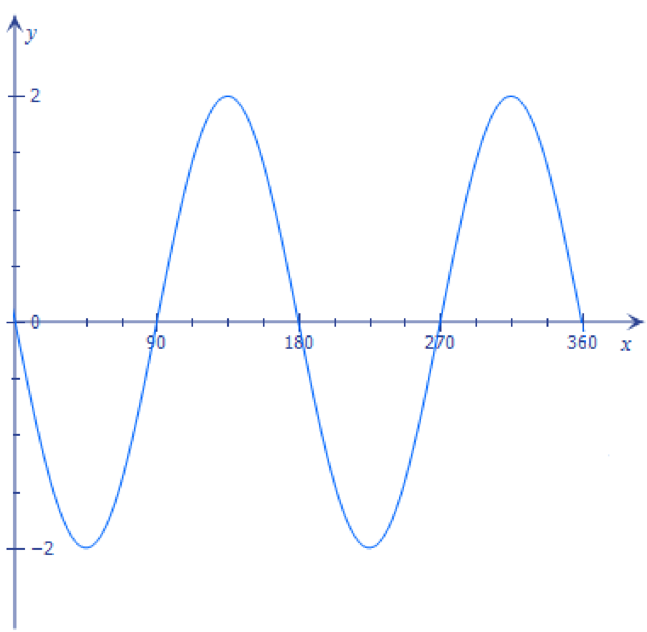
#38: Graph each function over a two-period interval. Give the period and amplitude.

$y = -\frac{2}{3}\sin\left(\frac{\pi}{4}x\right)$



Period: $\frac{2\pi}{\frac{\pi}{4}} = 2\pi\left(\frac{4}{\pi}\right) = 8$, Amplitude: $\frac{2}{3}$

#42: Each function graphed is of the form $y = a\sin bx$ or $y = a\cos bx$, where $b > 0$. Determine the equation of the graph.

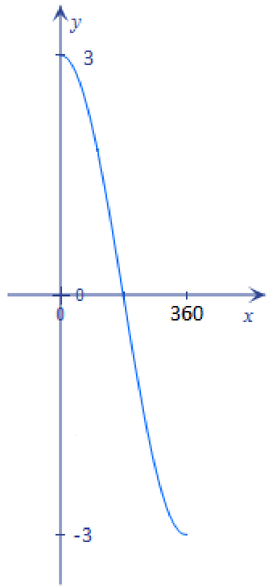


$\frac{2\pi}{b} = \pi \quad \rightarrow \quad b = \frac{2\pi}{\pi} = 2$

$a = 2$ and the graph of sine is flipped over the x-axis. Thus,

$y = -2\sin(2x)$

#44: Each function graphed is of the form $y = a\sin bx$ or $y = a\cos bx$, where $b > 0$. Determine the equation of the graph.



$$\frac{2\pi}{b} = 4\pi \quad \rightarrow \quad b = \frac{2\pi}{4\pi} = \frac{1}{2} \quad \rightarrow \quad a = 3 \quad \rightarrow \quad y = 3\cos\left(\frac{1}{2}x\right)$$

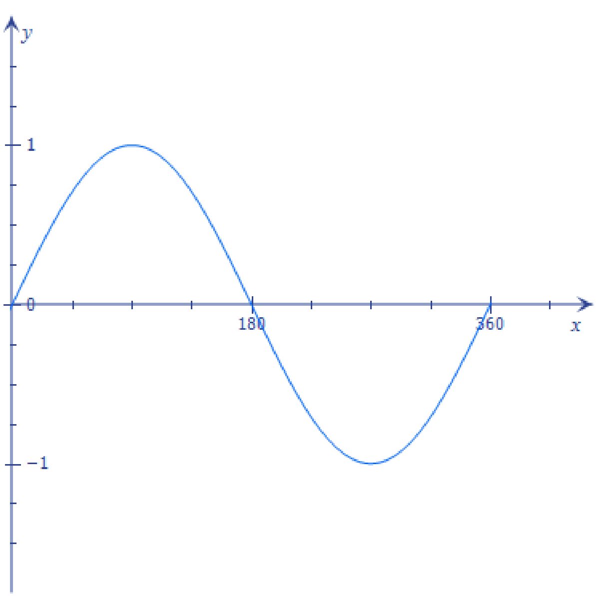
Section 4.2: Translation of Graphs of Sine and Cosine Functions:

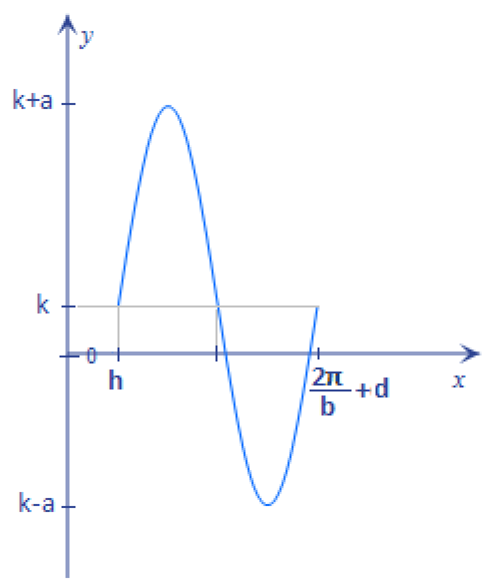
Phase Shift: The horizontal translation

Example:

$f(x) = k + a\sin b(x - h)$ where $a, b, h,$ and k are positive

Period: $\frac{2\pi}{b}$, Amplitude: $|a|$, Phase Shift: h , Vertical Translation: k



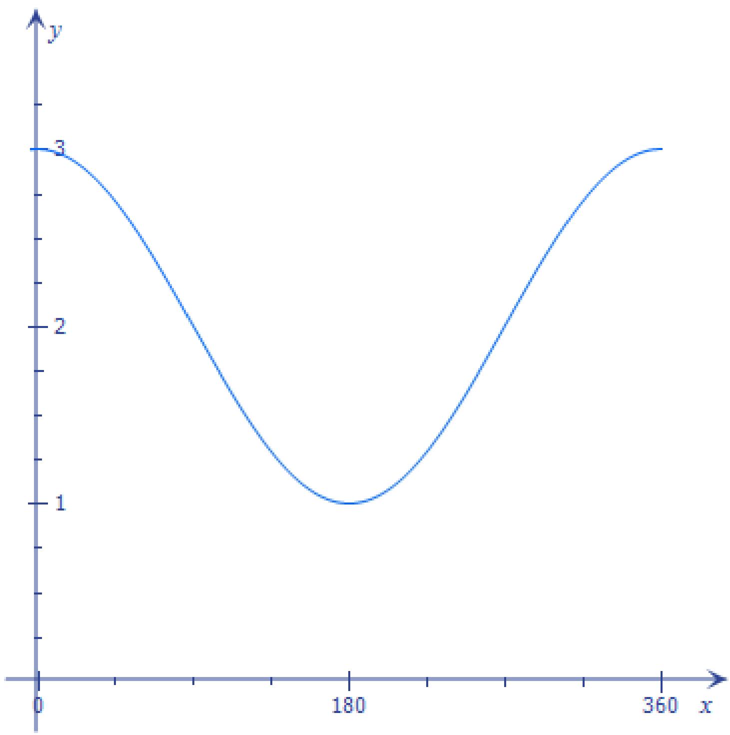


#16: Math each function in Column I with the appropreate description in Column II.

$y = 2\sin(3x - 4)$

$\text{Period: } \frac{2\pi}{3}$, $\text{Amplitude: } 2$, $\text{Phase Shift: } \frac{4}{3}$

#22: Each function graphed is of the form $y = c + \cos x$, $y = c + \sin x$, or $y = \sin(x - d)$, where d is the least positive value. Determine the equation of the graph.



$\text{Period: } 2\pi \quad \rightarrow \quad b = 1$

$\text{Phase Shift: } 0 \quad \rightarrow \quad h = 0$

$\text{Amplitude: } 1 \quad \rightarrow \quad a = 1$

$\text{Vertical Shift: } 2 \quad \rightarrow \quad k = 2$

So the equation is: $y = 2 + \cos x$

#28: Find the amplitude, the period, any vertical translation, and any phase shift of the graph of each function.

$y = -\frac{1}{2}\sin\left(\frac{1}{2}x + \pi\right)$

$\text{Amplitude: } \frac{1}{2}$, $\text{Period: } \frac{2\pi}{\frac{1}{2}} = 2\pi\left(\frac{2}{1}\right) = 4\pi$

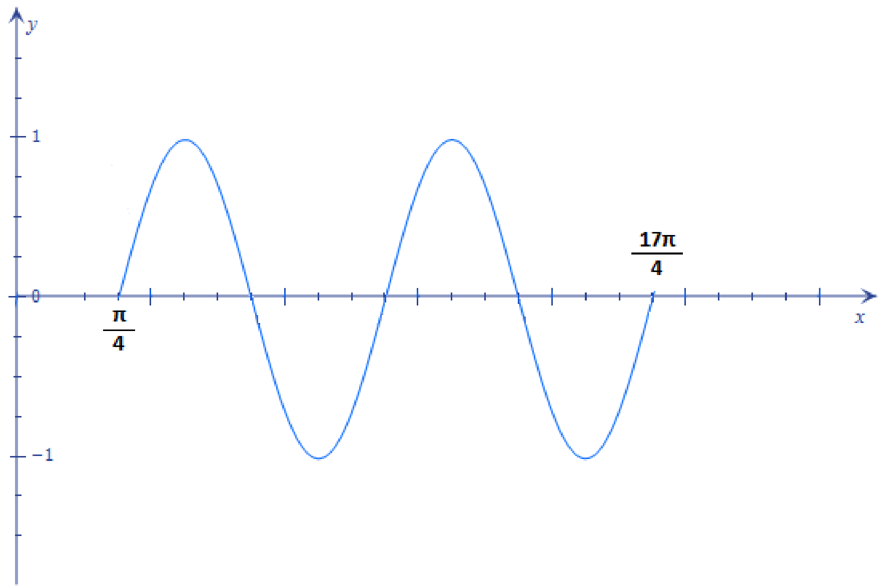
$\text{Phase Shift: } -\frac{\pi}{\frac{1}{2}} = -\pi\left(\frac{2}{1}\right) = -2\pi$, $\text{Vertical Translation: } 0$

#34: Graph each function over a two-period interval.

$y = \sin\left(x - \frac{\pi}{4}\right)$

Amplitude: 1 , Period: $\frac{2\pi}{1} = 2\pi$

Phase Shift: $\frac{\frac{\pi}{4}}{1} = \frac{\pi}{4}$, Vertical Translation: 0

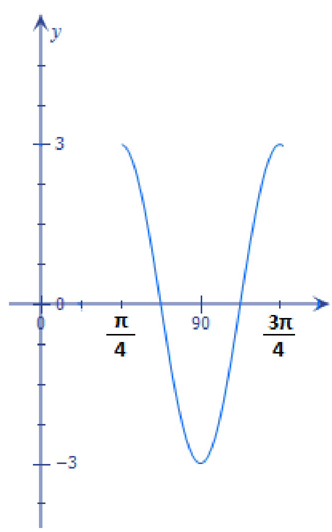


#42: Graph each function over a one-period interval.

$y = 3\cos(4x + \pi)$

Amplitude: 3 , Period: $\frac{2\pi}{4} = \frac{\pi}{2}$

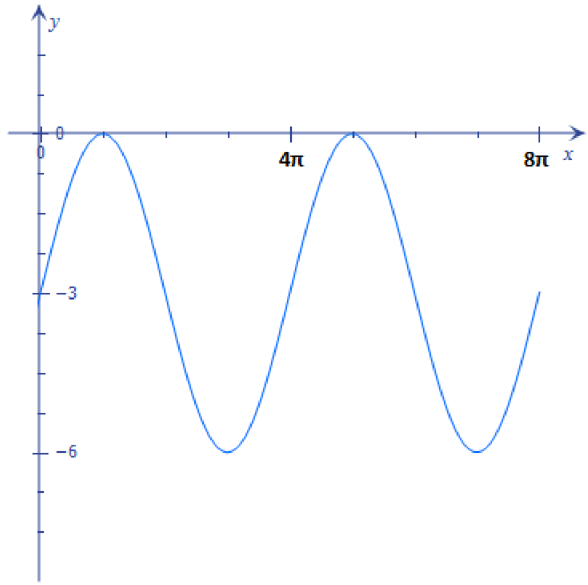
Phase Shift: $\frac{\pi}{4}$, Vertical Translation: 0



#50: Graph each function over a two-period interval.

$$y = -3 + 3\sin\left(\frac{1}{2}x\right)$$

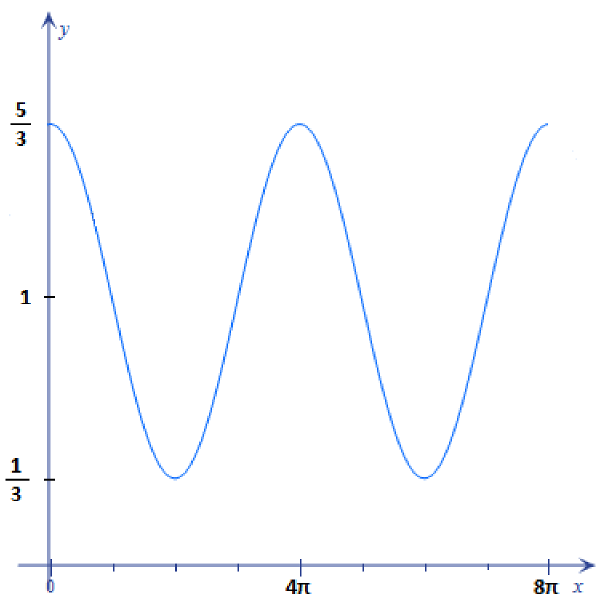
Amplitude: 3 , Period: $\frac{2\pi}{\frac{1}{2}} = 4\pi$, Phase Shift: 0 , Vertical Translation: -3



#52: Graph each function over a two-period interval.

$$y = 1 + \frac{2}{3}\cos\left(\frac{1}{2}x\right)$$

Amplitude: $\frac{2}{3}$, Period: $\frac{2\pi}{\frac{1}{2}} = 4\pi$, Phase Shift: 0 , Vertical Translation: 1



Section 4.3: Graph of Tangent and Cotangent

$f(x) = \tan x$

Domain: $\{x | x \neq n\pi + \frac{\pi}{2}, \text{ where } n \text{ is an integer}\}$

Range: $(-\infty, \infty)$

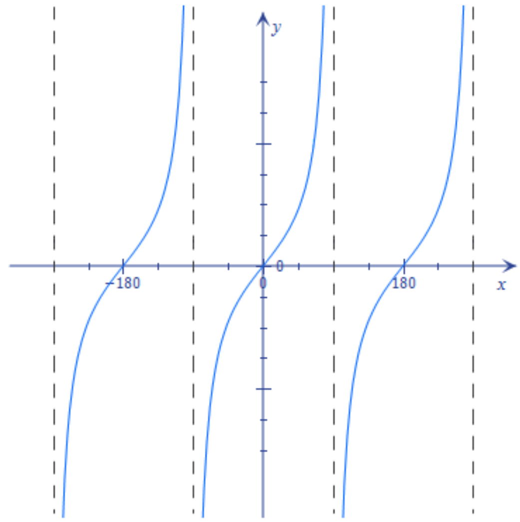
Discontinuous at $x = n\pi + \frac{\pi}{2}$

x-intercept: $x = n\pi$

Amplitude: none

Period: π

Symmetric with respect to the origin



$g(x) = \cot x$

Domain: $\{x | x \neq n\pi, \text{ where } n \text{ is an integer}\}$

Range: $(-\infty, \infty)$

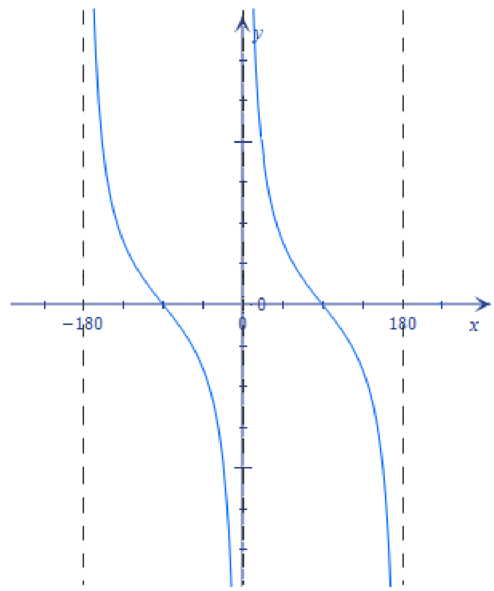
Discontinuous at $x = n\pi$

x-intercept: $x = n\pi + \frac{\pi}{2}$

Amplitude: none

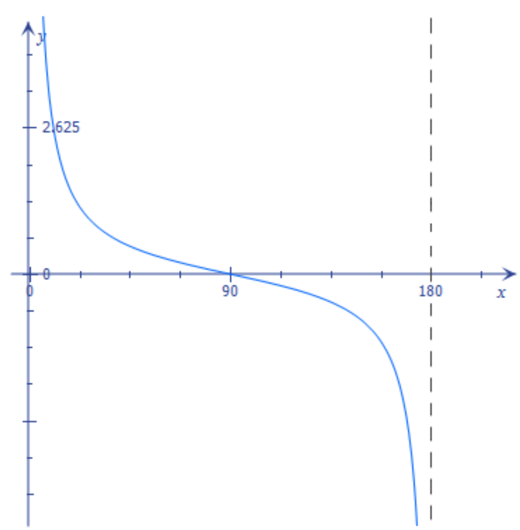
Period: π

Symmetric with respect to the origin



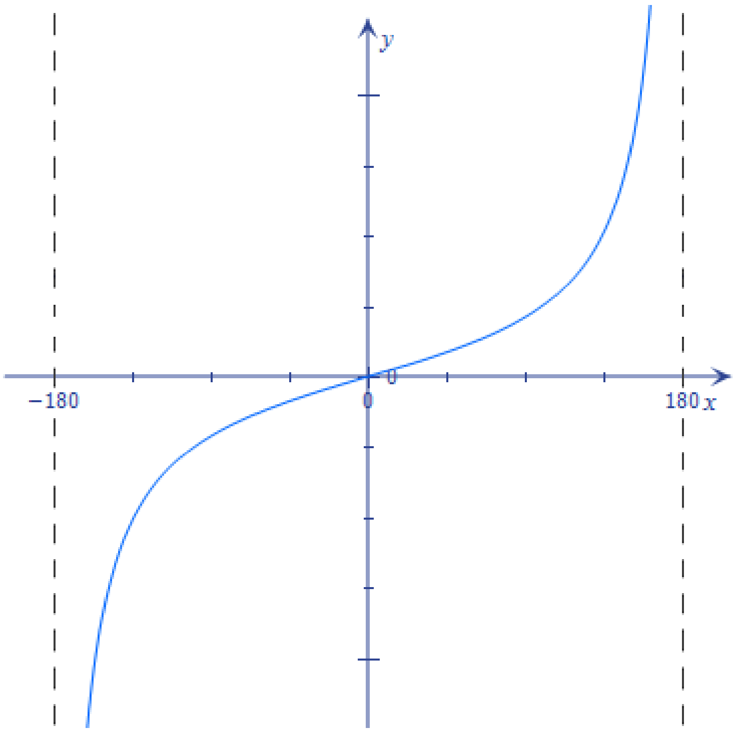
#12: Graph each function over a one-period interval.

$y = \frac{1}{2} \cot x$



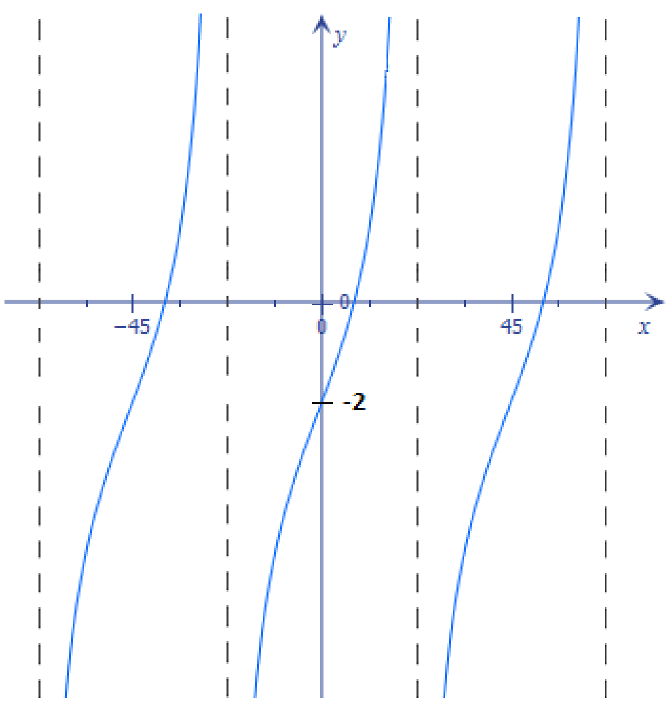
#16: Graph each function over a one-period interval.

$$y = 3\tan\left(\frac{1}{2}x\right)$$



#30: Graph each function over a two-period interval.

$$y = -2 + 3\tan(4x + \pi)$$



Period: $\frac{\pi}{4}$, Phase Shift: $\frac{\pi}{4}$

Section 4.4: Graph of Secant and Cosecant

$f(x) = \csc x$

Domain:

$\{x | x \neq n\pi, \text{ where } n \text{ is an integer}\}$

Range: $(-\infty, -1] \cup [1, \infty)$

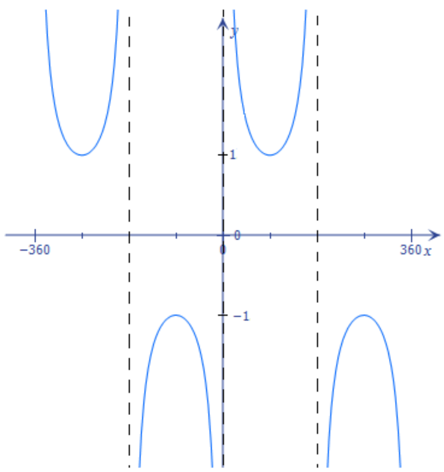
Discontinuous at $x = n\pi$

x-intercept: none

Amplitude: none

Period: 2π

Symmetric with respect to the origin



$g(x) = \sec x$

Domain:

$\{x | x \neq n\pi + \frac{\pi}{2}, \text{ where } n \text{ is an integer}\}$

Range: $(-\infty, -1] \cup [1, \infty)$

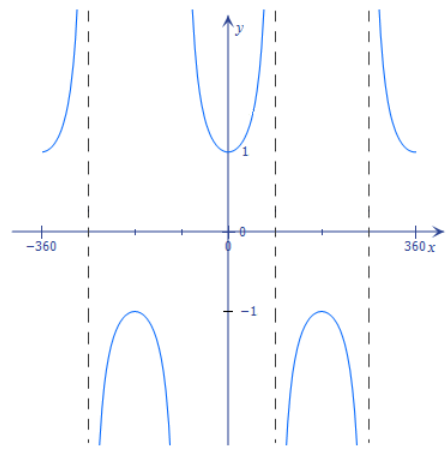
Discontinuous at $x = n\pi + \frac{\pi}{2}$

x-intercept: none

Amplitude: none

Period: 2π

Symmetric with respect to the y-axis



Section 5.1:

Pythagorean Identities:

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\cot^2\theta + 1 = \csc^2\theta$$

Negative Angle Identities:

$\sin(-\theta) = -\sin\theta$ $\tan(-\theta) = -\tan\theta$ $\sec(-\theta) = \sec\theta$

$\cos(-\theta) = \cos\theta$ $\csc(-\theta) = -\csc\theta$ $\cot(-\theta) = -\cot\theta$

#14: Find $\sin\theta$.

$$\tan\theta = -\frac{\sqrt{7}}{2}\ , \qquad \sec\theta > 0$$

$$\tan^2\theta + 1 = \sec^2\theta \quad\rightarrow\quad \sec^2\theta = \left(-\frac{\sqrt{7}}{2}\right)^2 + 1 = \frac{7}{4} + 1 = \frac{11}{4} \quad\rightarrow\quad \sec\theta = \pm\sqrt{\frac{11}{4}} = \pm\frac{\sqrt{11}}{2}$$

$$\cos\theta = \frac{2}{\sqrt{11}} = \frac{2\sqrt{11}}{11} \quad\rightarrow\quad \tan\theta = \frac{\sin\theta}{\cos\theta} \quad\rightarrow\quad \sin\theta = \tan\theta\cos\theta = -\frac{\sqrt{7}}{2}\times\frac{2\sqrt{11}}{11} = -\frac{\sqrt{77}}{11}$$

#34: Find the remaining five trigonometric functions of θ.

$$\csc\theta = -\frac{5}{2}\ , \qquad \theta \text{ is in quadrant III}$$

$$\sin\theta = \frac{1}{\csc\theta} = -\frac{2}{5}$$

$$\sin^2\theta + \cos^2\theta = 1 \quad\rightarrow\quad \cos\theta = \pm\sqrt{1 - \sin^2\theta}$$

$$\cos\theta = -\sqrt{1 - \left(-\frac{2}{5}\right)^2} = -\sqrt{\frac{25}{25} - \frac{4}{25}} = -\sqrt{\frac{21}{25}} = -\frac{\sqrt{21}}{5}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-\frac{2}{5}}{-\frac{\sqrt{21}}{5}} = \left(-\frac{2}{5}\right)\left(-\frac{5}{\sqrt{21}}\right) = \frac{2}{\sqrt{21}} = \frac{2\sqrt{21}}{21}$$

$$\cot\theta = \frac{\sqrt{21}}{2}\ , \qquad \sec\theta = -\frac{5}{\sqrt{21}} = -\frac{5\sqrt{21}}{21}$$

#38: Find the remaining five trigonometric functions of θ.

$$\cos\theta = -\frac{1}{4}\ , \qquad \sin\theta > 0$$

$$\sin^2\theta = 1 - \cos^2\theta \quad \rightarrow \quad \sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\left(-\frac{1}{4}\right)^2} = \sqrt{\frac{16}{16}-\frac{1}{16}} = \sqrt{\frac{15}{16}} = \frac{\sqrt{15}}{4}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{\sqrt{15}}{4}}{-\frac{1}{4}} = \frac{\sqrt{15}}{4}\times\left(-\frac{4}{1}\right) = -\sqrt{15}$$

$$\cot\theta = -\frac{1}{\sqrt{15}} = -\frac{\sqrt{15}}{15}$$

$$\csc\theta = \frac{4}{\sqrt{15}} = \frac{4\sqrt{15}}{15}$$

$$\sec\theta = -4$$

#60: Write each expression in terms of since and cosine, and simplify so that no quotients appear in the final expression and all functions are of θ only.

$$\tan\theta\cos\theta = \frac{\sin\theta}{\cos\theta}\cos\theta = \sin\theta$$

#66: Write each expression in terms of since and cosine, and simplify so that no quotients appear in the final expression and all functions are of θ only.

$$\cot^2\theta\,(1+\tan^2\theta) = \frac{\cos^2\theta}{\sin^2\theta}\left(1+\frac{\sin^2\theta}{\cos^2\theta}\right) = \frac{\cos^2\theta}{\sin^2\theta} + \frac{\sin^2\theta\cos^2\theta}{\sin^2\theta\cos^2\theta} = \frac{\cos^2\theta}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta}$$

$$= \frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta} = \csc^2\theta$$

#74: Write each expression in terms of since and cosine, and simplify so that no quotients appear in the final expression and all functions are of θ only.

$$\csc\theta - \sin\theta = \frac{1}{\sin\theta} - \sin\theta = \frac{1-\sin^2\theta}{\sin\theta} = \frac{\cos^2\theta}{\sin\theta} = \frac{\cos\theta}{\sin\theta}\cos\theta = \cot\theta\cos\theta$$

#84: Write each expression in terms of since and cosine, and simplify so that no quotients appear in the final expression and all functions are of θ only.

$$-\sec^2(-\theta) + \sin^2(-\theta) + \cos^2(-\theta) = -\frac{1}{\cos^2(-\theta)} + 1 = \frac{-1+\cos^2(-\theta)}{\cos^2(-\theta)} = -\frac{1-\cos^2(-\theta)}{\cos^2(-\theta)}$$

$$= -\frac{\sin^2(-\theta)}{\cos^2(-\theta)} = -\left(\frac{\sin(-\theta)}{\cos(-\theta)}\right)^2 = -\left(\frac{-\sin\theta}{\cos\theta}\right)^2$$

$$= -\frac{\sin^2\theta}{\cos^2\theta} = -\tan^2\theta$$

Section 5.2:

#16: Factor each trigonometric expression.

$$(\tan x + \cot x)^2 - (\tan x - \cot x)^2$$
$$= \big((\tan x + \cot x) + (\tan x - \cot x)\big)\big((\tan x + \cot x) - (\tan x - \cot x)\big)$$
$$= (\tan x + \cot x + \tan x - \cot x)(\tan x + \cot x - \tan x + \cot x)$$
$$= (2\tan x)(2\cot x) = 4\tan x \cot x = 4\frac{\sin x}{\cos x}\frac{\cos x}{\sin x} = 4$$

#20: Factor each trigonometric expression.

$$\cot^4 x + 3\cot^2 x + 2 = (\cot^2 x + 1)(\cot^2 x + 2) = \csc^2 x\,(\tan^2 x + 2)$$

#28: Each expression simplifies to a constant, a single function, or a power of a function. Use fundamental identities to somplify each expression.

$$\frac{\csc\theta\sec\theta}{\cot\theta} = \frac{\frac{1}{\sin\theta}\frac{1}{\cos\theta}}{\frac{\cos\theta}{\sin\theta}} = \frac{1}{\sin\theta\cos\theta}\frac{\sin\theta}{\cos\theta} = \frac{1}{\cos^2\theta} = \sec^2\theta$$

#32: Each expression simplifies to a constant, a single function, or a power of a function. Use fundamental identities to somplify each expression.

$$\frac{1}{\tan^2\alpha} + \cot\alpha\tan\alpha = \frac{1}{\frac{\sin^2\alpha}{\cos^2\alpha}} + \frac{\cos\alpha}{\sin\alpha}\frac{\sin\alpha}{\cos\alpha} = \frac{\cos^2\alpha}{\sin^2\alpha} + 1 = \frac{\cos^2\alpha}{\sin^2\alpha} + \frac{\sin^2\alpha}{\sin^2\alpha} = \frac{\sin^2\alpha + \cos^2\alpha}{\sin^2\alpha}$$
$$= \frac{1}{\sin^2\alpha} = \csc^2\alpha$$

#38: Verify that each trigonometric equation is an identity.

$$\frac{\tan^2\alpha + 1}{\sec\alpha} = \frac{\frac{\sin^2\alpha}{\cos^2\alpha} + 1}{\frac{1}{\cos\alpha}} = \frac{\frac{\sin^2\alpha + \cos^2\alpha}{\cos^2\alpha}}{\frac{1}{\cos\alpha}} = \frac{\sin^2\alpha + \cos^2\alpha}{\cos^2\alpha}\frac{\cos\alpha}{1} = \frac{1}{\cos\alpha} = \sec\alpha$$

#46: Verify that each trigonometric equation is an identity.

$$\sec^4 x - \sec^2 x = \sec^2 x\,(\sec^2 x - 1) = \sec^2 x \tan^2 x = (\tan^2 x + 1)\tan^2 x = \tan^4 x + \tan^2 x$$

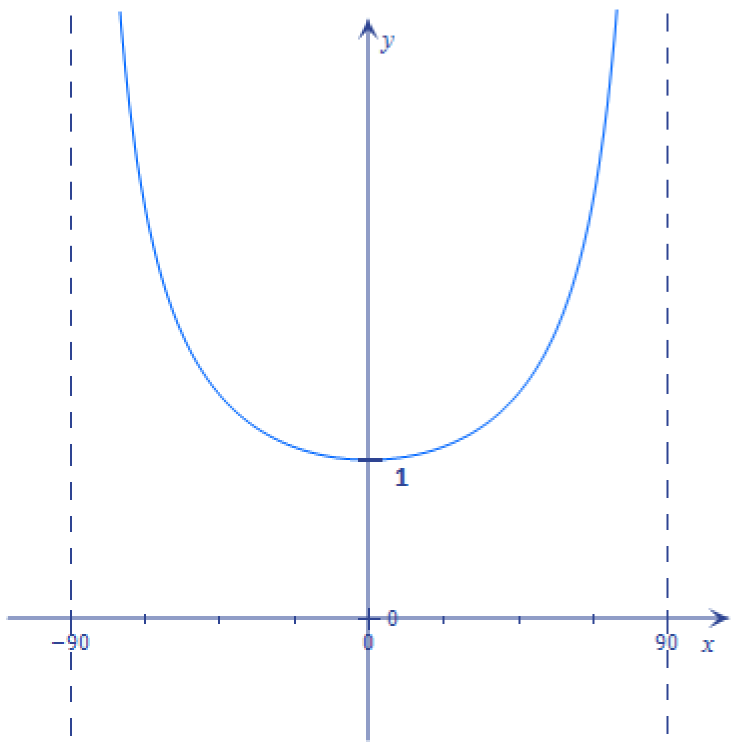
#54: Verify that each trigonometric equation is an identity.

$$\frac{\csc\theta + \cot\theta}{\tan\theta + \sin\theta} = \frac{\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}}{\frac{\sin\theta}{\cos\theta} + \frac{\sin\theta\cos\theta}{\cos\theta}} = \frac{\frac{1+\cos\theta}{\sin\theta}}{\frac{\sin\theta + \sin\theta\cos\theta}{\cos\theta}} = \frac{1+\cos\theta}{\sin\theta}\frac{\cos\theta}{\sin\theta + \sin\theta\cos\theta}$$
$$= \frac{\cos\theta\,(1+\cos\theta)}{\sin^2\theta\,(1+\cos\theta)} = \frac{\cos\theta}{\sin\theta}\frac{1}{\sin\theta} = \cot\theta\csc\theta$$

#82: Graph each expression and use the graph to make a conjecture, predicting what might be an identity. Then verify your conjecture algebraically.

$\tan\theta\sin\theta+\cos\theta$

θ	$\tan\theta\sin\theta+\cos\theta$	$(\theta\,,y)$
0	$\tan 0\sin 0+\cos 0=0\times 0+1=1$	$(0\,,1)$
$\frac{\pi}{6}$	$\tan\left(\frac{\pi}{6}\right)\sin\left(\frac{\pi}{6}\right)+\cos\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{3}\times\frac{1}{2}+\frac{\sqrt{3}}{2}=\frac{2\sqrt{3}}{3}$	$\left(\frac{\pi}{6}\,,\frac{2\sqrt{3}}{3}\right)$
$\frac{\pi}{4}$	$\tan\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{4}\right)+\cos\left(\frac{\pi}{4}\right)=1\times\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}=\sqrt{2}$	$\left(\frac{\pi}{4}\,,\sqrt{2}\right)$
$\frac{\pi}{3}$	$\tan\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{3}\right)+\cos\left(\frac{\pi}{3}\right)=\sqrt{3}\times\frac{\sqrt{3}}{2}+\frac{1}{2}=2$	$\left(\frac{\pi}{3}\,,2\right)$
$\frac{\pi}{2}$	$\tan\left(\frac{\pi}{2}\right)$ is undefined, thus $\tan\left(\frac{\pi}{2}\right)\sin\left(\frac{\pi}{2}\right)+\cos\left(\frac{\pi}{2}\right)$ is undefiend	vertical asymptote at $\frac{\pi}{2}$



Is is possibly the graph of $\sec\theta$?

$$\tan\theta\sin\theta+\cos\theta=\frac{\sin\theta}{\cos\theta}\sin\theta+\cos\theta=\frac{\sin^2\theta}{\cos\theta}+\frac{\cos^2\theta}{\cos\theta}=\frac{\sin^2\theta+\cos^2\theta}{\cos\theta}=\frac{1}{\cos\theta}=\sec\theta$$

Section 5.3:

<u>Cosine of a Sum or Difference:</u>

$$\cos(A+B)=\cos A\cos B-\sin A\sin B$$

$$\cos(A-B)=\cos A\cos B+\sin A\sin B$$

#8: Use identities to find each exact value. (Do not use a calculator.)

$$\cos(-15°) = \cos(30° - 45°) = \cos(30°)\cos(45°) + \sin(30°)\sin(45°)$$

$$= \left(\frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2} \times \frac{\sqrt{2}}{2}\right) = \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

#12: Use identities to find each exact value. (Do not use a calculator.)

$$\cos\left(\frac{\pi}{12}\right) = \cos\left(\frac{4\pi}{12} - \frac{3\pi}{12}\right) = \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \cos\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)$$

$$= \left(\frac{1}{2} \times \frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \frac{\sqrt{2}+\sqrt{6}}{4}$$

#16: Use identities to find each exact value. (Do not use a calculator.)

$$\cos\frac{7\pi}{9}\cos\frac{2\pi}{9} - \sin\frac{7\pi}{9}\sin\frac{2\pi}{9} = \cos\left(\frac{7\pi}{9} + \frac{2\pi}{9}\right) = \cos\left(\frac{9\pi}{9}\right) = \cos\pi = -1$$

#32: Use identities to fill in each blank with appropriate trigonometric function name.

$$\sin\left(\frac{2\pi}{3}\right) = \underline{\qquad\qquad}\left(-\frac{\pi}{6}\right)$$

$$\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$$

$$x - \left(\frac{2\pi}{3}\right) = -\frac{\pi}{6} \quad \rightarrow \quad x = \frac{2\pi}{3} - \frac{\pi}{6} = \frac{4\pi}{6} - \frac{\pi}{6} = \frac{3\pi}{6} = \frac{\pi}{2} \quad \rightarrow \quad \frac{\pi}{2} - \left(\frac{2\pi}{3}\right) = \left(-\frac{\pi}{6}\right)$$

$$\sin\left(\frac{2\pi}{3}\right) = \cos\left(\frac{\pi}{2} - \frac{2\pi}{3}\right) = \cos\left(-\frac{\pi}{6}\right)$$

#38: Find one angle θ that satisfies each of the following.

$$\sin\theta = \cos(2\theta + 30°)$$

$$\sin\theta = \cos(90° - \theta) \quad \rightarrow \quad \cos(90° - \theta) = \cos(2\theta + 30°)$$

$$90° - \theta = 2\theta + 30° \quad \rightarrow \quad 3\theta = 60° \quad \rightarrow \quad \theta = 20°$$

#42: Find one angle θ that satisfies each of the following.

$$\cot(\theta - 10°) = \tan(2\theta - 20°)$$

$$\cot\theta = \tan(90° - \theta) \quad \rightarrow \quad \tan\big(90° - (\theta - 10°)\big) = \tan(2\theta - 20°)$$

$$90° - (\theta - 10°) = 2\theta - 20° \quad \rightarrow \quad 90° - \theta + 10° = 2\theta - 20° \quad \rightarrow \quad 3\theta = 120° \quad \rightarrow \quad \theta = 40°$$

#46: Use the identities for the cosine of a sum or difference to write each expression as a function of θ.

$$\cos(\theta - 270°) = \cos\theta\cos 270° + \sin\theta\sin 270° = (\cos\theta \times 0) + \big(\sin\theta \times (-1)\big) = -\sin\theta$$

#68: Verify that each equation is an identity.

$$\sec(\pi - x) = \frac{1}{\cos(\pi - x)} = \frac{1}{\cos\pi\cos x + \sin\pi\sin x} = \frac{1}{\big((-1)\times\cos x\big) + (0 \times \sin x)} = -\frac{1}{\cos x}$$

$$= -\sec x$$

Section 5.4:

Sine of a Sum or Differece:

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

Tangent of Sum or Difference:

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

#16: Use identities to find each exact value.

$$\sin\left(-\frac{5\pi}{12}\right) = \sin\left(-\frac{3\pi}{12} - \frac{2\pi}{12}\right) = \sin\left(-\frac{\pi}{4} - \frac{\pi}{6}\right) = \sin\left(-\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) - \cos\left(-\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right)$$

$$= \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) = -\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = -\frac{\sqrt{6}+\sqrt{2}}{4}$$

#18: Use identities to find each exact value.

$$\tan\left(-\frac{7\pi}{12}\right) = \tan\left(-\frac{3\pi}{12} - \frac{4\pi}{12}\right) = \tan\left(-\frac{\pi}{4} - \frac{\pi}{3}\right) = \frac{\tan\left(-\frac{\pi}{4}\right) - \tan\left(\frac{\pi}{3}\right)}{1 + \tan\left(-\frac{\pi}{4}\right)\tan\left(\frac{\pi}{3}\right)} = \frac{-1-\sqrt{3}}{1+\left(-1\times\sqrt{3}\right)}$$

$$= \frac{-1-\sqrt{3}}{1-\sqrt{3}} = \frac{(-1-\sqrt{3})(1+\sqrt{3})}{(1-\sqrt{3})(1+\sqrt{3})} = \frac{-1-\sqrt{3}-\sqrt{3}-3}{1+\sqrt{3}-\sqrt{3}-3} = \frac{-4-2\sqrt{3}}{-2}$$

$$= \frac{-2(2+\sqrt{3})}{-2} = 2+\sqrt{3}$$

#20: Use identities to find each exact value.

$$\sin 40° \cos 50° + \cos 40° \sin 50° = \sin(40° + 50°) = \sin 90° = 1$$

#52: Find each exact value. Use an appropreate sum or difference identity.

$$\sin 255° = \sin(120° + 135°) = \sin 120° \cos 135° + \cos 120° \sin 135°$$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) + \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = -\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$$

Section 5.5:

Double-Angle Identities:

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\sin 2A = 2 \sin A \cos A$$

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$$

#10: Use identities to find values of the sine and cosine functions for each angle measure.

$$2x\ , \qquad \text{given } \tan x = \frac{5}{3}\ \ \&\ \sin x < 0$$

$$\tan x = \frac{opposite}{adjacent} = \frac{5}{3} \quad \rightarrow \quad hypotenuse = \sqrt{(5)^2 + (3)^2} = \sqrt{25+9} = \sqrt{34}$$

$$\sin 2x = 2\sin x \cos x = 2\left(\frac{5}{\sqrt{34}}\right)\left(\frac{3}{\sqrt{34}}\right) = \frac{30}{34} = \frac{15}{17}$$

$$\cos 2x = \cos^2 x - \sin^2 x = \left(\frac{3}{\sqrt{34}}\right)^2 - \left(\frac{5}{\sqrt{34}}\right)^2 = \frac{9}{34} - \frac{25}{34} = -\frac{16}{34} = -\frac{8}{17}$$

#14: Use identities to find values of the sine and cosine functions for each angle measure.

$$\theta\ , \qquad \text{given } \cos 2\theta = \frac{3}{4}\ \ \&\ \ \theta \text{ terminates in quadrant III}$$

$$\cos 2\theta = 2\cos^2\theta - 1 \quad \rightarrow \quad \cos^2\theta = \frac{\cos 2\theta + 1}{2} \quad \rightarrow \quad \cos\theta = \pm\sqrt{\frac{\cos 2\theta + 1}{2}}$$

$$\cos\theta = -\sqrt{\frac{\cos 2\theta + 1}{2}} = -\sqrt{\frac{\left(\frac{3}{4}\right) + 1}{2}} = -\sqrt{\frac{\frac{7}{4}}{2}} = -\sqrt{\frac{7}{8}} = -\frac{\sqrt{14}}{4}$$

$$\cos 2\theta = 1 - 2\sin^2\theta \quad \rightarrow \quad \sin^2\theta = \frac{1 - \cos 2\theta}{2} \quad \rightarrow \quad \sin\theta = \pm\sqrt{\frac{1 - \cos 2\theta}{2}}$$

$$\sin\theta = -\sqrt{\frac{1 - \left(\frac{3}{4}\right)}{2}} = -\sqrt{\frac{\frac{1}{4}}{2}} = -\sqrt{\frac{1}{8}} = -\frac{\sqrt{2}}{4}$$

#20: Verify that each equation is an identity.

$$(\cos 2x - \sin 2x)^2 = 1 - \sin 4x$$

$$(\cos 2x - \sin 2x)^2 = (\cos 2x - \sin 2x)(\cos 2x - \sin 2x) = \cos^2 2x - 2\sin 2x \cos 2x + \sin^2 2x$$

$$= \sin^2 2x + \cos^2 2x - \sin 2(2x) = 1 - \sin 4x$$

#24: Verify that each equation is an identity.

$$\tan 2\theta = \frac{-2\tan\theta}{\sec^2\theta - 2}$$

$$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{2\sin\theta\cos\theta}{2\cos^2\theta - 1} = \frac{\frac{2\sin\theta\cos\theta}{\cos^2\theta}}{\frac{2\cos^2\theta}{\cos^2\theta} - \frac{1}{\cos^2\theta}} = \frac{\frac{2\sin\theta}{\cos\theta}}{2 - \frac{1}{\cos^2\theta}} = \frac{2\tan\theta}{2 - \sec^2\theta}$$

$$= -\frac{2\tan\theta}{\sec^2\theta - 2} = \frac{-2\tan\theta}{\sec^2\theta - 2}$$

#40: Use an identity to write each expression as a single trigonometric function value or as a single number.

$$1-2\sin^2 22\frac{1}{2}^\circ = \cos 2\left(22\frac{1}{2}^\circ\right) = \cos 45^\circ = \frac{\sqrt{2}}{2}$$

#50: Express each function as a trigonometric function of x.

$$\cos 3x = \cos(2x+x) = \cos 2x\cos x - \sin 2x \sin x$$

$$= (\cos^2 x - \sin^2 x)\cos x - (2\sin x\cos x)\sin x$$

$$= \cos^3 x - \sin^2 x\cos x - 2\sin^2 x\cos x$$

$$= \cos^3 x - 3\sin^2 x\cos x$$

$$= \cos^3 x - 3\cos x\,(1-\cos^2 x)$$

$$= \cos^3 x - 3\cos x + 3\cos^3 x = 4\cos^3 x - 3\cos x$$

Section 5.6:

Half-Angle Identities:

$$\cos\frac{A}{2} = \pm\sqrt{\frac{1+\cos A}{2}} \qquad \text{or} \qquad \cos^2\left(\frac{A}{2}\right) = \frac{1+\cos A}{2}$$

$$\sin\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos A}{2}} \qquad \text{or} \qquad \sin^2\left(\frac{A}{2}\right) = \frac{1-\cos A}{2}$$

$$\tan\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos A}{1+\cos A}} = \frac{\sin A}{1+\cos A} = \frac{1-\cos A}{\sin A}$$

#20: Find each of the following.

$$\sin\frac{x}{2}\ , \qquad \text{given } \cos x = -\frac{5}{8} \text{ with } \frac{\pi}{2} < x < \pi$$

$$\sin\frac{x}{2} = \sqrt{\frac{1-\left(-\frac{5}{8}\right)}{2}} = \sqrt{\frac{\frac{8}{8}+\frac{5}{8}}{2}} = \sqrt{\frac{13}{16}} = \frac{\sqrt{13}}{4}$$

#26: Find each of the following.

$$\cot\frac{\theta}{2}\ , \qquad \text{given } \tan\theta = -\frac{\sqrt{5}}{2} \text{ with } 90^\circ < x < 180^\circ$$

$$r = \sqrt{(-5)^2+(2)^2} = \sqrt{5+4} = \sqrt{9} = 3 \quad \rightarrow \quad \sin\theta = \frac{\sqrt{5}}{3} \quad \& \quad \cos\theta = -\frac{2}{3}$$

$$\cot\frac{\theta}{2} = \frac{1}{\tan\frac{\theta}{2}} = \frac{1}{\frac{\sin\theta}{1+\cos\theta}} = \frac{1+\cos\theta}{\sin\theta} = \frac{1-\frac{2}{3}}{\frac{\sqrt{5}}{3}} = \frac{\frac{1}{3}}{\frac{\sqrt{5}}{3}} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

#30: Find each of the following.

$$\sin x \quad , \qquad \text{given } \cos 2x = \frac{2}{3} \text{ with } \pi < x < \frac{3\pi}{2}$$

$$\sin x = \sin\frac{2x}{2} = -\sqrt{\frac{1-\cos 2x}{2}} = -\sqrt{\frac{1-\frac{2}{3}}{2}} = -\sqrt{\frac{\frac{1}{3}}{2}} = -\sqrt{\frac{1}{6}} = -\frac{\sqrt{6}}{6}$$

#34: Use an identity to write each as a single trigonometric function.

$$\sqrt{\frac{1+\cos 76^\circ}{2}} = \cos\left(\frac{76^\circ}{2}\right) = \cos 38^\circ$$

#40: Use an identity to write each as a single trigonometric function.

$$\pm\sqrt{\frac{1+\cos 20\alpha}{2}} = \cos\left(\frac{20\alpha}{2}\right) = \cos 10\alpha$$

#44: Use an identity to write each as a single trigonometric function.

$$\pm\sqrt{\frac{1-\cos\frac{3\theta}{5}}{2}} = \sin\left(\frac{\frac{3\theta}{5}}{2}\right) = \sin\frac{3\theta}{10}$$

#50: Verify that each equation is an identity.

$$\tan\frac{\theta}{2} = \csc\theta - \cot\theta$$

$$\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta} = \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta} = \csc\theta - \cot\theta$$

#52: Verify that each equation is an identity.

$$\cos x = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$$

$$\cos x = \cos^2\frac{x}{2} - \sin^2\frac{x}{2} = \left(\cos^2\frac{x}{2} - \sin^2\frac{x}{2}\right)\left(\frac{\frac{1}{\cos^2\frac{x}{2}}}{\frac{1}{\cos^2\frac{x}{2}}}\right) = \frac{\frac{\cos^2\frac{x}{2}}{\cos^2\frac{x}{2}} - \frac{\sin^2\frac{x}{2}}{\cos^2\frac{x}{2}}}{\frac{1}{\cos^2\frac{x}{2}}} = \frac{1-\tan^2\frac{x}{2}}{\frac{1}{\cos^2\frac{x}{2}}}$$

$$= \frac{1-\tan^2\frac{x}{2}}{\frac{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}}{\cos^2\frac{x}{2}}} = \frac{1-\tan^2\frac{x}{2}}{\frac{\sin^2\frac{x}{2}}{\cos^2\frac{x}{2}}+1} = \frac{1-\tan^2\frac{x}{2}}{\tan^2\frac{x}{2}+1} = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$$

#54: Extra Credit

Secion 6.1: Inverse Circular Functions:

$y = \sin^{-1} x \quad \text{or} \quad y = \arcsin x$

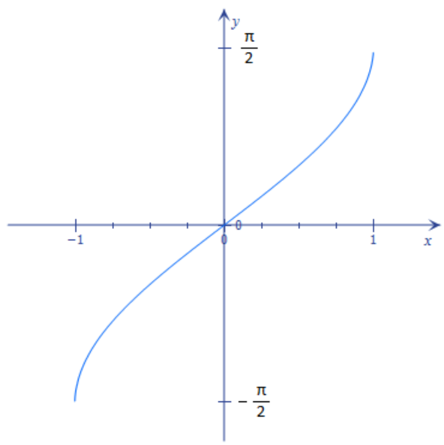
Domain: $[-1,1]$

Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Increasing and continuous on its domain

x-intercept: 0, y-intercept: 0

Symmetric with respect to the origin



$y = \cos^{-1} x \quad \text{or} \quad y = \arccos x$

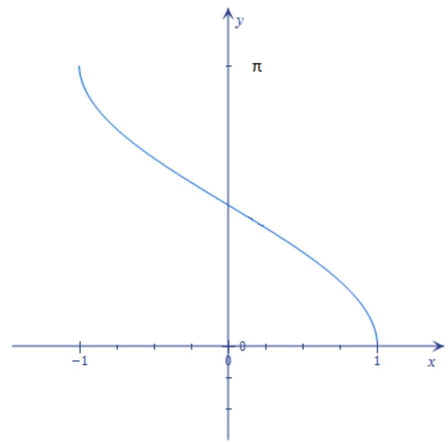
Domain: $[-1,1]$

Range: $[0, \pi]$

Decreasing and continuous on its domain

x-intercept: 1, y-intercept: $\frac{\pi}{2}$

Not Symmetric



$y = \tan^{-1} x \quad \text{or} \quad y = \arctan x$

Domain: $(-\infty, \infty)$

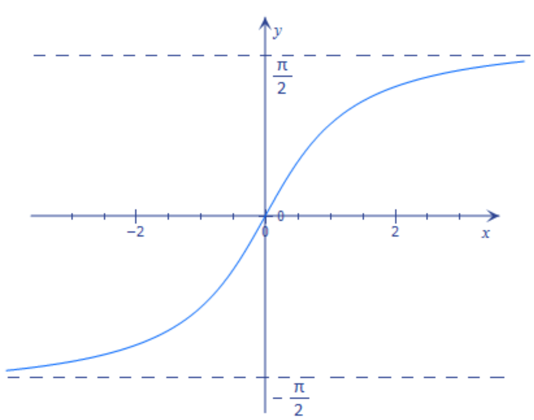
Range: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Increasing and continuous on its domain

x-intercept: 0, y-intercept: 0

Symmetric with respect to the origin

Horizontal Asymptotes at $y = -\frac{\pi}{2}, \ \frac{\pi}{2}$



$y = \cot^{-1} x \quad \text{or} \quad y = \operatorname{arccot} x$

Domain: $(-\infty, \infty)$

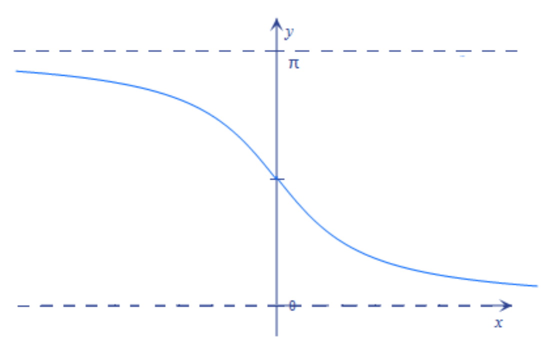
Range: $(0, \pi)$

Decreasing and continuous on its domain

x-intercept: none, y-intercept: $\frac{\pi}{2}$

Not Symmetric

Horizontal Asymptotes at $y = 0, \ \pi$



$y = \csc^{-1} x$ or $y = \text{arccsc}\, x$

Domain: $(-\infty, -1] \cup [1, \infty)$

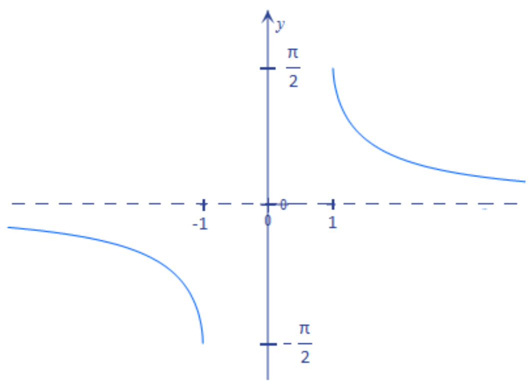
Range: $\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$

Decreasing on its domain

x-intercept: none, y-intercept: none

Symmetric with respect to the origin

Horizontal Asymptotes at $y = 0$



$y = \sec^{-1} x$ or $y = \text{arcsec}\, x$

Domain: $(-\infty, -1] \cup [1, \infty)$

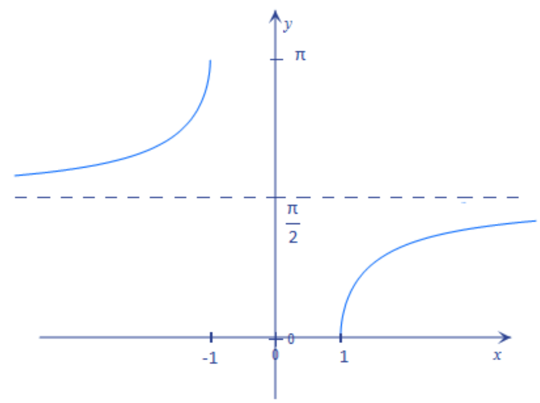
Range: $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$

Increasing on its domain

x-intercept: 1, y-intercept: none

Not Symmetric

Horizontal Asymptotes at $y = \frac{\pi}{2}$



#14: Find the exact value of each real number y if it exists. Do not use a calculator.

$$y = \sin^{-1}(-1) \quad \to \quad \sin y = -1 \quad \to \quad \sin\left(-\frac{\pi}{2}\right) = -1 \quad \to \quad y = \sin^{-1}(-1) = -\frac{\pi}{2}$$

#20: Find the exact value of each real number y if it exists. Do not use a calculator.

$$y = \tan^{-1}(-1) \quad \to \quad \tan y = -1 \quad \to \quad \tan\left(-\frac{\pi}{4}\right) = -1 \quad \to \quad y = \tan^{-1}(-1) = -\frac{\pi}{4}$$

#24: Find the exact value of each real number y if it exists. Do not use a calculator.

$$y = \cos^{-1}\left(-\frac{1}{2}\right) \quad \to \quad \cos y = -\frac{1}{2} \quad \to \quad \cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2} \quad \to \quad y = \cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$$

#40: Give the degree measure of θ if it exists. Do not use a calculator.

$$\theta = \arcsin\left(-\frac{\sqrt{2}}{2}\right) \quad \to \quad \sin\theta = -\frac{\sqrt{2}}{2} \quad \to \quad \sin(-45°) = -\frac{\sqrt{2}}{2} \quad \to \quad \theta = \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = -45°$$

#44: Give the degree measure of θ if it exists. Do not use a calculator.

$$\theta = \cot^{-1}\left(\frac{\sqrt{3}}{3}\right) \quad \to \quad \cot\theta = \frac{\sqrt{3}}{3} \quad \to \quad \cot(60°) = \frac{\sqrt{3}}{3} \quad \to \quad \theta = \cot^{-1}\left(\frac{\sqrt{3}}{3}\right) = 60°$$

#46: Give the degree measure of θ if it exists. Do not use a calculator.

$$\theta = \csc^{-1}(-1) \quad \to \quad \csc\theta = -1 \quad \to \quad \csc(-90°) = -1 \quad \to \quad \theta = \csc^{-1}(-1) = -90°$$

#57: Use a calculator to give each value in decimal degrees.

$$\theta = \tan^{-1}(-7.7828641) = -82.678329°$$

#66: Use a calculator to give each real number value.

$$y = \cot^{-1}(-36.874610)$$

$$\cot y = -36.874610 \quad \rightarrow \quad \tan y = -\frac{1}{36.874610} \quad \rightarrow \quad y = \tan^{-1}\left(-\frac{1}{36.874610}\right) = 3.1144804$$

#84: Give the exact value of each expression without using a calculator.

$$\cos\left(2\sin^{-1}\left(\frac{1}{4}\right)\right) = 1 - 2\sin^2\left(\sin^{-1}\left(\frac{1}{4}\right)\right) = 1 - 2\left(\frac{1}{4}\right)^2 = 1 - 2\left(\frac{1}{16}\right) = 1 - \frac{1}{8} = \frac{8}{8} - \frac{1}{8} = \frac{7}{8}$$

#88: Give the exact value of each expression without using a calculator.

$$\cos(2\tan^{-1}(-2))$$

Let $\theta = \tan^{-1}(-2)$, then $\tan\theta = -2$

So, $y = -2$ and $x = 1$. Thus, $r = \sqrt{(1)^2 + (-2)^2} = \sqrt{1+4} = \sqrt{5} \quad \rightarrow \quad \cos\theta = \frac{1}{\sqrt{5}}$

So, $\cos(2\tan^{-1}(-2)) = \cos 2\theta = 2\cos^2\theta - 1 = 2\left(\frac{1}{\sqrt{5}}\right)^2 - 1 = 2\left(\frac{1}{5}\right) - 1 = \frac{2}{5} - \frac{5}{5} = -\frac{3}{5}$

Section 6.2: Solving Trigonometric Equations

#14: Solve each equation for exact solutions over the interval $[0, 2\pi)$.

$$2\sec x + 1 = \sec x + 3 \quad \rightarrow \quad 2\sec x - \sec x = 3 - 1 \quad \rightarrow \quad \sec x = 2 \quad \rightarrow \quad \frac{1}{\cos x} = \frac{2}{1}$$

$$\cos x = \frac{1}{2} \quad \rightarrow \quad \cos\frac{\pi}{3} = \frac{1}{2} \text{ and } \cos\left(\frac{5\pi}{3}\right) = \frac{1}{2} \quad \rightarrow \quad x = \frac{\pi}{3}, \frac{5\pi}{3}$$

#18: Solve each equation for exact solutions over the interval $[0, 2\pi)$.

$$(\csc x + 2)(\csc x - \sqrt{2}) = 0$$

$$\csc x + 2 = 0 \quad \rightarrow \quad \csc x = -2 \quad \rightarrow \quad \frac{1}{\sin x} = -\frac{2}{1} \quad \rightarrow \quad \sin x = -\frac{1}{2}$$

$$\sin\frac{7\pi}{6} = -\frac{1}{2} \text{ and } \sin\left(\frac{11\pi}{6}\right) = -\frac{1}{2} \quad \rightarrow \quad x = \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$\csc x - \sqrt{2} = 0 \quad \rightarrow \quad \csc x = \sqrt{2} \quad \rightarrow \quad \frac{1}{\sin x} = \frac{\sqrt{2}}{1} \quad \rightarrow \quad \sin x = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} \text{ and } \sin\left(\frac{3\pi}{4}\right) = \frac{\sqrt{2}}{2} \quad \rightarrow \quad x = \frac{\pi}{4}, \frac{3\pi}{4} \quad \rightarrow \quad \text{So,} \quad x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

#24: Solve each equation for solutions over the interval $[0°, 360°)$. Give solutions to the nearest tenth as appropreate.

$(\tan\theta - 1)(\cos\theta - 1) = 0 \quad \rightarrow \quad \tan\theta - 1 = 0 \quad \rightarrow \quad \tan\theta = 1 \quad \rightarrow \quad \theta = 45°\,, 135°$

$\cos\theta - 1 = 0 \quad \rightarrow \quad \cos\theta = 1 \quad \rightarrow \quad \theta = 0° \quad \rightarrow \quad \text{So,} \quad \theta = 0°\,, 45°\,, 135°$

#30: Solve each equation for solutions over the interval $[0°, 360°)$. Give solutions to the nearest tenth as appropreate.

$\sin^2\theta\cos\theta = \cos\theta \quad \rightarrow \quad \sin^2\theta\cos\theta - \cos\theta = 0 \quad \rightarrow \quad \cos\theta\,(\sin^2\theta - 1) = 0$

$\cos\theta\,(-\cos^2\theta) = 0 \quad \rightarrow \quad -\cos^3\theta = 0 \qquad \cos^3\theta = 0 \quad \rightarrow \quad \cos\theta = 0 \quad \rightarrow \quad \theta = 90°\,, 270°$

#34: Solve each equation for solutions over the interval $[0°, 360°)$. Give solutions to the nearest tenth as appropreate.

$\cos^2\theta - \sin^2\theta = 0 \quad \rightarrow \quad \cos 2\theta = 0 \quad \rightarrow \quad 2\theta = 90°\,, 270°\,, 450°\,, 630°$

$$\theta = \frac{90°}{2}\,, \frac{270°}{2}\,, \frac{450°}{2}\,, \frac{630°}{2} \quad \rightarrow \quad \theta = 45°\,, 135°\,, 225°\,, 315°$$

#46: Solve each equation (x in radians and θ in degrees) for all exact solutions where appropreate. Round approximate answers in radians to four decimal places and approximate answers in degrees to the nearest tenth. Write answers using the least possible non-negative angle measure.

$$\cot x + \sqrt{3} = 0 \quad \rightarrow \quad \cot x = -\sqrt{3} \quad \rightarrow \quad \tan x = -\frac{1}{\sqrt{3}} = -\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$$

$$x = \left\{\frac{5\pi}{6} + n\pi\,, \text{where } n \text{ is an integer}\right\}$$

Section 6.3:

#8: Solve each equation in x for exact solutions over the interval $[0, 2\pi)$ and each equation in θ for exact solutions over the interval $[0°, 360°)$.

$$\cos 2x = -\frac{1}{2} \quad \rightarrow \quad 2x = \frac{2\pi}{3}\,, \frac{4\pi}{3}\,, \frac{8\pi}{3}\,, \frac{10\pi}{3} \quad \rightarrow \quad x = \frac{2\pi}{6}\,, \frac{4\pi}{6}\,, \frac{8\pi}{6}\,, \frac{10\pi}{6}$$

$$x = \frac{\pi}{3}\,, \frac{2\pi}{3}\,, \frac{4\pi}{3}\,, \frac{5\pi}{3}$$

#18: Solve each equation in x for exact solutions over the interval $[0, 2\pi)$ and each equation in θ for exact solutions over the interval $[0°, 360°)$.

$\cos 2x - \cos x = 0 \quad \rightarrow \quad 2\cos^2 x - 1 - \cos x = 0 \quad \rightarrow \quad 2\cos^2 x - \cos x - 1 = 0$

$(2\cos x + 1)(\cos x - 1) = 0$

$$2\cos x + 1 = 0 \quad \rightarrow \quad 2\cos x = -1 \quad \rightarrow \quad \cos x = -\frac{1}{2} \quad \rightarrow \quad x = \frac{2\pi}{3}\,, \frac{4\pi}{3}$$

$$\cos x - 1 = 0 \quad \rightarrow \quad \cos x = 1 \quad \rightarrow \quad x = 0 \quad \rightarrow \quad \text{So,} \quad x = 0\,, \frac{2\pi}{3}\,, \frac{4\pi}{3}$$

#24: Solve each equation in x for exact solutions over the interval $[0, 2\pi)$ and each equation in θ for exact solutions over the interval $[0°, 360°)$.

$$\sin x \cos x = \frac{1}{4} \quad \rightarrow \quad 2(\sin x \cos x) = 2\left(\frac{1}{4}\right) \quad \rightarrow \quad 2\sin x \cos x = \frac{1}{2} \quad \rightarrow \quad \sin 2x = \frac{1}{2}$$

$$2x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6} \quad \rightarrow \quad x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$$

#28: Solve each equation (x in radians and θ in degrees) for all exact solutions where appropreate. Round approximate answers in radians to four decimal places and approximate answers in degrees to the nearest tenth. Write answers using the least possible non-negative angle measure.

$$\sin\frac{\theta}{2} = 1 \quad \rightarrow \quad \frac{\theta}{2} = 90° + 360°n \quad \rightarrow \quad \theta = 180° + 720°n$$

#32: Solve each equation (x in radians and θ in degrees) for all exact solutions where appropreate. Round approximate answers in radians to four decimal places and approximate answers in degrees to the nearest tenth. Write answers using the least possible non-negative angle measure.

$$\cos\theta - 1 = \cos 2\theta \quad \rightarrow \quad \cos\theta - 1 = 2\cos^2\theta - 1 \quad \rightarrow \quad 2\cos^2\theta - 1 - \cos\theta + 1 = 0$$

$$2\cos^2\theta - \cos\theta = 0 \quad \rightarrow \quad \cos\theta\,(2\cos\theta - 1) = 0 \quad \rightarrow \quad \cos\theta = 0 \quad \rightarrow \quad \theta = 90°\ ,\ 270°$$

$$2\cos\theta - 1 = 0 \quad \rightarrow \quad 2\cos\theta = 1 \quad \rightarrow \quad \cos\theta = \frac{1}{2} \quad \rightarrow \quad \theta = 60°\ ,\ 300°$$

$$\theta = \{60° + 360°n\ ,\ 90° + 360°n\ ,\ 270° + 360°n\ ,\ 300° + 360°n\}$$

$$\theta = \{60° + 360°n\ ,\ 90° + 180°n\ ,\ 300° + 360°n\}$$

Section 6.4:

#8: Solve each equation for x where x is restricted to the given interval.

$$y = \frac{1}{12}\sec x, \text{ for } x \text{ in } \left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$$

$$12y = \sec x \quad \rightarrow \quad x = \operatorname{arcsec} 12y$$

#16: Solve each equation for x where x is restricted to the given interval.

$$y = \tan(2x - 1), \text{ for } x \text{ in } \left(\frac{1}{2} - \frac{\pi}{4}, \frac{1}{2} + \frac{\pi}{4}\right)$$

$$\tan^{-1} y = 2x - 1 \quad \rightarrow \quad 2x = 1 + \tan^{-1} y \quad \rightarrow \quad x = \frac{1}{2} + \frac{1}{2}\tan^{-1} y$$

#20: Solve each equation for x where x is restricted to the given interval.

$$y = 4 + 3\cos x, \text{ for } x \text{ in } (0, \pi)$$

$$y - 4 = 3\cos x \quad \rightarrow \quad \frac{y-4}{3} = \cos x \quad \rightarrow \quad x = \cos^{-1}\left(\frac{y-4}{3}\right)$$

#26: Solve each equation for exact solution.

$$6\arccos x = 5\pi \quad \rightarrow \quad \arccos x = \frac{5\pi}{6} \quad \rightarrow \quad x = \cos\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}$$

#30: Solve each equation for exact solution.

$$\arccos\left(x - \frac{\pi}{3}\right) = \frac{\pi}{6} \quad \rightarrow \quad x - \frac{\pi}{3} = \cos\frac{\pi}{6} \quad \rightarrow \quad x - \frac{\pi}{3} = \frac{\sqrt{3}}{2} \quad \rightarrow \quad x = \frac{\sqrt{3}}{2} + \frac{\pi}{3} = \frac{3\sqrt{3} + 2\pi}{6}$$

#34: Solve each equation for exact solution.

$$\cot^{-1} x = \tan^{-1}\frac{4}{3} \quad \rightarrow \quad x = \cot\left(\tan^{-1}\left(\frac{4}{3}\right)\right)$$

Let $\theta = \tan^{-1}\left(\frac{4}{3}\right)$, then the $opposite - side = 4$, and the $adjacent - side = 3$, and so,

the $hypotenuse - side = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$

Now, $x = \cos\left(tan^{-1}\left(\frac{4}{3}\right)\right) = \cos\theta = \dfrac{adjacent - side}{hypotenuse - side} = \dfrac{3}{5}$

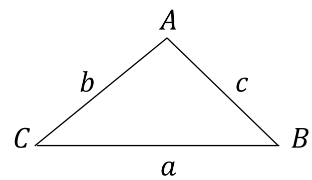
#36: Solve each equation for exact solution.

$$\sin^{-1} x + \tan^{-1}\sqrt{3} = \frac{2\pi}{3} \quad \rightarrow \quad \sin^{-1} x + \frac{\pi}{3} = \frac{2\pi}{3} \quad \rightarrow \quad \sin^{-1} x = \frac{2\pi}{3} - \frac{\pi}{3} \quad \rightarrow \quad \sin^{-1} x = \frac{\pi}{3}$$

$$x = \sin\frac{\pi}{3} \quad \rightarrow \quad x = \frac{\sqrt{3}}{2}$$

Section 7.1:

Law of Sines: Given the triangle ABC,



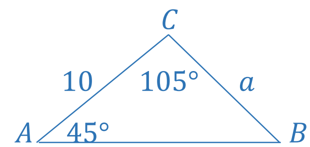
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \text{or} \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Area of a triangle: *(Refer to Page 293 & 294 for Proof)*

$$\mathcal{A} = \frac{1}{2}bc\sin A = \frac{1}{2}ac\sin B = \frac{1}{2}ab\sin C$$

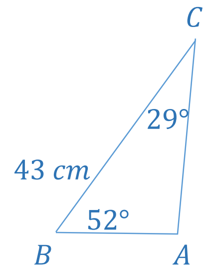
#4: Find the length of each side a. Do not use a calculator.

$B = 180° - 105° - 45° = 30°$



$$\frac{a}{\sin 45°} = \frac{10}{\sin 30°} \quad \rightarrow \quad a = \frac{10\sin 45°}{\sin 30°} = \frac{10\left(\frac{\sqrt{2}}{2}\right)}{\frac{1}{2}} = 10\sqrt{2}$$

#6: Determine the remaining sides and angles of each triangle ABC.



$A = 180° - 52° - 29° = 99°$

$$\frac{b}{\sin 52°} = \frac{43}{\sin 99°} \quad \to \quad b = \frac{43 \sin 52°}{\sin 99°} = 34\ cm$$

$$\frac{c}{\sin 29°} = \frac{43}{\sin 99°} \quad \to \quad c = \frac{43 \sin 29°}{\sin 99°} = 21\ cm$$

#10: Determine the remaining sides and angles of each triangle ABC.

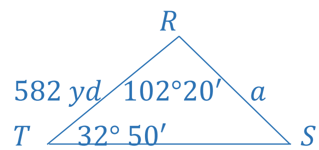
$C = 74.08°$, $B = 69.38°$, $c = 45.38\ m$

$A = 180° - 74.08° - 69.38° = 36.54°$

$$\frac{a}{\sin 36.54°} = \frac{45.38}{\sin 74.08°} \quad \to \quad a = \frac{45.38 \sin 36.54°}{\sin 74.08°} = 28.10\ m$$

$$\frac{b}{\sin 69.38°} = \frac{45.38}{\sin 74.08°} \quad \to \quad b = \frac{45.38 \sin 69.38°}{\sin 74.08°} = 44.17\ m$$

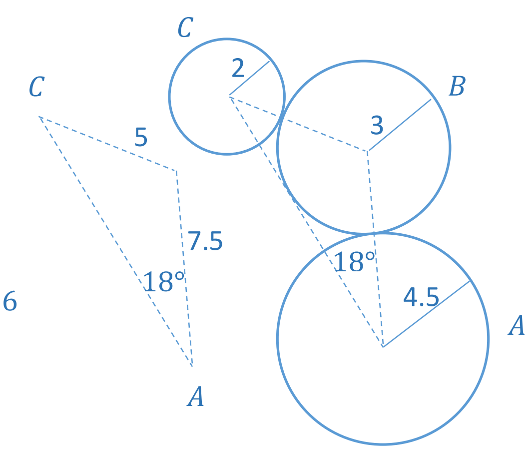
#26: To determine the distance RS across a deep canyon, Rhonda lays off a distance $TR = 582\ yd$. She then finds that $T = 32°\ 50'$ and $R = 102°\ 20'$. Find RS.



$S = 180° - 102°20' - 32°50' = 44°50'$

$$\frac{RS}{\sin 32°50'} = \frac{582}{\sin 44°50'} \quad \to \quad RS = \frac{582 \sin 32°50'}{\sin 44°50'} = 448\ yd$$

#34: Three atoms with atomic radii of 2.0, 3.0, and 4.5 are arranged as in the figure. Find the distance between the center of the atoms A and C.



$$\frac{\sin C}{7.5} = \frac{\sin 18°}{5} \quad \to \quad \sin C = \frac{7.5 \sin 18°}{5} = 0.4635$$

$C = \sin^{-1}(0.4635) = 27.6°$

$B = 180° - 18° - 27.6° = 134.4°$

$$\frac{AC}{\sin 134.4°} = \frac{5}{\sin 18°} \quad \to \quad AC = \frac{5 \sin 134.4°}{\sin 18°} = 11.56$$

#44: Find the area of each triangle ABC.

$C = 72.2°$, $b = 43.8$ ft , $a = 35.1$ ft

$$\mathcal{A} = \frac{1}{2}(35.1)(43.8) \sin 72.2° = 732\ \text{ft}^2$$

#48: Find the area of each triangle ABC.

$A = 34.97°$, $b = 35.29\ m$, $c = 28.67\ m$

$$\mathcal{A} = \frac{1}{2}(35.29)(28.67) \sin 34.97° = 289.9\ m^2$$

Section 7.2: The Ambiguous Case of the Law of Sines:

Given the angle A,

- If $\sin B > 1$, then no triangle is possible
- If $\sin B = 1$, then one right triangle is possible
- If $0 < \sin B < 1$, then
  - If $B_2 + A < 180°$ $(B_2 = 180° - B)$, then two triangles are possible, ABC and AB_2C
  - If $B_2 + A \geq 180°$ $(B_2 = 180° - B)$, then only one triangle is possible

#6: Determine the number of triangles ABC possible with the given parts.

$a = 35$, $b = 30$, $A = 40°$

$$\frac{\sin B}{30} = \frac{\sin 40°}{35} \quad \rightarrow \quad \sin B = \frac{30 \sin 40°}{35} = 0.55 \quad \rightarrow \quad B = \sin^{-1}(0.55) = 33°$$

$B_2 = 180° - 33° = 147°$ and $B_2 + A = 147° + 40° = 187° > 180°$

So, only one triangle is possible with $B = 33°$

#10: Determine the number of triangles ABC possible with the given parts.

$b = 60$, $a = 82$, $B = 100°$

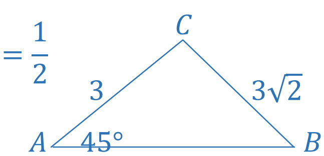
$$\frac{\sin A}{82} = \frac{\sin 100°}{60} \quad \rightarrow \quad \sin A = \frac{82 \sin 100°}{60} = 1.3 > 1$$

So, no triangle is possible.

#12: Find each angle B. Do not use a calculator.

$$\frac{\sin B}{3} = \frac{\sin 45°}{3\sqrt{2}} \quad \rightarrow \quad \sin B = \frac{3 \sin 45°}{3\sqrt{2}} = \frac{3\left(\frac{\sqrt{2}}{2}\right)}{3\sqrt{2}} = \frac{\frac{\sqrt{2}}{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{2}$$

$$B = \sin^{-1}\left(\frac{1}{2}\right) = 30° \quad \text{and} \quad C = 180° - 45° - 30° = 105°$$



#16: Find the unknown angles in triangle ABC for each triangle that exists.

$B = 48°50'$, $a = 3850$ in , $b = 4730$ in

$$\frac{\sin A}{3850} = \frac{\sin 48°50'}{4730} \rightarrow \sin A = \frac{3850 \sin 48°50'}{4730} = 0.61 \rightarrow A = \sin^{-1}(0.61) = 37.8° \approx 37°50'$$

$B = 180° - 37°50' - 48°50' = 93°20'$

#24: Solve each triangle ABC that exists.

$C = 68.5°$, $c = 258\ cm$, $b = 386\ cm$

$$\frac{\sin B}{386} = \frac{\sin 68.5°}{258} \quad \rightarrow \quad \sin B = \frac{386 \sin 68.5°}{258} = 1.39$$

So, no triangle is possible.

#28: Solve each triangle ABC that exists.

$C = 88.70°$, $b = 56.87\ m$, $c = 112.4\ m$

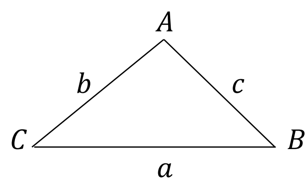
$$\frac{\sin B}{56.87} = \frac{\sin 88.70°}{112.4} \rightarrow \sin B = \frac{56.87 \sin 88.70°}{112.4} = 0.506 \rightarrow B = \sin^{-1}(0.506) = 30.4°$$

$B_2 = 180° - 30.4° = 149.6°$ But $B_2 + C = 149.6° + 88.70° = 238.3° > 180°$

So, only one triangle is possible with $B = 30.4°$. $A = 180° - 88.70° - 33.4° = 60.9°$

$$\frac{a}{\sin 60.9°} = \frac{112.4}{\sin 88.70°} \rightarrow a = \frac{112.4 \sin 60.9°}{\sin 88.70°} = 98.2\ m$$

Section 7.3:



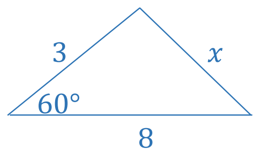
Law of Cosines: Given the triangle ABC,

$a^2 = b^2 + c^2 - 2bc \cos A$

$b^2 = a^2 + c^2 - 2ac \cos B$

$c^2 = a^2 + b^2 - 2ab \cos C$

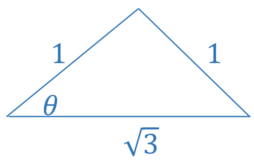
#10: Find the length of the remaining side of each triangle. Do not use a calculator.



$x^2 = 3^2 + 8^2 - 2(3)(8)\cos 60° = 9 + 64 - 48\left(\frac{1}{2}\right) = 73 - 24 = 49$

$x = \sqrt{49} = 7$

#12: Find the measure of θ in each triangle. Do not use a calculator.

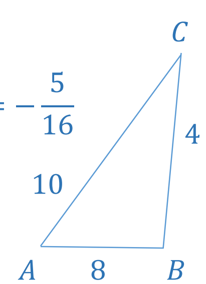


$1^2 = 1^2 + \left(\sqrt{3}\right)^2 - 2(1)\left(\sqrt{3}\right)\cos\theta = 1 + 3 - 2\sqrt{3}\cos\theta$

$1 = 4 - 2\sqrt{3}\cos\theta \rightarrow -3 = -2\sqrt{3}\cos\theta \rightarrow \frac{3}{2\sqrt{3}} = \cos\theta$

$$\cos\theta = \frac{3\sqrt{3}}{2\sqrt{3}\sqrt{3}} = \frac{3\sqrt{3}}{2(3)} = \frac{\sqrt{3}}{2} \rightarrow \theta = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = 30°$$

#16: Solve each triangle. Approximate values to the nearest tenth.



$10^2 = 4^2 + 8^2 - 2(4)(8)\cos B = 16 + 64 - 64\cos B$

$100 = 80 - 64\cos B \rightarrow 20 = -64\cos B \rightarrow \cos B = -\frac{20}{64} = -\frac{5}{16}$

$B = \cos^{-1}\left(-\frac{5}{16}\right) = 108.2°$

$4^2 = 10^2 + 8^2 - 2(10)(8)\cos A = 100 + 64 - 160\cos A$

$16 = 164 - 160\cos A \rightarrow -148 = -160\cos A \rightarrow \cos A = \frac{-148}{-160} = \frac{37}{40}$

$A = \cos^{-1}\left(\frac{37}{40}\right) = 22.3°$ and $C = 180° - 22.3° - 108.2° = 49.5°$

#20: Solve each triangle.

$C = 28.3°$, $b = 5.71$ in , $a = 4.21$ in

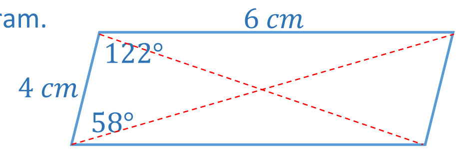
$$c^2 = (4.21)^2 + (5.71)^2 - 2(4.21)(5.71)\cos 28.3° = 17.7241 + 32.6041 - 48.0782(0.88048)$$

$$c^2 = 7.9963 \quad \rightarrow \quad c = 2.83 \text{ in}$$

$$\frac{\sin A}{4.21} = \frac{\sin 28.3°}{2.83} \quad \rightarrow \quad \sin A = \frac{4.21 \sin 28.3°}{2.83} = 0.7053 \quad \rightarrow \quad A = \sin^{-1}(0.7053) = 44.9°$$

$$B = 180° - 44.9° - 28.3° = 106.8°$$

#42: The sides of a parallelogram are 4.0 cm and 6.0 cm. One angle is $58°$ while another is $122°$. Find the lengths of the diagonals of the parallelogram.



Let l be the longer diagonal,

and s be the shorter diagonal.

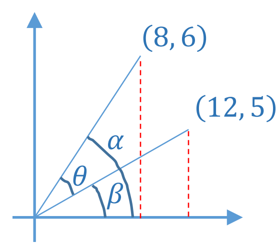
$$l^2 = 6^2 + 4^2 - 2(6)(4)\cos 122° = 36 + 16 - 48(-0.52992) = 52 + 25.43612 = 77.43612$$

$$l = \sqrt{77.43612} = 8.8\ cm$$

$$s^2 = 6^2 + 4^2 - 2(6)(4)\cos 58° = 36 + 16 - 48(0.52992) = 52 - 25.43612 = 26.56388$$

$$s = \sqrt{26.56388} = 5.2\ cm$$

#62: Find the measure of each angle θ to two decimal places.



$$\theta = |\alpha - \beta|$$

$$\tan\alpha = \frac{6}{8} = \frac{3}{4} \quad \rightarrow \quad \alpha = \tan^{-1}\left(\frac{3}{4}\right) = 36.87°$$

$$\tan\beta = \frac{5}{12} \quad \rightarrow \quad \beta = \tan^{-1}\left(\frac{5}{12}\right) = 22.62°$$

$$\theta = 36.87° - 22.62° = 14.25°$$

Section 8.1:

Complex Number: A complex number is of the form $a + bi$, where a and b are any real number and i is $\sqrt{-1}$. Conjugate of $a + bi$ is $a - bi$.

Note: In the complex number $a + bi$, a is the real part of the number, and b is the imaginary part.

Powers of i:

$$i^2 = \left(\sqrt{-1}\right)^2 = -1$$

$$i^3 = i^2 \times i = -1 \times i = -i$$

$$i^4 = i^2 \times i^2 = (-1) \times (-1) = 1$$

$$i^5 = i^4 \times i = 1 \times i = i$$

Algebraic Operations over Complex Numbers:

Addition: $(a+bi)+(c+di)=(a+c)+(b+d)i$

Subtraction: $(a+bi)-(c+di)=(a-c)+(b-d)i$

Multiplication:

$$(a+bi)(c+di)=a(c+di)+bi(c+di)=ac+adi+bci+bdi^2$$
$$=ac+(ad+bc)i+bd(-1)=(ac-bd)+(ad+bc)i$$

Division:

$$\frac{a+bi}{c+di}=\frac{(a+bi)(c-di)}{(c+di)(c-di)}=\frac{ac-adi+bci-bdi^2}{c^2-cdi+cdi-d^2i^2}=\frac{(ac+bd)+(bc-ad)i}{c^2+d^2}$$
$$=\left(\frac{ac+bd}{c^2+d^2}\right)+\left(\frac{bc-ad}{c^2+d^2}\right)i$$

#18: Write each number as the product of a real number and i.

$\sqrt{-36}=\sqrt{36}\sqrt{-1}=6i$

#26: Solve each quadratic equation and express all nonreal complex solutions in terms of i.

$x^2=-36 \quad \rightarrow \quad x=\pm\sqrt{-36}=\pm 6i$

#30: Solve each quadratic equation and express all nonreal complex solutions in terms of i.

$2x^2+3x=-2 \quad \rightarrow \quad 2x^2+3x+2=0$

$$x=\frac{-(3)\pm\sqrt{(3)^2-4(2)(2)}}{2(2)}=\frac{\left(-3\pm\sqrt{9-16}\right)}{4}=\frac{-3\pm\sqrt{-7}}{4}=-\frac{3}{4}\pm\frac{\sqrt{7}}{4}i$$

#38: Multiply or divide, as indicated. Simplify each answer.

$\sqrt{-17}\sqrt{-17}=\left(i\sqrt{17}\right)\left(i\sqrt{17}\right)=i^2\left(\sqrt{17}\right)^2=(-1)(17)=-17$

#50: Write each number in standard form $a+bi$.

$$\frac{-9-\sqrt{-18}}{3}=\frac{-9-3i\sqrt{2}}{3}=-\frac{9}{3}-\frac{3i\sqrt{2}}{3}=-3-i\sqrt{2}$$

#58: Find each sum or difference. Write the answer in standard form.

$(-3+2i)-(-4+2i)=-3+2i+4-2i=1$

#68: Find each product. Write the answer in standard form.

$(2+i)^2=(2+i)(2+i)=4+2i+2i+i^2=4+4i-1=3+4i$

#74: Find each product. Write the answer in standard form.

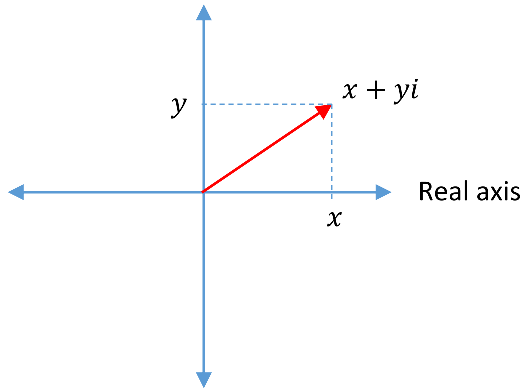
$\left(\sqrt{2}-4i\right)\left(\sqrt{2}+4i\right)=\left(\sqrt{2}\right)^2+4i\sqrt{2}-4i\sqrt{2}-16i^2=2+\left((-16)\times(-1)\right)=2+16=18$

#86: Find each quotient. Write the answer in standard form $a + bi$.

$$\frac{-3+4i}{2-i} = \frac{(-3+4i)(2+i)}{(2-i)(2+i)} = \frac{-6-3i+8i+4i^2}{4+2i-2i-i^2} = \frac{-6+5i+4(-1)}{4-(-1)} = \frac{-6+5i-4}{4+1}$$

$$= \frac{-10+5i}{5} = -\frac{10}{5} + \frac{5}{5}i = -2 + i$$

Section 8.2: Complex Plane:



Trigonometric (Polar) form of a complex number:

For a complex number $x + yi$, $r(\cos\theta + i\sin\theta)$ or $r\, cis\, \theta$ is the trigonometric form, where $x = r\cos\theta$ and $y = r\sin\theta$. Note that $r = \sqrt{x^2 + y^2}$ and $\tan\theta = \frac{y}{x}$ $(x \neq 0)$.

#26: Write each complex number in rectangular form.

$$4(\cos 60° + i\sin 60°) = 4\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = 2 + 2i\sqrt{3}$$

#32: Write each complex number in rectangular form.

$$2\, cis\, 30° = 2(\cos 30° + i\sin 30°) = 2\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) = \sqrt{3} + i$$

#42: Write each complex number in trigonometric form $r(\cos\theta + i\sin\theta)$, with θ in the interval $[0°, 360°)$.

$4\sqrt{3} + 4i$

$x = 4\sqrt{3}$ and $y = 4$

$$r = \sqrt{\left(4\sqrt{3}\right)^2 + (4)^2} = \sqrt{(16 \times 3) + 16} = \sqrt{48 + 16} = \sqrt{64} = 8$$

$$\tan\theta = \frac{4}{4\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} \quad \rightarrow \quad \theta = 30° \quad \rightarrow \quad 4\sqrt{3} + 4i = 8\, cis\, 30°$$

#48: Write each complex number in trigonometric form $r(\cos\theta + i\sin\theta)$, with θ in the interval $[0°, 360°)$.

$-2i$

$-2i$ is two units down on the i-axis on the graph, which implies that $\theta = 270°$.

$r = \sqrt{0^2 + (-2)^2} = \sqrt{4} = 2 \quad \rightarrow \quad -2i = 2\ cis\ 270°$

Section 8.3:

Product Theorem: If $r_1(\cos\theta_1 + i\sin\theta_1)$ and $r_2(\cos\theta_2 + i\sin\theta_2)$ are any two complex numbers, then,

$$\big(r_1(\cos\theta_1 + i\sin\theta_1)\big)\cdot\big(r_2(\cos\theta_2 + i\sin\theta_2)\big) = r_1 r_2(\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)).$$

That is,

$$(r_1\ cis\ \theta_1)\cdot(r_2\ cis\ \theta_2) = r_1 r_2\ cis\ (\theta_1+\theta_2).$$

Quotient Theorem: If $r_1(\cos\theta_1 + i\sin\theta_1)$ and $r_2(\cos\theta_2 + i\sin\theta_2)$ are any two complex numbers, then,

$$\frac{r_1(\cos\theta_1 + i\sin\theta_1)}{r_2(\cos\theta_2 + i\sin\theta_2)} = \frac{r_1}{r_2}(\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2)).$$

That is,

$$\frac{r_1\ cis\ \theta_1}{r_2\ cis\ \theta_2} = r_1 r_2\ cis\ (\theta_1-\theta_2).$$

#8: Find each product and write it in rectangular form.

$$[8(\cos 210° + i\sin 210°)][2(\cos 330° + i\sin 330°)]$$
$$= (8)(2)[\cos(210°+330°) + i\sin(210°+330°)] = 16(\cos 540° + i\sin 540°)$$
$$= 16(\cos 180° + i\sin 180°) = 16(-1+0i) = -16$$

#14: Find each quotient and write it in rectangular form.

$$\frac{24(\cos 150° + i\sin 150°)}{2(\cos 30° + i\sin 30°)} = \frac{24}{2}(\cos(150°-30°) + i\sin(150°-30°))$$
$$= 12(\cos 120° + i\sin 120°) = 12\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = -6 + 6i\sqrt{3}$$

#18: Find each quotient and write it in rectangular form.

$$\frac{16\ cis\ 310°}{8\ cis\ 70°} = \frac{16}{8}\ cis\ (310°-70°) = 2\ cis\ 240° = 2(\cos 240° + i\sin 240°) = 2\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$$
$$= -1 - i\sqrt{3}$$

#20: Find each quotient and write it in rectangular form.

$$\frac{2i}{-1-i\sqrt{3}}$$

For the numerator: $2i$ is two units up on the i-axis. Thus, $r = 2$ and $\theta = 90°$.

For the denominator: $r = \sqrt{(-1)^2 + \left(-\sqrt{3}\right)^2} = \sqrt{1+3} = \sqrt{4} = 2$ and

$$\tan\theta = \frac{-\sqrt{3}}{-1} = \sqrt{3} \quad \rightarrow \quad \theta = \tan^{-1}\left(\sqrt{3}\right) = 60° \quad \text{and since } -1-i\sqrt{3}$$

is on the third quadrant, $\theta = 240°$.

$$\frac{2i}{-1-i\sqrt{3}} = \frac{2(\cos 90° + i\sin 90°)}{2(\cos 240° + i\sin 240°)} = 1(\cos(90° - 240°) + i\sin(90° - 240°))$$

$$= \cos(-150°) + i\sin(-150°) = \cos 210° + i\sin 210° = -\frac{\sqrt{3}}{2} - \frac{1}{2}i$$

#24: Find each quotient and write it in rectangular form.

$$\frac{-3\sqrt{2}+3i\sqrt{6}}{\sqrt{6}+i\sqrt{2}}$$

For the numerator: $r = \sqrt{\left(-3\sqrt{2}\right)^2 + \left(3\sqrt{6}\right)^2} = \sqrt{9(2)+9(6)} = \sqrt{18+54} = \sqrt{72} = 6\sqrt{2}$

$$\text{and} \quad \tan\theta = \frac{3\sqrt{6}}{-3\sqrt{2}} = -\sqrt{3} \quad \rightarrow \quad \theta = \tan^{-1}\left(-\sqrt{3}\right) = -60°$$

and since $-3\sqrt{2}+3i\sqrt{6}$ is on the third quadrant, $\theta = 120°$.

For the denominator: $r = \sqrt{\left(\sqrt{6}\right)^2 + \left(\sqrt{2}\right)^2} = \sqrt{6+2} = \sqrt{8} = 2\sqrt{2}$ and

$$\tan\theta = \frac{\sqrt{2}}{\sqrt{6}} = \frac{1}{\sqrt{3}} \quad \rightarrow \quad \theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30°$$

$$\frac{-3\sqrt{2}+3i\sqrt{6}}{\sqrt{6}+i\sqrt{2}} = \frac{6\sqrt{2}(\cos 120° + i\sin 120°)}{2\sqrt{2}(\cos 30° + i\sin 30°)} = \frac{6\sqrt{2}}{2\sqrt{2}}(\cos(120° - 30°) + i\sin(120° - 30°))$$

$$= 3(\cos 90° + i\sin 90°) = 3(0+1i) = 3i$$

Section 8.4:

DeMoivre's Theorem: If $r\ cis\ \theta$ is a complex number and n is a real number, then

$$(r\ cis\ \theta)^n = r^n\ cis\ (n\theta).$$

n^{th} root: If $(a+bi)^n = x+yi$, then n^{th} root of $x+yi$ is $a+bi$. That is,

$$\sqrt[n]{x+yi} = \sqrt[n]{(a+bi)^n} = a+bi$$

n^{th} root Theorem: If n is a positive integer, r is a positive real number, and θ is in degrees, then $r\ cis\ \theta$ has exactly n roots:

$$\sqrt[n]{r}\ cis\ \alpha, \quad \text{where} \quad \alpha = \frac{\theta + 360°k}{n} \quad \text{for } k = 0, 1, 2, \ldots, n-1$$

#2: Find each power. Write each answer in rectangular form.

$$[2(\cos 135° + i \sin 135°)]^4 = 2^4\left[\cos\big(4(135°)\big) + i \sin\big(4(135°)\big)\right] = 16(\cos 540° + i \sin 540°)$$

$$= 16(\cos 180° + i \sin 180°) = 16(-1+0i) = -16$$

#10: Find each power. Write each answer in rectangular form.

$$\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right)^8$$

$$r = \sqrt{\left(\frac{\sqrt{2}}{2}\right)^2 + \left(-\frac{\sqrt{2}}{2}\right)^2} = \sqrt{\frac{2}{4}+\frac{2}{4}} = \sqrt{\frac{4}{4}} = \sqrt{1} = 1$$

$$\tan\theta = \frac{-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = -1 \quad \rightarrow \quad \theta = \tan^{-1}(-1) = -45°$$

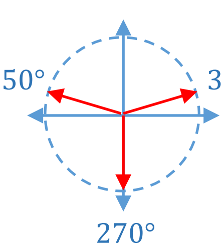
$$\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right)^8 = \big(1\ cis\ (-45°)\big)^8 = cis\ \big(8(-45°)\big) = cis\ (-360°) = cis\ 0° = \cos 0° + i \sin 0°$$

$$= 1 + 0i = 1$$

#18: (a) Find all cube roots of each complex number. Leave answers in trigonometric form. (b) Graph each cube root as a vector in the complex plane.

$$\sqrt[3]{27i}$$

$27i$ is 27 units up on the i-axis. So, $r = 27$ and $\theta = 90°$.

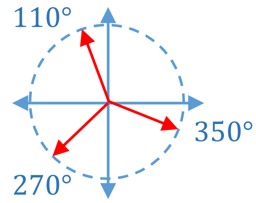


$$\sqrt[3]{27i} = \sqrt[3]{27\ cis\ 90°}$$

$$= \sqrt[3]{27}\ cis\ \frac{90°}{3}, \quad \sqrt[3]{27}\ cis\ \frac{90° + 1(360°)}{3}, \quad \sqrt[3]{27}\ cis\ \frac{90° + 2(360°)}{3}$$

$$= 3\ cis\ 30°, \quad 3\ cis\ 150°, \quad 3\ cis\ 270°$$

<u>#24:</u> (a) Find all cube roots of each complex number. Leave answers in trigonometric form. (b) Graph each cube root as a vector in the complex plane.



$$\sqrt[3]{\sqrt{3}-i}$$

$$r=\sqrt{\left(\sqrt{3}\right)^2+(-1)^2}=\sqrt{3+1}=\sqrt{4}=2$$

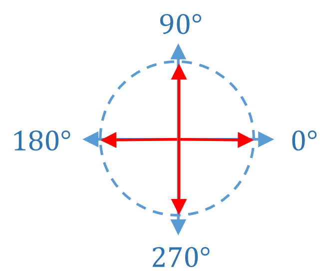
$$\tan\theta=\frac{-1}{\sqrt{3}}=-\frac{1}{\sqrt{3}}\quad\rightarrow\quad\theta=\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)=-30°=330°$$

$$\sqrt[3]{\sqrt{3}-i}=\sqrt[3]{2\ cis\ 330°}$$

$$=\sqrt[3]{2}\ cis\frac{330°}{3}\ ,\ \sqrt[3]{2}\ cis\frac{330°+1(360°)}{3}\ ,\ \sqrt[3]{2}\ cis\frac{330°+2(360°)}{3}$$

$$=\sqrt[3]{2}\ cis\ 110°\ ,\ \sqrt[3]{2}\ cis\ 230°\ ,\ \sqrt[3]{2}\ cis\ 350°$$

<u>#26:</u> Find and graph all fourth root of 1.



$$\sqrt[4]{1}$$

1 is one unit to the right on the real-axis (x-axis).

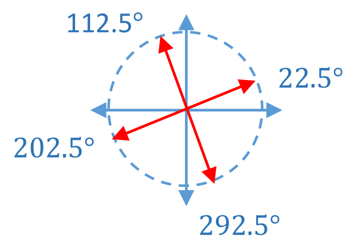
So, $r=1$ and $\theta=0°$.

$$\sqrt[4]{1}=\sqrt[4]{1\ cis\ 0°}$$

$$=\sqrt[4]{1}\ cis\frac{0°}{4}\ ,\ \sqrt[4]{1}\ cis\frac{0°+1(360°)}{4}\ ,\ \sqrt[4]{1}\ cis\frac{0°+2(360°)}{4}\ ,\ \sqrt[4]{1}\ cis\frac{0°+3(360°)}{4}$$

$$=cis\ 0°\ ,\quad cis\ 90°\ ,\quad cis\ 180°\ ,\quad cis\ 270°$$

<u>#30:</u> Find and graph all fourth root of i.



$$\sqrt[4]{i}$$

i is one unit up on the i-axis.

So, $r=1$ and $\theta=90°$.

$$\sqrt[4]{i}=\sqrt[4]{1\ cis\ 90°}$$

$$=\sqrt[4]{1}\ cis\frac{90°}{4}\ ,\ \sqrt[4]{1}\ cis\frac{90°+1(360°)}{4}\ ,\ \sqrt[4]{1}\ cis\frac{90°+2(360°)}{4}\ ,\ \sqrt[4]{1}\ cis\frac{90°+3(360°)}{4}$$

$$=cis\ 22.5°\ ,\quad cis\ 112.5°\ ,\quad cis\ 202.5°\ ,\quad cis\ 292.5°$$

<u>#38:</u> Find all complex number solutions of each equation. Leave answers in trigonometric form.

$$x^4+16=0\quad\rightarrow\quad x^4=-16$$

$$x=\sqrt[4]{-16}=\sqrt[4]{16\ cis\ 180°}=2\ cis\ 45°\ ,\ 2\ cis\ 135°\ ,\ 2\ cis\ 225°\ ,\ 2\ cis\ 315°$$

#42: Find all complex number solutions of each equation. Leave answers in trigonometric form.

$$x^4 - \left(8 + 8i\sqrt{3}\right) = 0 \quad \rightarrow \quad x^4 = 8 + 8i\sqrt{3} \quad \rightarrow \quad x = \sqrt[4]{8 + 8i\sqrt{3}}$$

$$r = \sqrt{(8)^2 + \left(8\sqrt{3}\right)^2} = \sqrt{64 + 64(3)} = \sqrt{64 + 192} = \sqrt{256} = 16$$

$$\tan\theta = \frac{8\sqrt{3}}{8} = \sqrt{3} \quad \rightarrow \quad \theta = \tan^{-1}\left(\sqrt{3}\right) = 60°$$

$$x = \sqrt[4]{16\, cis\, 60°}$$

$$= \sqrt[4]{16}\, cis \frac{60°}{4}\ ,\ \sqrt[4]{16}\, cis \frac{60° + 360°}{4}\ ,\ \sqrt[4]{16}\, cis \frac{60° + 360°}{4}\ ,\ \sqrt[4]{16}\, cis \frac{60° + 360°}{4}$$

$$= 2\, cis\, 15°\ ,\ \sqrt[4]{16}\, cis\, 105°\ ,\ \sqrt[4]{16}\, cis\, 195°\ ,\ \sqrt[4]{16}\, cis\, 285°$$

Extra Credit: Solve for x.

$$x^4 - 4x^2 - 5 = 0$$