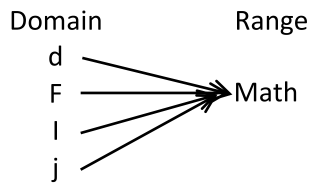
Section1.1 Worksheet	**Date:_______________ Name:____________________________**
Concept	**Meaning in words and/or examples as required**
1. Relation	
2. Function	
3. Domain of a function	
4. Range of a function	
5. All the different ways to represent a function (give examples you make up not the ones that appear in the book)	
6. One-to-one function	
7. Give an example of a relation that is not a function	
8. Given an example of a function that is not one-to-one.	
Difficulties encountered in the section:	

Exercises 1.1

1. Determine the following. (For graphs, the direction of the relation is from x to y.)
   i. Is the relation a function or not.
   ii. If the relation is a function, is it one-to-one?
   iii. Domain and Range of the relation.

A.

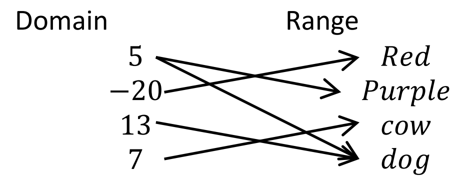


○ Function	○ One-to-One
○ Not a Function	○ Not One-to-One

Domain:

Range:

B.



○ Function	○ One-to-One
○ Not a Function	○ Not One-to-One

Domain:

Range:

C.

$$\{(-1,b),(2,8),(5,-3),(2,-3)\}$$

○ Function	○ One-to-One
○ Not a Function	○ Not One-to-One

Domain:

Range:

D.

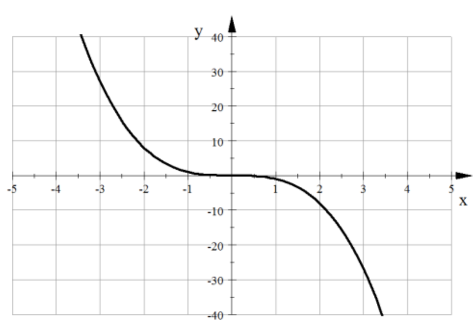
$$\{(2,-1),(2,1),(c,2),(d,4)\}$$

○ Function	○ One-to-One
○ Not a Function	○ Not One-to-One

Domain:

Range:

E.

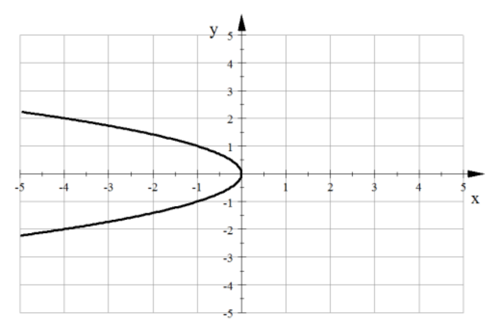


○ Function	○ One-to-One
○ Not a Function	○ Not One-to-One

Domain:

Range:

F.



○ Function	○ One-to-One
○ Not a Function	○ Not One-to-One

Domain:

Range:

G. $f(x) = 3\sqrt{x-1}$

○ Function	○ One-to-One
○ Not a Function	○ Not One-to-One

Domain:

Range:

H. $x^2 + y^2 = 9$ (Imagine the graph!)

○ Function	○ One-to-One
○ Not a Function	○ Not One-to-One

Domain:

Range:

I. $Temp(t) = \frac{5}{9}(t - 32)$

○ Function	○ One-to-One
○ Not a Function	○ Not One-to-One

Domain:

Range:

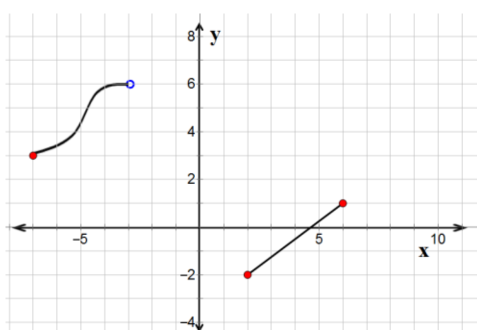
J. $y^5 = x$

○ Function	○ One-to-One
○ Not a Function	○ Not One-to-One

Domain:

Range:

K.

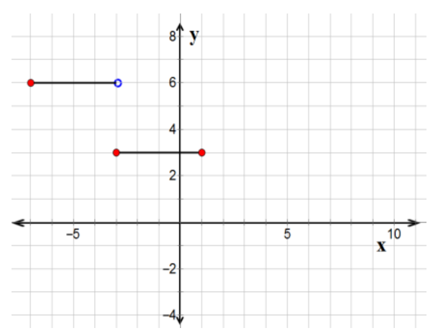


○ Function	○ One-to-One
○ Not a Function	○ Not One-to-One

Domain:

Range:

L.

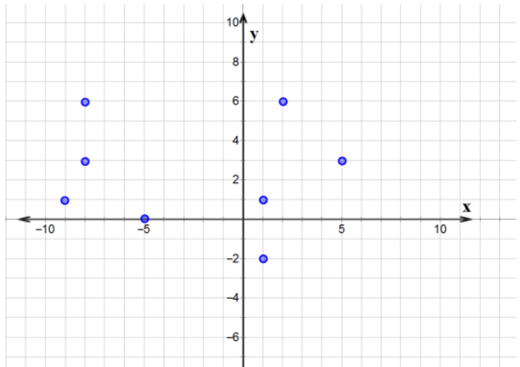


○ Function	○ One-to-One
○ Not a Function	○ Not One-to-One

Domain:

Range:

M.

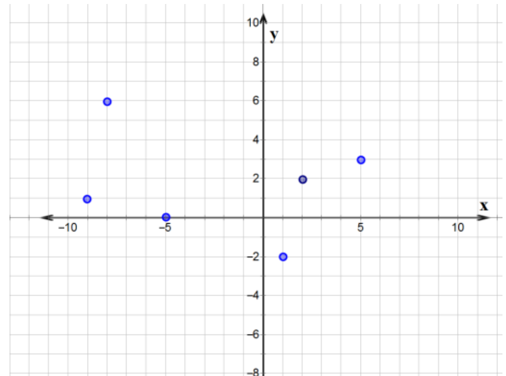


○ Function	○ One-to-One
○ Not a Function	○ Not One-to-One

Domain:

Range:

N.



○ Function	○ One-to-One
○ Not a Function	○ Not One-to-One

Domain:

Range:

2. Draw the graph of $y = f(x)$ where the domain is $-3 \leq x \leq 5$ and the Range is $-1 \leq y \leq 3$ and $f(0) = 2$ and the function is one-to-one.

3. Draw the graph of $y = g(x)$ where the domain is $-3 \leq x \leq -1 \cup 1 \leq x \leq 3$ and the Range is $y \in \{-2, 3\}$ and $f(2) = -2$. Is your function one-to-one?

6. For each if the following equations, determine whether:

In column 1, is y a function of x?	In column 2, is x a function of y?
A. $y^2 = 2x$ ☐ Function ☐ Not a function	B. $7x - 5y = 10$ ☐ Function ☐ Not a function
C. $2x - 7y = 9$ ☐ Function ☐ Not a function	D. $x = 3y^2$ ☐ Function ☐ Not a function
E. $6x + \lvert y \rvert = 3$ ☐ Function ☐ Not a function	F. $x^3 = y$ ☐ Function ☐ Not a function
G. $x^2 + 2 = y$ ☐ Function ☐ Not a function	H. $x^2y^2 = 9$ ☐ Function ☐ Not a function
I. $x^2 + 2y^2 = 8$ ☐ Function ☐ Not a function	J. $xy = 5$ ☐ Function ☐ Not a function

7. The function h is defined by the following rule: $h(x) = 4x + 5$. Complete the following table.

x	$h(x)$
-4	
-2	
2	
4	
5	

8. The functions f and g are defined as follows:

$f(x) = -3x + 2, \quad g(x) = 3x^3 + 5.$

Find $f(3)$ and $g(-3)$. Simplify your answers as much as possible.

$f(3) =$

$g(-3) =$

9. The function f is defined as follows: $f(x) = \frac{4x}{3x-15}$.

Find $f(4)$. Simplify your answer as much as possible.

10. The function h is defined as follows:

$h(x) = \frac{x^2-3x-10}{x^2-14x+45}$.

Find $h(6)$. Simplify your answer as much as possible.

11. Fill the table using the function rule $f(x) = \sqrt{x} + 6$. Simplify your answers as much as possible.

x	$f(x)$
-9	
0	
$1/4$	
4	
25	

12. The function h is defined as follows:

$h(x) = \sqrt[3]{x - 1}$.

Evaluate each output value and simplify your answers as much as possible.

$h(9) =$

$h(126) =$

$h(-26) =$

13. Simplify the expressions below for $f(x) = 2x^2 + 3x - 5$.

$f(-6) =$

$f(a) =$

$f(a+h) =$

14. With $M(x)$ and $F(x)$ being the mother and father of person x, try to evaluate:

$M(yourself) =$

$F\big(M(yourself)\big) =$

$M(F(yourself) =$

$M\Big(F\big(M(yourself)\big)\Big) =$

15. Describe one or more functions and what the relations mean through using each of four methods: by a formula; by a table; by a graph; and through a sentence or two of words. State the domain and range of your function. (You could do this all for a single function or you can come up with different functions for each method.)

16. Create a function $y = f(x)$ that satisfies the following criteria.
    a. Is One-to-one
    b. Has domain all real numbers
    c. $f(2) = -1$

Questions about your function-
- i) What is the domain of $f(x)$?
- ii) What is the range of $f(x)$?

17. Create a function $y = f(x)$ that satisfies the following criteria.

   a. Has domain $[-2, 1]$
   b. For at least two x's in the domain of f, $f(x) = 4$

Questions about your function-

i) What is the domain of $f(x)$?
ii) What is the range of $f(x)$?
iii) Is your function one-to-one?

18. Create a function $y = f(x)$ that satisfies the following criteria.

   a. Is One-to-one
   b. Has domain $[-2, 1] \cup [3,10]$
   c. $f(-2) = 1$
   d. $f(3) = 0$
   e. $f(0) = 5$
   f. For at least one x in the domain of f, $f(x) = -4$

Questions about your function-

i) What is the domain of $f(x)$?
ii) What is the range of $f(x)$?

Section1.2 Worksheet	**Date:_______________ Name:____________________________**
Concept	**Meaning in words and/or examples as required**
1. **One-to-One function**	
2. **Inverse function**	
3. **Relationship of domain and range of inverse functions to the original function**	
4. **Polynomial functions**	
5. **Rational functions**	
6. **Exponential functions**	
7. **Logarithmic function**	
8. **Describe the difference between the exponential and logarithmic functions and the relationship between them.**	
Difficulties encountered in the section:	

Exercises 1.2

1. The function f is defined by the following rule: $f(x) = 8^x$. Find $f(x)$ for each x-value in the table.

x	$f(x) = 8^x$
-2	
-1	
0	
1	
2	

2. The tables below give exponential functions in the form $y = a^x$. Write the equation for each of the functions.

x	$f(x)$
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4

x	$g(x)$
-2	$\frac{1}{e^2}$
-1	$\frac{1}{e}$
0	1
1	e
2	e^2

$f(x) =$________ $g(x) =$_________

3. Evaluate each logarithmic function below.
   a. $\log_2 16 =$
   b. $\log_5 25 =$
   c. $\log_2 \frac{1}{8} =$
   d. $\log_3 \sqrt{3} =$
   e. $\log_6 1 =$
   f. $\log_9 3 =$
   g. $\log_{10} 0.00001 =$

4. Convert each logarithmic expression to its exponential format and then evaluate the unknown value x.
   a. $\log_2 32 = x$
   b. $\log_{16} x = 1.5$
   c. $\log_5 x = -3$
   d. $\log_6 x = 3$
   e. $\log_x 16 = 2$
   f. $\log_x 25 = 1/2$

5. The function f is defined as follows: $f(x) = \begin{cases} \frac{3}{4}x + 2 & if\ x \neq -1 \\ 2 & if\ x = -1 \end{cases}$ Find the following. $f(-3) =$ $f(-1) =$ $f(-2) =$	6. The function g is defined by $g(x) = \frac{3x-4}{x+5}$. Find the following: $g(-3) =$ $g(x+5) =$ The domain of g.

7. $f(x) = \sqrt{x-2}$

a. Find $f(6)$
b. Find $f(2)$
c. Find domain and range of the function.
d. Is f one-to-one?
e. If you answered yes to part d, find the inverse function.

8. Find the domain and range of all the relations below either in interval notation or set notation as appropriate.

a) $f(x) = \frac{1}{x-4}$

Domain:

Range:

b) $g(x) = \sqrt{x-4}$

Domain:

Range:

c) $h(x) = \sqrt{4-x}$

Domain:

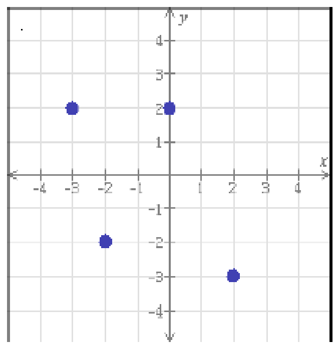
Range:

d) $r(x) = \sqrt{x-4} + 2$

Domain:

Range:

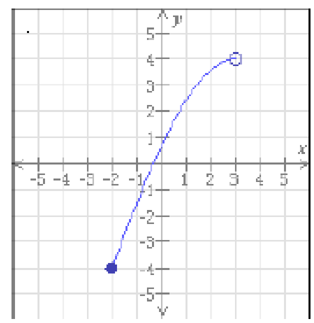
e)



Domain of T:

Range of T:

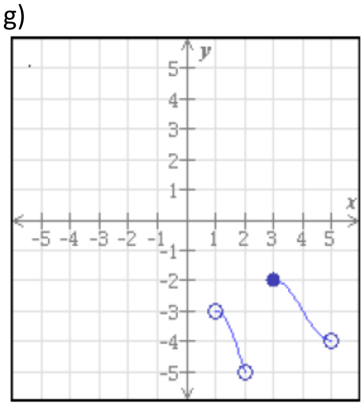
f)



Domain of h:

Range of h:

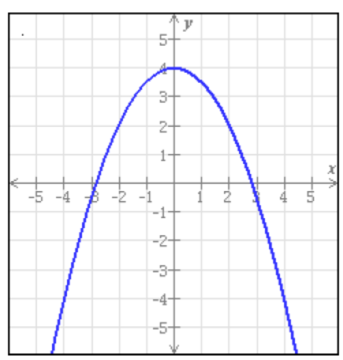
g)



Domain of S:

Range of S:

h)



Domain of f:

Range of f:

Evaluate $f(0) =$____________

Find the values of x, for which $f(x) = 0$

______ , _______

i)

$$g(x) = \begin{cases} 2 & if\ x = 1 \\ -2 & if\ \ x < 1 \\ 3 & if\ x > 1 \end{cases}$$

Domain of g:

Range of g:

j)

$$p(x) = \begin{cases} 3\sqrt{x} & if\ x > 4 \\ 2x - 1 & if\ \ x < -4 \end{cases}$$

Domain of p:

Range of p:

9. A species of bacteria doubles every 30 minutes at room temperature. If you initially started with 30000 bacteria at $t = 0$:
   a. Find the number of bacteria after 2 hours.

   b. Find the number of bacteria at 4 hours.

   c. See if you can come up with a formula for an exponential function that describes this where t is the input in minutes and y is the output being the number of bacteria present at time t.

10. The radioactive substance Uranium-240 has a half-life of 14 hours. The amount $A(t)$ of a sample of Uranium-240 remaining in grams after t hours is given by the exponential function $A(t) = 3000\left(\frac{1}{2}\right)^{\frac{t}{14}}$.
    a. Find the initial amount in the sample.

    b. Find the amount remaining after 30 hours. Round your answer to the nearest gram.

    c. Determine how many grams remain after two years.

11. Suppose Rahul places $\$2000$ in an account that pays 5% interest compounded each year. Assume that no withdrawals are made from the account. Round to the nearest cent.
    a. Find the amount in the account at the end of 1 year.

    b. Find the amount in the account at the end of 2 years.

    c. Try to find a formula for the account value after t years.
       $A(t) =$

12. A car is purchased for $\$26{,}500$. After each year, the resale value decreases by 15%.
    a. What will the resale value be after 5 years? Round your answer to the nearest dollar.

    b. Try to find a formula for the value after t years.
    $V(t) =$

13. Devon deposited $4000 into an account with 4.4% interest compounded quarterly.
    a. Assuming the no withdrawals are made, how much will she have in the account after 10 years? Round your answer to the nearest dollar.

    b. Try to find a formula for the value after t years.
    $V(t) =$

14. At the beginning of a population study, a city had 360,000 people. Each year since, the population has grown by 2.6%. Let t be the number of years since the start of the study. Let $P(t)$ represent the city's population at time t.
    a. Predict the population 5 years after the start of the study.

    b. Try to write an exponential function for the population $P(t)$, t years after the study starts.

15. Compare the function $f(x) = 3^x$ and $g(x) = 50x^2$ by completing parts a. and b.
    a. Fill in the table below. Note that the table is already filled in for $x = 6$

x	$f(x) = 3^x$	$g(x) = 50x^2$
6	$3^6 = 729$	$50(6^2) = 1800$
7		
8		
9		
10		

    b. For all x, $x \geq 8$ please check which of the following are true statements
    - ☐ $f(x) \geq g(x)$
    - ☐ $f(x) = g(x)$
    - ☐ $f(x) \leq g(x)$

    c. Explain your answer in part b.

16. An organic farmer raises free-range chickens and will deliver eggs to your home for a fee. The farmer sells a one-year contract for delivery of 25 to 100 dozen eggs to your home. The annual cost in dollars for the service is given by $y = C(n) = 25 + 4n$, where n is the number of dozen eggs you will use over the year.

Domain:	Range
a. State the domain variable and what it stands for.	a. State the Range variable and what it stands for.
b. What is the domain here, assuming eggs are available only in full dozen quantities?	b. What is the Range here, assuming eggs are available only in full dozen quantities?
c. Determine the value $y = C(50)$ and what it means.	c. Determine the value of n if your contract payment was \$325.

17. A construction crew needs to pave a road that is 204 miles long. The crew paves 9 miles of the road each day. The length, L (in miles), that remains to be paved after d days is given by the following function. $L(d) = 204 - 9d$.

Answer the following questions.

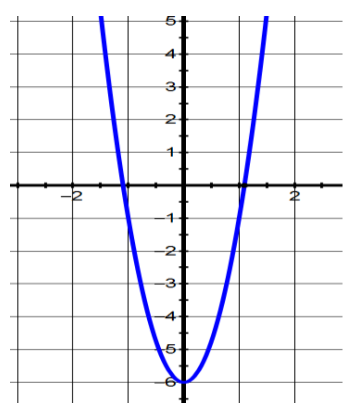
a. How many miles of the road the crew have left to pave after 13 days?

b. After how many days will there be only 114 miles left to pave?

18. Find the difference quotient $\frac{f(x+h)-f(x)}{h}$ where $h \neq 0$ for the function $f(x) = 5x^2 - 6$. The graph of this function is given as well. Simplify your answer as much as possible.

$\frac{f(x+h)-f(x)}{h} =$

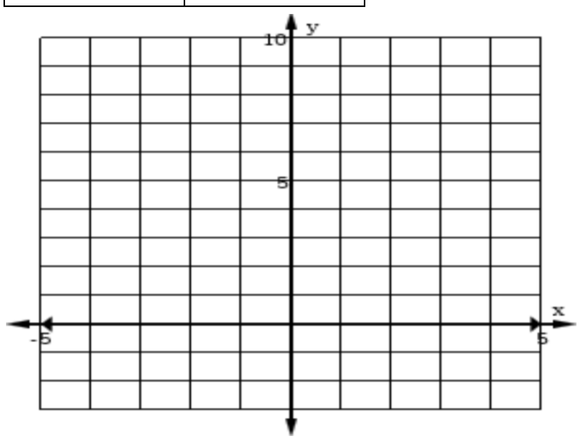
Try to explain what the quotient represents.



19. Sketch the graph of the functions and relations below. Explain clearly how you decided the graph was the shape you drew. Can you determine the domain and range of the functions and relations that you graphed based on the graphs.

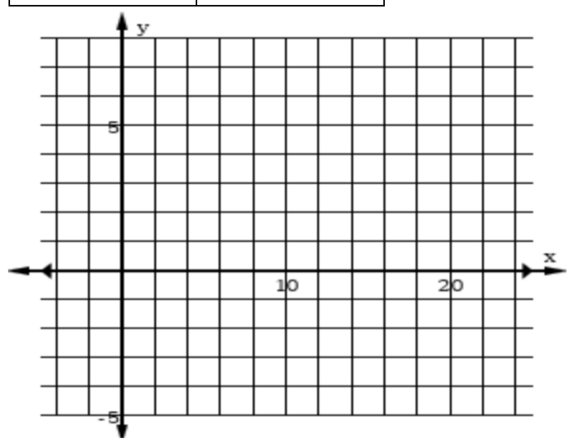
a. $y = 2^x$

x	y



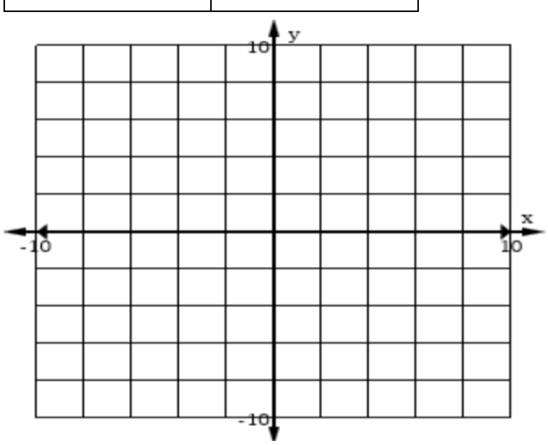
b. $y = \sqrt{x}$

x	y



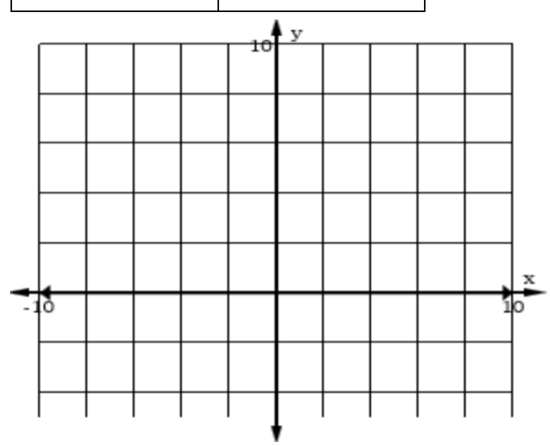
c. $y = x^3$

x	y



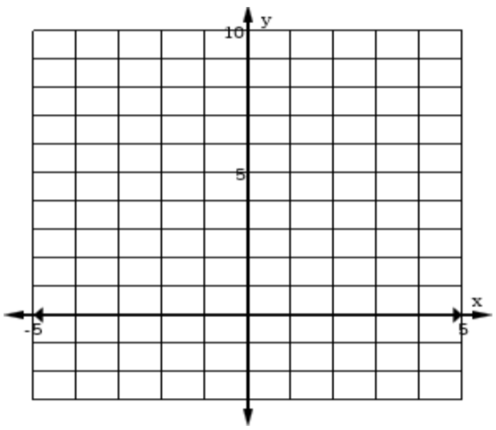
d. $y = x^4$

x	y



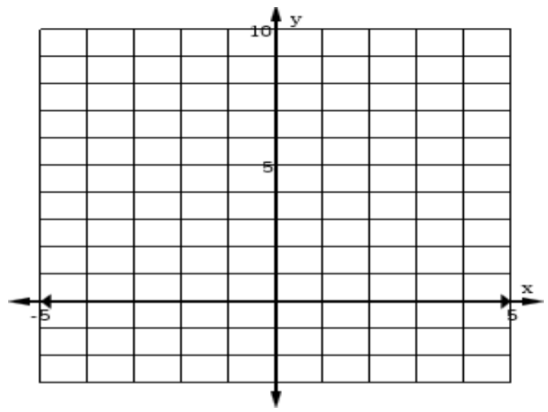
e. $y = 4^x$

x	y



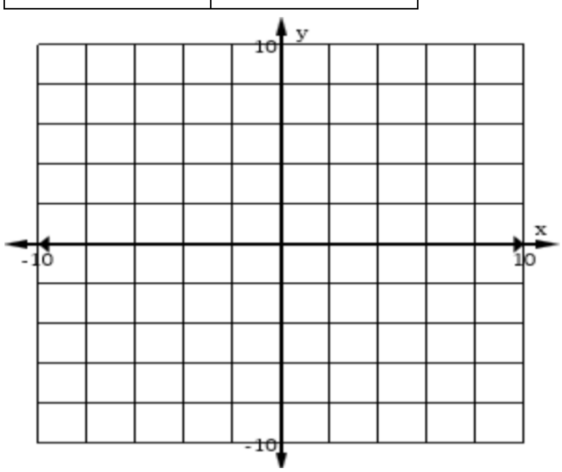
f. $y = \left(\frac{1}{4}\right)^x$

x	y



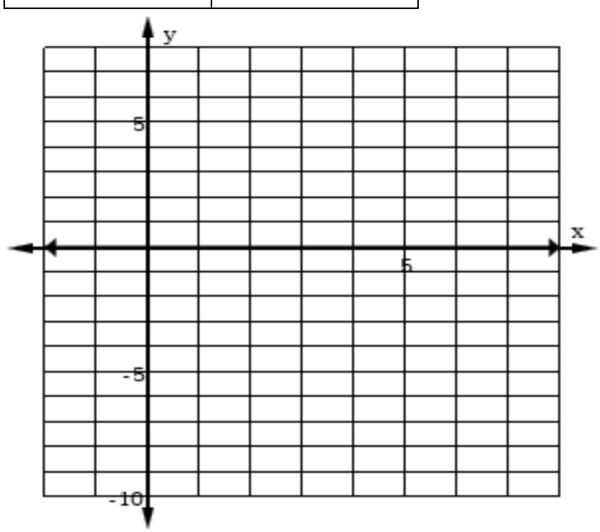
g. $y = \frac{3}{x}$

x	y



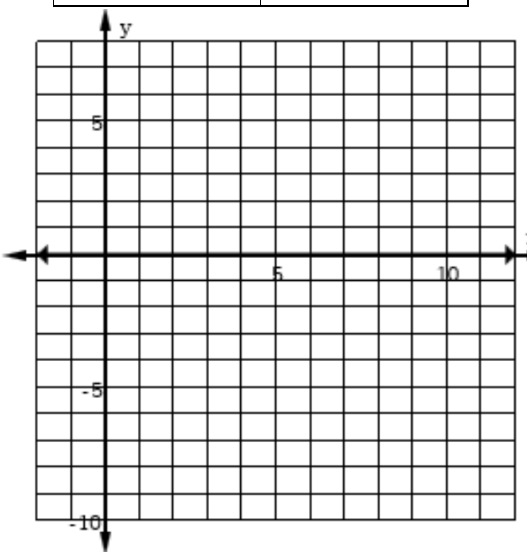
h. $y = \log_4(x)$

x	y
4	
16	
1/4	
1	
1/16	



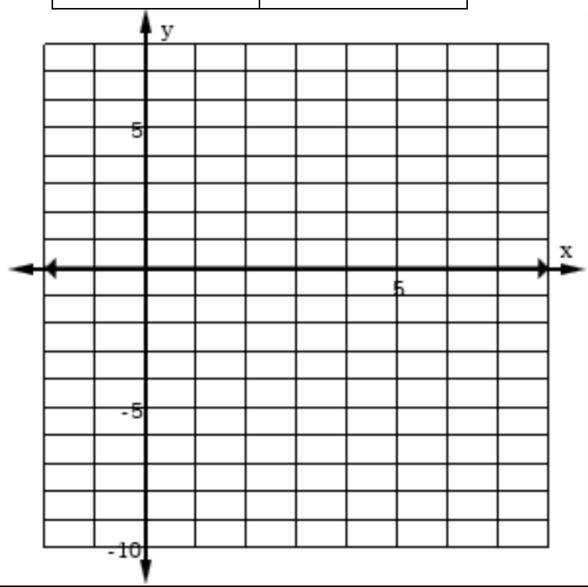
i. $y = \log_{10}(x)$

x	y

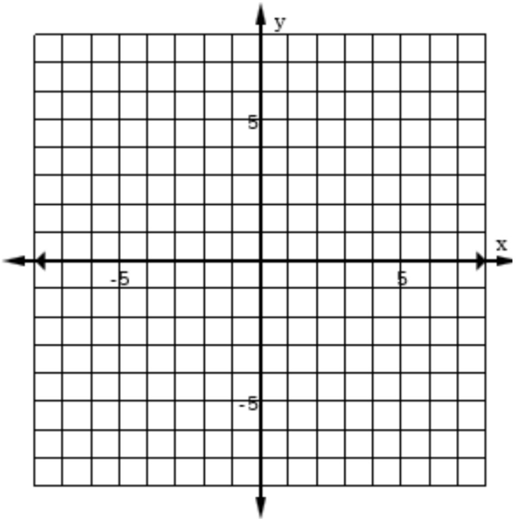


j. $y = log_3(x)$

x	y
3	
9	
1/3	
1/9	
1	

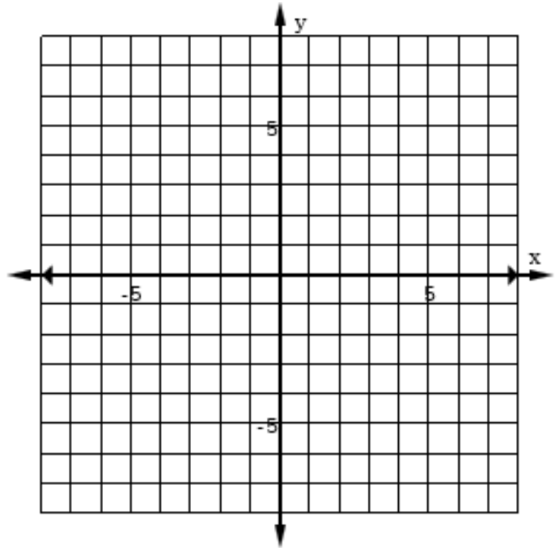


k. $y = \begin{cases} 2, & if\ x > 1 \\ -2, & if\ x < 1 \\ 0, & if\ x = 1 \end{cases}$



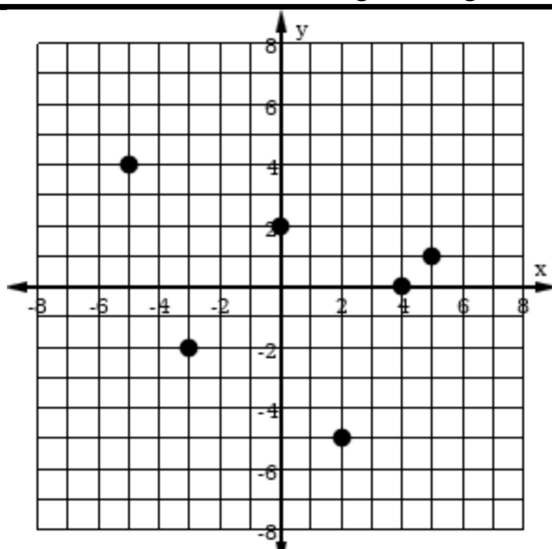
x	y

l. $y = \begin{cases} x + 2, & if\ x > 2 \\ x^2, & if\ x \leq 2 \end{cases}$

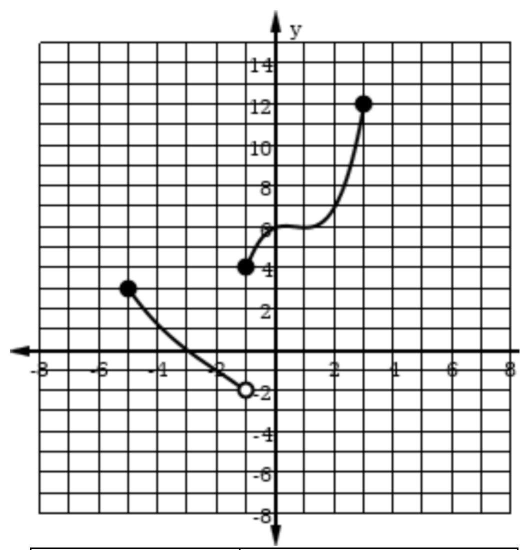


x	y

20. Evaluate the following for the given one-to-one functions below.



a. $f(-5)$	b. $f^{-1}(-5)$
c. $f(4)$	d. $f^{-1}(4)$



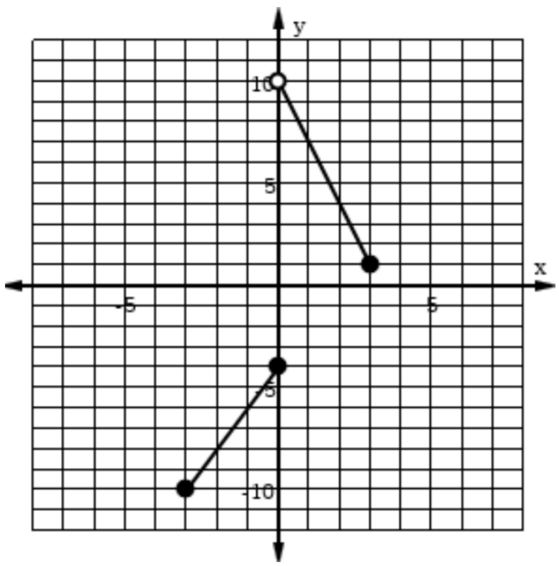
a. $f(2)$	b. $f^{-1}(0)$
c. $f(-5)$	d. $f^{-1}(-2)$

21. Find the inverses of the following one-to-one functions. Then find the domains and ranges of the functions and their inverses.			
a. $f(x) = \frac{7x+1}{2x-1}$		b. $g(x) = \sqrt{2x-1}$ for $x \geq \frac{1}{2}$	
Domain of f	Range of f^{-1}	Domain of g	Range of g^{-1}
Domain of f^{-1}	Range of f	Domain of g^{-1}	Range of g
c. $h(x) = 4^x$		d. $p(x) = log_4 x$	
Domain of h	Range of h^{-1}	Domain of p	Range of p^{-1}
Domain of h^{-1}	Range of h	Domain of p^{-1}	Range of p

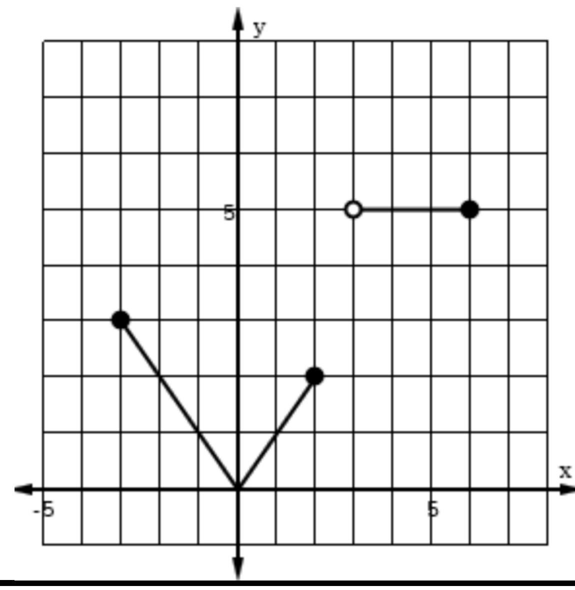
22. Given that $f^{-1}(x) = 3x^3 + 5$ find the formula for $y = f(x)$.

23. Obtain the piece-wise formula for the functions whose graphs are given below. If the functions below are one-to-one, please also find the formulas for their inverse functions and sketch their graphs.

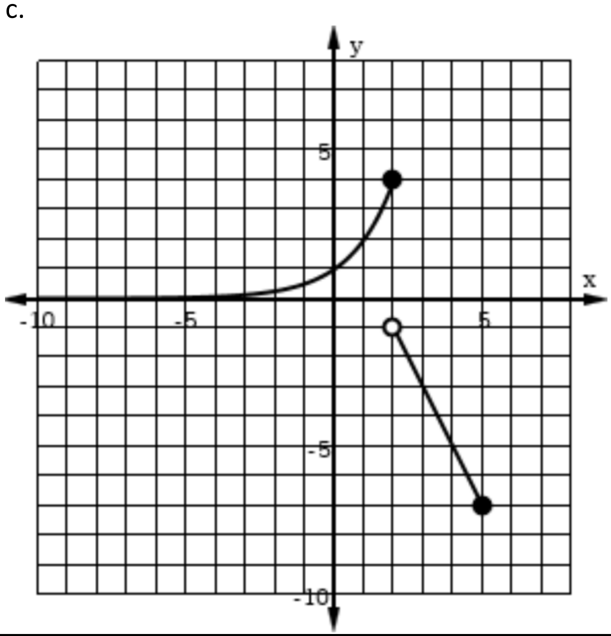
a.



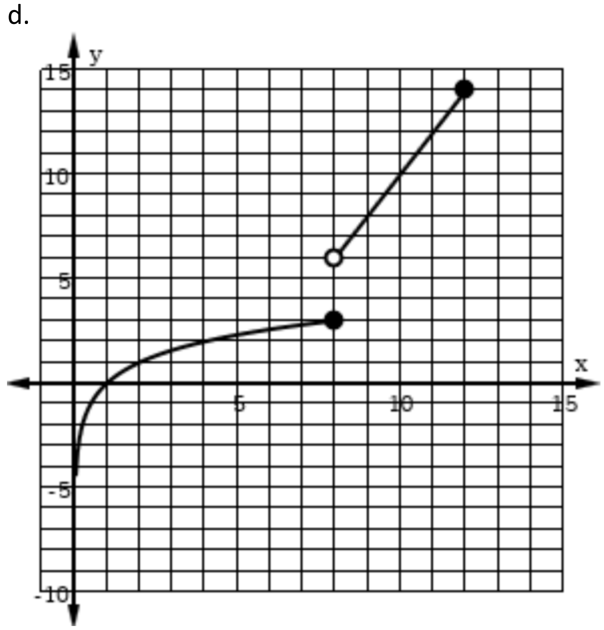
b.



c.



d.

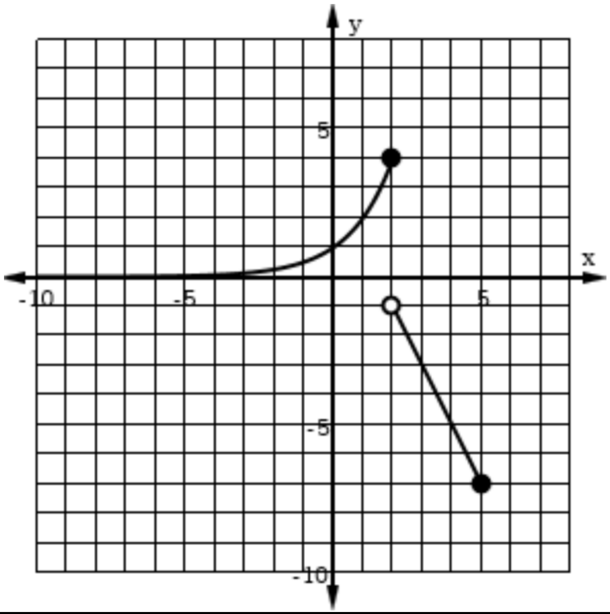


24. Find the original function whose inverse function is given below.

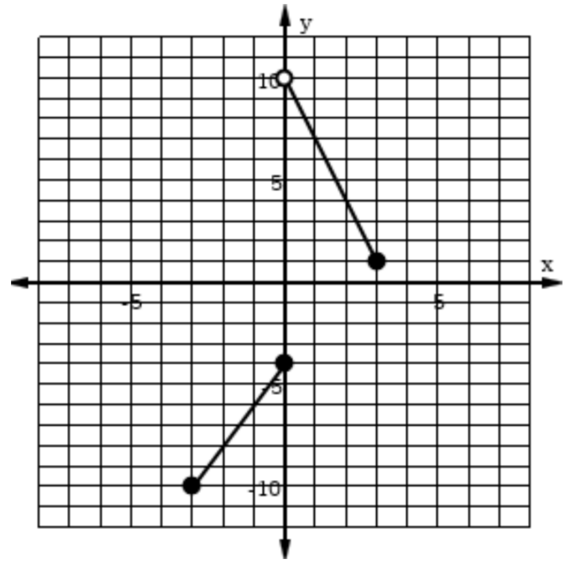
a. $f^{-1}(x) = \frac{7x+1}{2x-1}$

b. $g^{-1}(x) = \sqrt{1-x}$, for $x \leq 1$

c.



d.



25. Create functions with the given properties below and then answer the questions.

A. Create a function $y = f(x)$ that satisfies the following criteria.
   a. Is One-to-one
   b. Has domain $[-2, 1] \cup [3,10]$
   c. $f(-2) = 1$
   d. $f(3) = 0$
   e. $f(0) = 5$
   f. For at least one x in the domain of f, $f(x) = -4$

Questions about your function-

iii) What is the domain of $f(x)$?
iv) What is the range of $f(x)$?
v) What are the x-intercepts of $y = f(x)$?
vi) What is the y-intercepts of $y = f(x)$?
vii) What is $f^{-1}(x)$?

Worksheet Section1.3a	**Date:_______________ Name:____________________________**
Concept	**Meaning in words and/or examples as required**
1. Domain of Logarithmic Functions	
2. Domain of Exponential Functions	
3. Properties of logarithms	
4. Transcendental Number e	
5. Compounding Formula	
6. Continuous Compounding Formula	
Difficulties encountered in the section:	

Exercises 1.3a

1. List two uses each for exponential functions and logarithmic functions.
2. How are exponential and logarithmic functions related to each other?
3. How do we find domains of logarithmic functions?
4. In your own words describe how to change logarithmic equations into an exponential equation.
5. In your own words describe how to change exponential equations into logarithmic equations.
6. Rewrite the exponential equations in logarithmic form and logarithmic equations in exponential form. If possible simplify your answers.

Exponential Equation	Logarithmic Equation
$e^x = 5$	
$2^{x+1} = 8$	
	$log_2(x) = -1$
	$\log(x+1) = 2$
	$\ln(x+1) = 3$
$5^{1-x} = 3$	
	$log_{\frac{1}{2}}(x) = -3$

Exponential Equation	Logarithmic Equation
	$log_{\frac{1}{3}}(81) = -4$
$5^2 = 25$	
	$log_3\left(\frac{1}{9}\right) = -2$
$e^3 = x$	
	$\ln(e^2) = 2$
$10^{-2} = 0.01$	
	$log1000 = 3$

7. Find the domain of the functions below.

A. $f(x) = \log(x+1)$

Domain:

B. $g(x) = log\left(\frac{3}{x-4}\right)$

Domain:

C. $h(x) = 3^{x-1}$

Domain:

D. $k(x) = ln(1-x)$

Domain:

8. Properties of logarithms: Please fill out the missing values.

A. $log_a x + log_a y =$ ________________

B. $log_a x - log_a y =$ ____________________

C. $log_a(xy) =$ ____________________

D. $log_a\left(\frac{x}{y}\right) =$ ________________

E. $log_a(x^n) =$ ____________________

F. $nlog_a(x) =$ ____________________

G. Change of base formula: Write in terms of the common or natural logarithm function.

$$log_a(x) = \_\_\_\_\_\_\_\_$$

9. Use properties of logarithms to do the problems below.

F. Fill in the missing values to make the statement a true statement.

vi. $log_5 8 - log_5(\_\_\_\_\_) = log_5 4$

vii. $log_2 3 + log_2 5 = log_2(\_\_\_\_\_)$

viii. $3log_7 2 = log_7(\_\_\_\_)$

ix. $log_5 49 = (\_\_\_\_\_)log_5 7$

x. $\frac{ln5}{ln4} = log_4(\_\_\_\_\_)$

G. Expand the following. Each logarithm in your answer should involve only one variable. Assume that all variables are positive.

VI. $\log(x^3y^2) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

VII. $\log_2\left(\frac{x^3y^2}{\sqrt{z}}\right) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

VIII. $\log\left(\frac{x^3}{\sqrt{z^5y}}\right) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

IX. $\ln((4+x)(x-2)) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

X. $\ln\left(\frac{x^5\sqrt[3]{y}}{3z}\right) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

H. Write the following as one term.

i. $4log_2 x + 2log_2 y = \_\_\_\_\_\_\_\_\_\_$

ii. $\frac{1}{3}logx - 2logy + 3logz = \_\_\_\_\_\_\_\_\_\_\_$

I. Compute the values below exactly

i. $log_2(8) = \_\_\_\_\_\_\_\_\_\_$

ii. $\log(0.000001) = \_\_\_\_\_\_\_\_\_\_$

iii. $\ln(e^5) = \_\_\_\_\_\_\_\_\_$

iv. $\ln(\sqrt{e}) = \_\_\_\_\_\_\_$

v. $log_5\left(\frac{1}{25}\right) = \_\_\_\_\_\_\_\_$

J. Use a scientific calculator to evaluate the following, and round your answers to 2 digits.

i. $\frac{20000}{\log(1+\frac{0.02}{5})} = \_\_\_\_\_\_\_\_\_\_\_$

ii. $\frac{log3}{log2} = \_\_\_\_\_\_\_\_\_$

iii. $\frac{\ln(0.5)}{3} = \_\_\_\_\_\_\_\_\_$

iv. $log_3 11 = \_\_\_\_\_\_\_\_\_\_$

10. State a formula for the value of each compound interest account after t years.
    a. An account starts with \$5000 and earns 6.2%/yr interest compounded annually.

    b. An investment is initially worth \$20,000 and earns 6% interest/yr compounded quarterly.

    c. An investment of \$2000 earns interest at 7.5%/yr compounded continuously.

    d. A credit card debt has an interest rate of 18%/yr and is compounded monthly. Find the function for the debt after t years if it is initially at \$10,000 and no payments are made.

    e. You purchase a pair of shoes for \$200 with your credit card but make no payments. The interest rate then gets moved to 24% (2%/month). State a formula for the balance due after n years and state list the balance due at $\frac{1}{2}$, 1, 2, 3 and 10 years.

11. Population growth is modeled with exponential functions in several equivalent formats where P_0 is the population at $t = 0$.
    - If the percentage growth over a one-year period is known, then we treat it like annual compounding and get $P(t) = P_0(1 + r)^t$.
    - If the continuous growth rate is known, then we use $P(t) = P_0 e^{rt}$ just like continuous compounding interest problems.
    - If the time it takes a population to double is known, then we write the exponential function in terms of the base-two exponential as $P(t) = P_0 \cdot 2^{\frac{t}{T_d}}$

    State formulas for each population growth problem below.

    a. Wisconsin's population was 5,000,000 at $t = 0$ in 2010 and is growing 0.35% each year.

    b. The U.S. population was 310,000,000 in 2010 at $t = 0$ and has a continuous growth rate of 0.97% per year.

    c. A blood infecting bacteria grows rapidly and has a doubling time of 30 minutes. Find the exponential function that gives the number of bacteria t minutes after a puncture wound introduces 50 of the microbes into the blood stream.

12. Exponential decay models are modeled much like growth problems above but with the r being negative.
    - If the percentage decline over a one-year period is known, then we treat it like annual compounding and get $P(t) = P_0(1-r)^t$. E.g., 10% decline per year in the value of a car leads to the value function of a car originally worth $25,000 as $V(t) = 25000(0.9)^t$
    - If the continuous decay rate is known, then we use $P(t) = P_0e^{-rt}$ much like continuous compounding interest problems. E.g., A 20 gram sample of radioactive tritium declines continuously by 5.6% per year. The Amount after t years is $A(t) = 20e^{-0.056t}$.
    - If the time it takes for a quantity to decline to half its original amount (T_h), then we write the exponential function in terms of the base-$\left(\frac{1}{2}\right)$ exponential as $A(t) = A_0 \cdot (\frac{1}{2})^{\frac{t}{T_h}}$

    State formulas for each exponential decay problem below.

    a. Ukraine's population was 42,690,000 in 2016 and decreasing by 8.4% each year. State $P(t) =$_________________ where t is years after 2016.

    b. The light intensity under water in Lake Superior decreases continuously at 3% per foot of depth. If the surface intensity $(x = 0)$ is $180\frac{W}{m^2}$. State the intensity function as a function of the number of feet below the surface. $I(x) =$
    Also predict the light intensity at a depth of 100 feet.

    c. Radioactive Carbon-14 occurs naturally in all plant-derived materials. Once the plant dies, the carbon-14 atoms in the plant are slowly converted back to nitrogen-14. The half-life for this conversion is about 5,730 years. Give the exponential function that describes the amount of carbon-14 (t) years after the plant died if it started with 5 trillion atoms of carbon-14.
    Also predict how many atoms would remain after 200,000 years.

13. Consider the population growth function for the world $P(t) = 7.4e^{0.011t}$ where $t = 0$ corresponds to 2015.
    a. What is the continuous growth rate?
    b. Determine by what percent the population increases over the course of one year. Then give a formula for $P(t)$ in the "annual compounding" format.
    c. By guess and check or other methods, try to use $P(t) = 7.4e^{0.011t}$ to determine how many years it takes for the population to double and give the formula for $P(t)$ in the "doubling time" format.

14. Evaluate the measures of acidity, sound intensity and earthquake magnitude using the appropriate logarithmic definitions of these measures.
    a. Find the pH of blueberries which have a $[H^+]$ concentration of 0.00076 $\frac{mol}{liter}$
    b. Most foods have pH less than 7 and are thus acidic. Tofu has a pH of 7.2. Write down the pH statement for this and convert it to its exponential form to obtain the $[H^+]$ concentration of tofu.
    c. A very loud speaker outputs sound that you can feel a 5 feet from the speaker. The sound intensity there is at $\frac{5\ watts}{m^2}$. Determine the decibel level of this sound.
    d. A weed whipper produces noise at 95 dB for the operator. Determine the sound intensity level in $\frac{Watts}{m^2}$.
    e. Oklahoma has seen experienced unprecedented earthquakes since the advent of Fracking for natural gas. In 2015 there were more than 30 of magnitude 4 or greater on the Richter scale. Determine the seismograph reading in mm of a magnitude 4 and the most recent magnitude 9 quake that occurred 6 years ago in Japan. What is the ratio of the amount of movement between a level 4 and 9 earthquake?
15. Create an exponential function with $f(0) = 100$ and $f(1) = 200$. Do you think your function is unique? Explain your answer.
16. Create a logarithmic function with $f(1) = 100$, and $f(2) = 200$. Do you think your function is unique? Explain your answer.

Section1.3b Worksheet	Date:_______________ Name:____________________________

Concept	**Meaning in words and/or examples as required**
What is a sequence?	
What is an arithmetic sequence?	
What is a geometric sequence?	
Give an example of a sequence that is neither an arithmetic sequence nor a geometric sequence.	
What is a Fibonacci sequence?	
What is the golden ratio?	
Give an example of an arithmetic sequence.	
Give an example of an geometric sequence	
Given an example of a sequence that is neither arithmetic nor geometric.	
Difficulties encountered in the section:	

Exercises 1.3b

1. For the sequences below determine if they are arithmetic or geometric. Then find the formula for the a_n and then fill in the missing terms column with value of that term.

Sequence	Type	$f(n) = a_n = n^{th}$ term
$\{22, 26, 30, 34, \dots.\}$	☐ Arithmetic ☐ Geometric ☐ Neither	
$\{6, 12, 24, \dots\}$	☐ Arithmetic ☐ Geometric ☐ Neither	
$\{9, 13, 17, \dots.\}$	☐ Arithmetic ☐ Geometric ☐ Neither	
$\{7, 11, 19, \dots.\}$	☐ Arithmetic ☐ Geometric ☐ Neither	

2. Find the first 4 terms of the sequences given below.

$a_n = n^{th}$ term of the sequence	First term	Second term	Third term	Fourth term	
$a_n = 4\left(\frac{1}{3}\right)^{2n+1}$, $n = 0,1,2,\dots$					☐ Arithmetic ☐ Geometric ☐ Neither
$a_n = 3n + 5, n = 3,4,5,\dots$					☐ Arithmetic ☐ Geometric ☐ Neither
$a_n = \frac{2n}{n+3}, n = 3,4,5,\dots$					☐ Arithmetic ☐ Geometric ☐ Neither
$a_n = \frac{(-1)^n}{n}, n = 1,2,\dots$					☐ Arithmetic ☐ Geometric ☐ Neither

3. For a given arithmetic sequence, the 82^{nd} element, $a_{82} = -370$ and the 6^{th} element, $a_6 = 10$. Find the linear function for a_n and find the 33^{rd} element a_{33}.

4. For a given geometric sequence, the 7^{th} element $a_7 = \frac{23}{25}$ and the 10^{th} element $a_{10} = 115$. Find a formula for this exponential sequence and evaluate the14^{th} element a_{14}.

5. Find the 20th and 21st elements of the Fibonacci sequence. Also compute the ratio $\frac{a_{21}}{a_{20}}$ to show that this ratio is very near in value to the Golden Ratio.

6. Consider the sequence of continued fractions: $a_1 = 1$, $a_2 = 1 + \frac{1}{1}$, $a_3 = 1 + \frac{1}{1+\frac{1}{1}}$, $a_4 = 1 + \frac{1}{1+\frac{1}{1+\frac{1}{1}}}$, $\dots a_{n+1} = 1 + \frac{1}{a_n}$. Simplify the first 5 elements and express how the results relate to the Fibonacci sequence. What number does this sequence tend to?

7. Create your own sequence where $a_5 = 32$. Is your sequence unique? Explain your answer.

8. Create a sequence of each type listed below
    a. Geometric
    b. Arithmetic
    c. Constant
    d. Neither

Section1.3c Worksheet	Date:______________ Name:___________________________

Concept	**Meaning in words and/or examples as required**
What is an even function?	
What is an odd function?	
What kinds of functions are symmetric with respect to y-axis?	
What kinds of functions are symmetric with respect to x-axis?	
What kinds of functions are symmetric with respect to the origin?	
Give graphical examples of an a) **Even function** b) **Odd Function** c) **Neither**	
What does it mean algebraically when a functions an even function? Give examples and explain.	
What does it mean algebraically when a functions an odd function? Give examples and explain.	
Difficulties encountered in the section:	

Exercises 1.3c

1. Determine if the functions below are odd, even, neither.

a. $y = x^2 - x$

Odd Even Neither

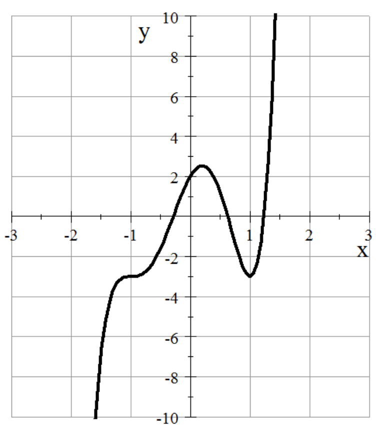
b. $y = |x|$

Odd Even Neither

c. $y = x^3 - 3x$

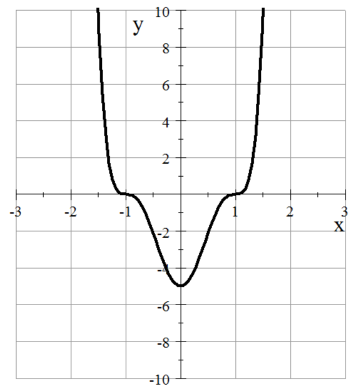
Odd Even Neither

d.



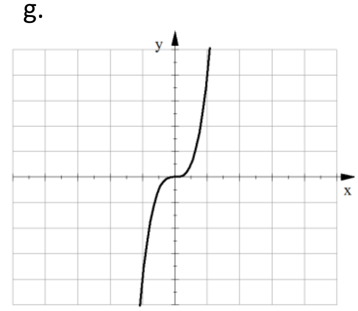
Odd Even Neither

f.



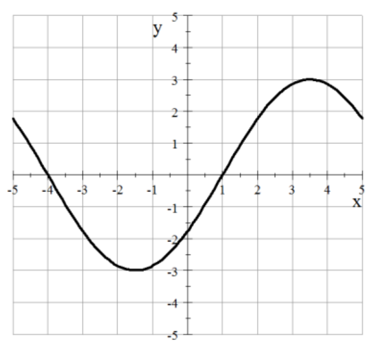
Odd Even Neither

g.



Odd Even Neither

h.



Odd Even Neither

i. $f(x) = 5x^2 + 3$

Odd Even Neither

j. $g(x) = \frac{3}{x}$

Odd Even Neither

2. Determine whether:
   a. An exponential function could be even or odd.

   b. A Linear function could be even or odd.

   c. Is there any function that could be both even and odd?

3. Consider the function $f(x) = x^2 - 3x + 5$ and try to write it as a sum of an even and an odd function, i.e., find formulas for $g(x) =$, and $h(x) =$ such that $g(x) + h(x) = (g + h)(x) = x^2 - 3x + 5$.

4.	Show that for $f(x) = x^2 - 3x + 5$ then $g(x) = \frac{f(x)+f(-x)}{2}$ is an even function.
5.	Create a function of each type listed below. a. Odd b. Even c. Neither Which of these functions are one-to-one? Do you think an even function can be one-to-one? Explain your answer

Section1.4a Worksheet	Date:_______________ Name:__________________________
Concept	**Meaning in words and/or examples as required**
How is the sum function $f \pm g$ defined? I.e. how do you compute outputs of this function?	
How do you determine the domain of the sum function $f \pm g$?	
For the sum function $f \pm g$, what is necessary about the units of the output for f and for g so that $f \pm g$ is meaningful?	
How is the product function $f \times g$ defined? I.e. how do you compute outputs of this function?	
How do you determine the domain of the product function $f \times g$?	
How is the quotient function $\frac{f}{g}$ defined? I.e. how do you compute outputs of this function?	
How do you determine the domain of the quotient function $\frac{f}{g}$?	
Difficulties encountered in the section:	

Exercises 1.4a

1 $f(x) = 3x - 1$ and $g(x) = x^2 + 2$ i. $(f + g)(x) =$ ______________ ii. Domain of $(f + g)$ iii. $(f + g)(3) =$ ______________ iv. $(fg)(x) =$ ______________ v. Domain of $(fg) =$ ____________ vi. $(fg)(0) =$ ______________	2 $f(x) = 5x^2 + 1$ and $g(x) = x - 2$ i. $\left(\frac{f}{g}\right)(x) =$ ______________ ii. Domain of $\left(\frac{f}{g}\right)$ iii. $\left(\frac{f}{g}\right)(3) =$ ______________ iv. $(f - g)(x) =$ ______________ v. Domain of $(f - g) =$ ____________ vi. $(f - g)(0) =$ ______________

3. If $f(x) = (2 + x)(-4 + x)$ and $g(x) = (1 - x)(1 + x)$. Find all values that are NOT in the domain of $\frac{f}{g}$. If there are more than one value separate them with commas.

4. EXTRA CREDIT: If $f(x) = \sqrt{x - 1}$ and $g(x) = \ln(x - 3)$. Evaluate the following

   A. Domain of $f + g$

   B. Domain of $\frac{f}{g}$

5. Let the number of bushels of corn produced in each county of Wisconsin at year t be denoted by $B_{county}(t)$ and the number of acres planted in corn in the county by $A_{county}(t)$.

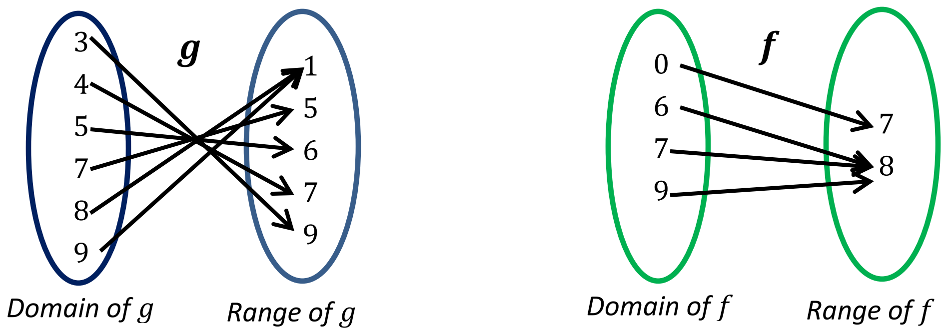
   a. Describe what the function $\frac{B_{county}(t)}{A_{county}(t)}$ represents.

   b. Describe what the sum over all the counties $B_{adams} + B_{dane} + B_{marathon} + \cdots$ over all the counties in the state function represents.

Section1.4b Worksheet	**Date:_______________ Name:__________________________**
Concept	**Meaning in words and/or examples as required**
What is the composite of two functions?	
How do you find domain and range of a composite function? Explain.	
What is the result of the composition of a one-to-one function and its inverse?	
Give example of a composite function that is its own inverse.	
Difficulties encountered in the section:	

Exercises 1.4b

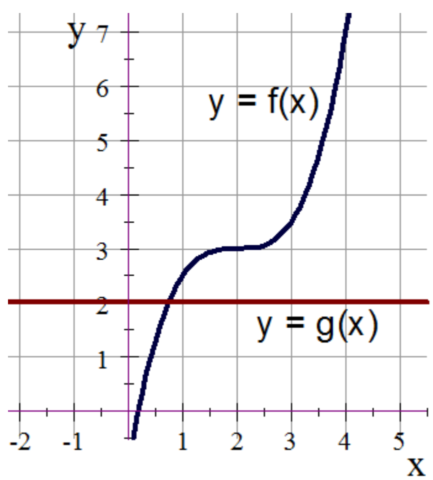
1. Two functions g and f are defined in the figure below. Find the domain and range of the compositions $f{\circ}g(x) = f(g(x))$, and $g{\circ}f(x) = g(f(x))$. Then evaluate the function values below.



Domain of f	Range of f
Domain of g	Range of g
Domain of $(f \circ g)(x) = f(g(x))$	Range of $(f \circ g)(x) = f(g(x))$
Domain of $(g \circ f)(x) = g(f(x))$	Range of $(g \circ f)(x) = g(f(x))$

a. $f \circ g(3)$	b. $g \circ f(9)$	c. $f \circ g(7)$	d. $g \circ f(0)$

2. Consider the functions $y = f(x)$ and $y = g(x)$ which are given by their graphs below.



a. Evaluate $f(4) =$

b. Evaluate $g(4) =$

c. Evaluate $(f + g)(4) =$

d. Evaluate $(f + g)(2) =$

e. Evaluate $(f + g)(1) =$

f. Plot the graph of $y = (f + g)(x)$

g. Evaluate $(fg)(3) =$

h. Evaluate $\frac{g}{f}(4) =$

i. Evaluate $f \circ g(4) =$

j. Evaluate $g \circ f(2) =$

k. Evaluate $f^{-1}(4) =$ (Approximately)

3. For the real valued functions $g(x) = \frac{x+6}{x-5}$, and $f(x) = 2x - 7$ find the compositions listed below and specify domains of these functions using interval notation. Then evaluate the values of the function listed below.	
$(f \circ g)(x) = f\big(g(x)\big) =$	$(g \circ f)(x) = g\big(f(x)\big) =$
Domain of f Domain of g	Range of f Range of g
Domain of $(f_\circ g)(x) = f(g(x))$ Domain of $(g_\circ f)(x) = g(f(x))$	Range of $(f_\circ g)(x) = f(g(x))$ Range of $(g_\circ f)(x) = g(f(x))$

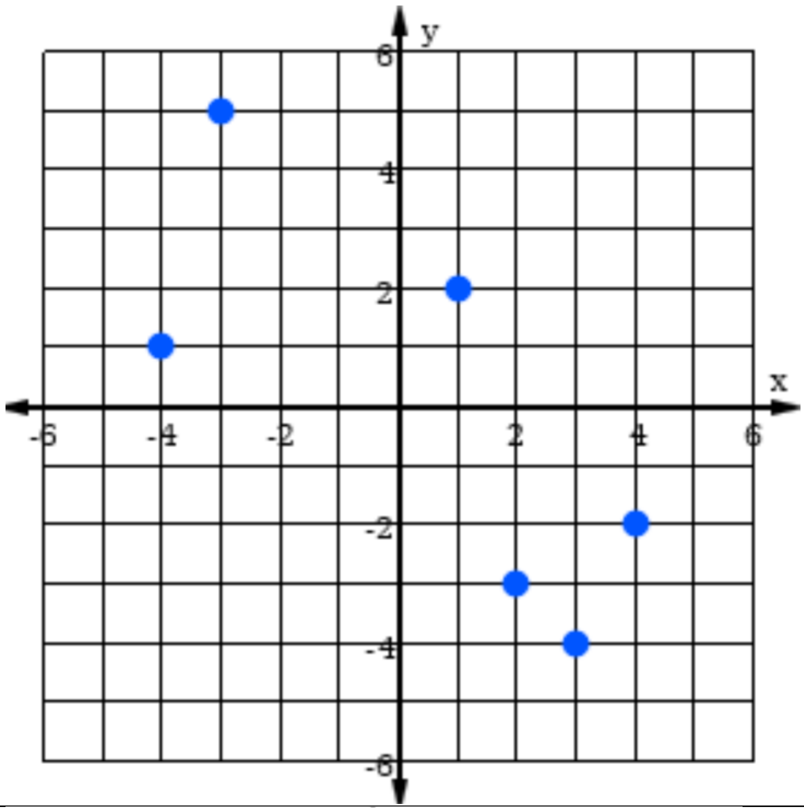
a. $(f_\circ g)(x)$	b. $(g_\circ f)(6)$	c. $(f_\circ g)(6)$	d. $(g_\circ f)(0)$

4. For the real valued functions $g(x) = \sqrt{x+8}$, and $f(x) = x^2 + 7$ find the compositions listed below and specify domains of these functions using interval notation. Then evaluate the values of the function listed below.	
$(f_\circ g)(x) = f\big(g(x)\big) =$	$(g_\circ f)(x) = g\big(f(x)\big) =$
Domain of f Domain of g	Range of f Range of g
Domain of $(f_\circ g)(x) = f(g(x))$ Domain of $(g_\circ f)(x) = g(f(x))$	Range of $(f_\circ g)(x) = f(g(x))$ Range of $(g_\circ f)(x) = g(f(x))$

e. $f_\circ g(x)$	f. $g_\circ f(1)$	g. $g_\circ f(x)$	h. $f_\circ g(1)$

5. For each of the pairs of functions below find $f(g(x))$ and $g(f(x))$. Then determine whether f and g are inverses of each other. Simplify your answers as much as possible. (Assume that your expressions are defined for all x in the domain of the composition. It is a good idea to write the domain of each of the functions to make sure you know what x values make sense in the functions.	
a. $f(x) = 6x + 3$ and $g(x) = 6x - 3$ $f(g(x))$ $g(f(x))$ ○ f and g are inverses of each other ○ f and g are not inverses of each other	b. $f(x) = \frac{2}{x}$ and $g(x) = \frac{2}{x}$ $f(g(x))$ $g(f(x))$ ○ f and g are inverses of each other ○ f and g are not inverses of each other
c. $f(x) = \sqrt{x}$ and $g(x) = x^2$, for $x \geq 0$ $f(g(x))$ $g(f(x))$ ○ f and g are inverses of each other ○ f and g are not inverses of each other	d. $f(x) = \sqrt{x}$ and $g(x) = x^2$ $f(g(x))$ $g(f(x))$ ○ f and g are inverses of each other ○ f and g are not inverses of each other
6. Given the formula for the composite function $f \circ g(x) = f\big(g(x)\big) = \sqrt{2x^2 - 4}$, state possible formulas for what each of the functions g and f. $g(x) =$ $f(x) =$	7. Come up with formulas for two functions f and g so that when you compose them, then $f\big(g(x)\big) = x$. $f(x) =$ $g(x) =$

8. Evaluate the following for each of the one-to-one functions given by their graphs below.



a. $f(1)$

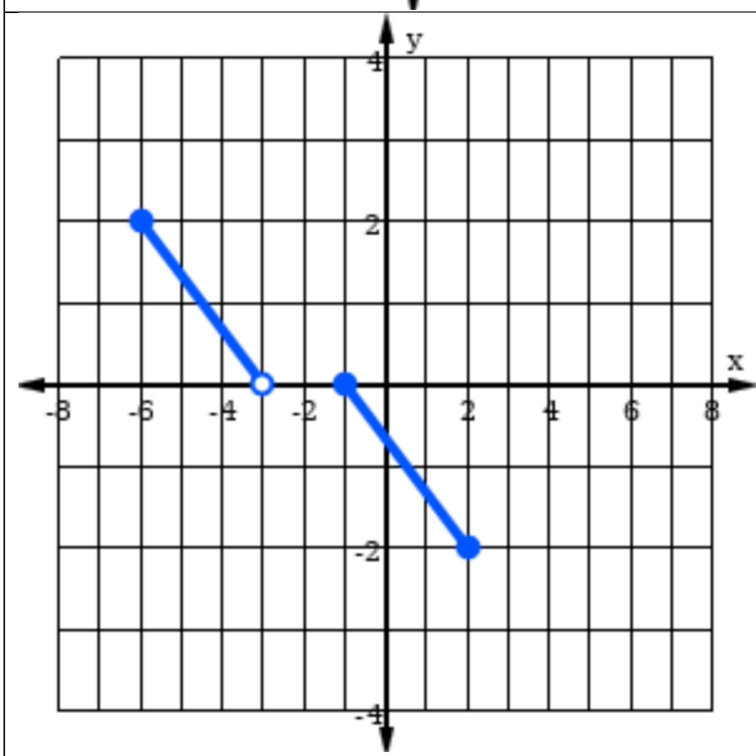
b. $f^{-1}(1)$

c. $f(2)$

d. $f^{-1}(-4)$

e. $f\left(f^{-1}(-3)\right)$

f. $f^{-1}\left(f(-4)\right)$



a. $f(2)$

b. $f^{-1}(0)$

c. $f(-5)$

d. $f^{-1}(-2)$

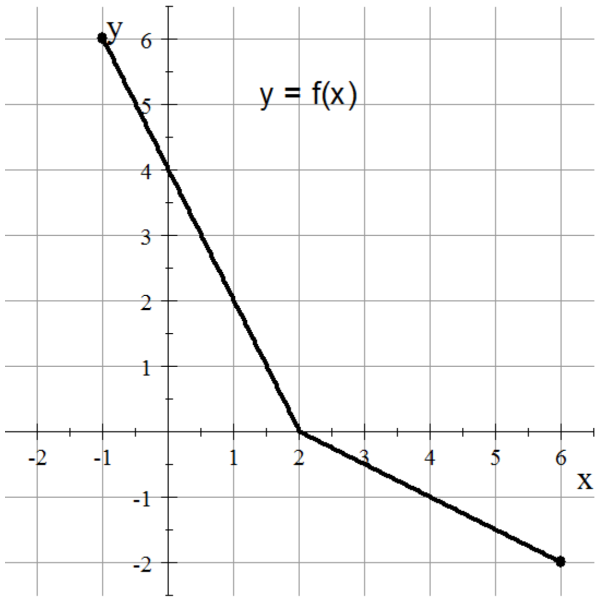
e. $f\left(f^{-1}(-2)\right)$

f. $f^{-1}\left(f(-5)\right)$

9. Find the inverses of the following one-to-one functions. Then find the domains and ranges of the functions and their inverses.

a) $f(x) = \frac{7x+1}{2x-1}$		b) $g(x) = \sqrt{2x-1}$ for $x \geq \frac{1}{2}$	
Domain of f	Range of f^{-1}	Domain of g	Range of g^{-1}
Domain of f^{-1}	Range of f	Domain of g^{-1}	Range of g
$(f \circ f^{-1})(x) =$	$(f^{-1} \circ f)(x) =$	$(g \circ g^{-1})(x) =$	$(g^{-1} \circ g)(x) =$
c) $h(x) = 2^x$		d) $p(x) = lnx$	
Domain of h	Range of h^{-1}	Domain of p	Range of p^{-1}
Domain of h^{-1}	Range of h	Domain of p^{-1}	Range of p
$(h \circ h^{-1})(x) =$	$(h^{-1} \circ h)(x) =$	$(p \circ p^{-1})(x) =$	$(p^{-1} \circ p)(x) =$

10. For the function $y = f(x)$ graphed below, Use the graph to evaluate:



a. $f(1) =$

b. $f(f(0)) =$

c. $f^{-1}(-1) =$

d. $f^{-1}(0) =$

e. Plot the graph of $y = f^{-1}(x)$ on the same grid as $y = f(x)$ to the left.

f. Find a formula for $y = f(x)$ (Piecewise defined!)

g. Find a formula for $y = f^{-1}(x)$

11. Show that $A = f(t) = 200e^{0.055t}$ and $g(A) = \frac{1}{0.055}\ln\frac{A}{200}$ are inverses of each other by showing that both $f \circ g(A) = A$ and also that $g \circ f(t) = t$.

Section2.1 Worksheet	**Date:_______________ Name:____________________________**
Concept	**Meaning in words and/or examples as required**
1. Vertical Shift	
2. Horizontal Shift	
3. Vertical Stretch	
4. Horizontal Stretch	
5. Reflection across x-axis	
6. Reflection across y-axis	
7. Transformation of a function	
Difficulties encountered in the section:	

Exercises 2.1

1. Please fill in the blanks below.
   a. The graph of the function $y = f(x) + k$ is shifted ________________________________ from the graph of the original function $y = f(x)$.
   b. The graph of the function $y = f(x) - k$ is shifted ________________________________ from the graph of the original function $y = f(x)$.
      a. Statements 1 and 2 can also be written as statements 3 and 4 below.
   c. The graph of the function $y - k = f(x)$ is shifted ________________________________ from the graph of the original function $y = f(x)$.
   d. The graph of the function $y + k = f(x)$ is shifted ________________________________ from the graph of the original function $y = f(x)$.
   e. The graph of the function $y = f(x - h)$ is shifted ________________________________ from the graph of the original function $y = f(x)$.
   f. The graph of the function $y = f(x + h)$ is shifted ________________________________ from the graph of the original function $y = f(x)$.
   g. The graph of the function $y = af(x), a > 1$ is ___________________________ from the graph of the original function $y = f(x)$.
   h. The graph of the function $y = cf(x), 0 < c < 1$ is ___________________________ from the graph of the original function $y = f(x)$.
   i. The graph of the function $y = -f(x)$ is ____ from the graph of the original function.
   j. The graph of the function $y = f(-x)$ is ____ from the graph of the original function.
   k. For quadratic functions, completing the squares allows you to find the vertex of the parabola. So if we had the function $y - k = a(x - h)^2$, the point (h, k) is referred to as the _______________ of the parabola.
   l. For quadratic functions, completing the squares allows you to find the vertex of the parabola. So if we had the function $y + k = a(x + h)^2$, the point $(-h, -k)$ is referred to as the _______________ of the parabola.
   m. An even function is symmetric with respect to the _______________.
   n. An odd function is symmetric with respect to the ________________________.

2. Please find the following for all functions below.
   I. Sketch the graph of the functions and relations below. Explain clearly how you decided the graph was the shape you drew. Show all relevant parts of the graph.
   II. Find the domain and range of the functions and relations that you graphed based on your graph.

a. $y = \lvert x\rvert + 5$	p. $y = \sqrt{-x}$
b. $y = \lvert x\rvert - 5$	q. $y = 2\sqrt{x-4} - 5$
c. $y = \lvert x+3\rvert$	r. $y = 2\sqrt{-x-4} - 5$
d. $y = \lvert x-3\rvert$	s. $y = -2\sqrt{-x+4} - 5$
e. $y = \lvert x-3\rvert + 5$	t. $y = x^2 + 3$
f. $y = \lvert x+3\rvert + 5$	u. $y = -2x^3$
g. $y = \lvert x-3\rvert - 5$	v. $y = -x^4+1$
h. $y = \lvert x-3\rvert - 5$	w. $y = 3(x-1)^2 - 4$
i. $y = 2\lvert x\rvert$	x. $y = \lvert 2x\rvert - 1$
j. $y = -2\lvert x\rvert$	y. $y = 2\sqrt{x+5} - 4$
k. $y = 2\lvert x\rvert + 5$	z. $y - 5 = -3(x-1)^2$
l. $y = -2\lvert x\rvert + 5$	aa. $y = -\frac{1}{2}(x-4)^2 + 5$
m. $y = 2\lvert x-3\rvert$	bb. $y = \sqrt{-\frac{1}{2}(x+2)} - 1$
n. $y = -2\lvert x+3\rvert - 5$	
o. $y = \sqrt{x-4} - 5$	

3. Please find the following for all functions below.
   I. Sketch the graph of the functions and relations below. Explain clearly how you decided the graph was the shape you drew. Show all relevant parts of the graph.
   II. Find the domain and range of the functions and relations that you graphed based on your graph.
   III. List all the vertical and horizontal asymptotes if any.

a. $y = 2^x + 3$

b. $y = 2^x - 3$

c. $y = 2^{x+3}$

d. $y = 2^{x-3}$

e. $y = e^x + 3$

f. $y = e^x - 3$

g. $y = e^{x+3}$

h. $y = e^{x-3}$

i. $y = 2^{x-1} + 3$

j. $y = e^{x-2} - 3$

k. $y = \left(\frac{1}{2}\right)^{x+1} - 3$

l. $y = 2e^{x-1} + 3$

m. $y = 2e^{-x} + 1$

n. $y = 300\left(\frac{1}{2}\right)^{\frac{t}{4}}$

o. $y = \frac{1}{2}e^{-2x} + 1$

p. $y = 300\left(\frac{1}{2}\right)^{\frac{t}{4}}$

q. $y = \log(x) + 3$

r. $y = log_2(x)$

s. $y = -3\log(x)$

t. $y = \ln(x+1)$

u. $y = -\log_2(x+3) + 4$

v. $y = 2\ln(-x) + 5$

w. $y = -2\log(x-1) + 4$

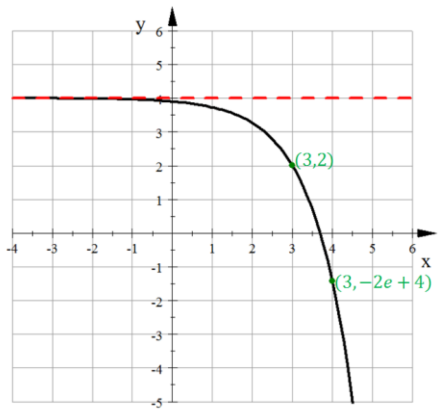
x. $y = \frac{1}{x-2}$

y. $y = \frac{1}{x-2} + 5$

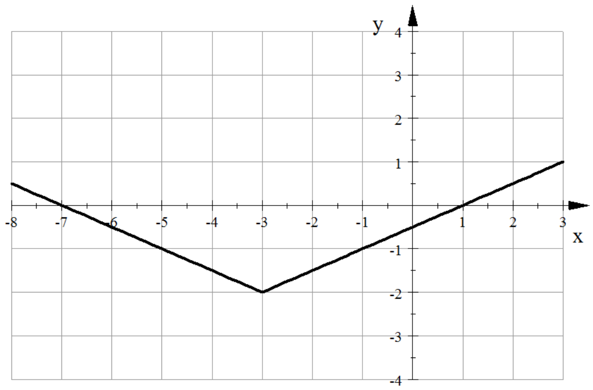
z. $y = -\frac{3}{x-2} - 5$

4. Write a formula for each of the functions a-d shown in the graphs below so that they are a transformation of one of the standard functions listed below. Explain your reasoning.
$y = x,\ y = x^2,\ y = x^3,\ y = \frac{1}{x},\ y = \sqrt{x},\ y = a^x,\ y = \log_a x\,,\ y = |x|,\ y = \frac{1}{x}.$

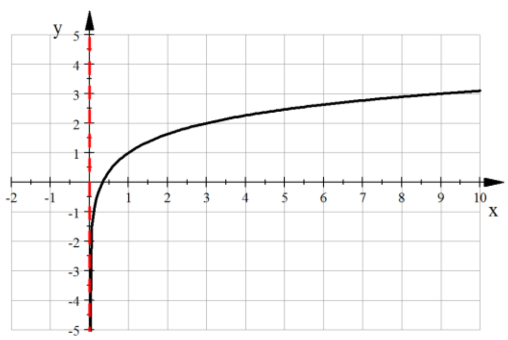
a.



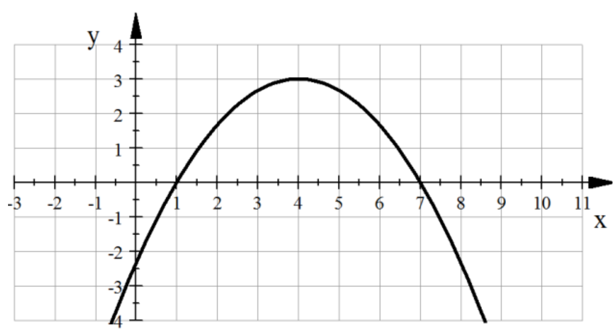
b.



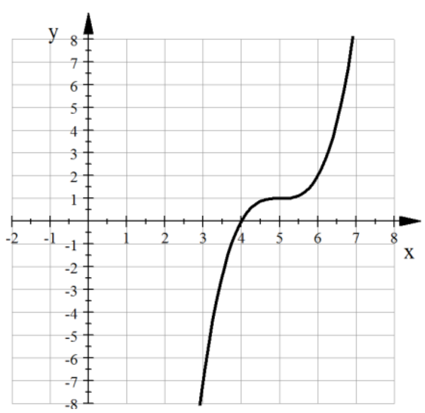
c.



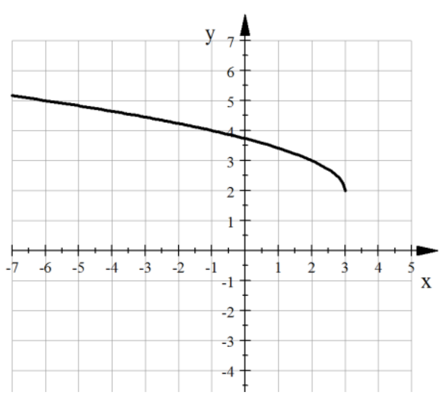
d.



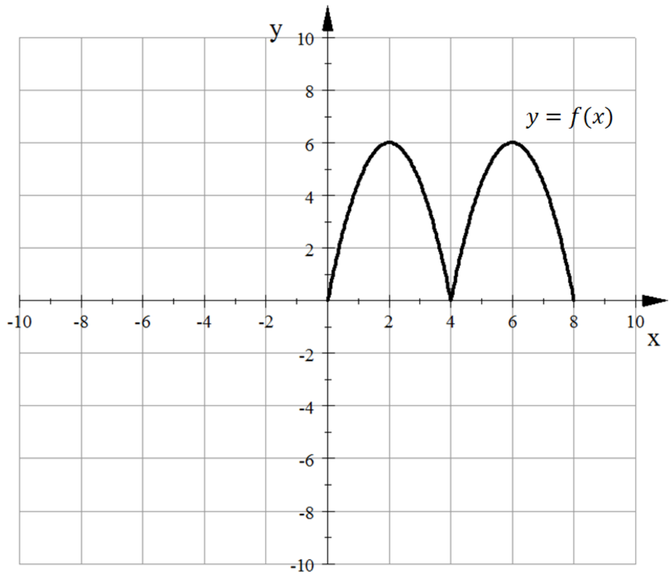
e.



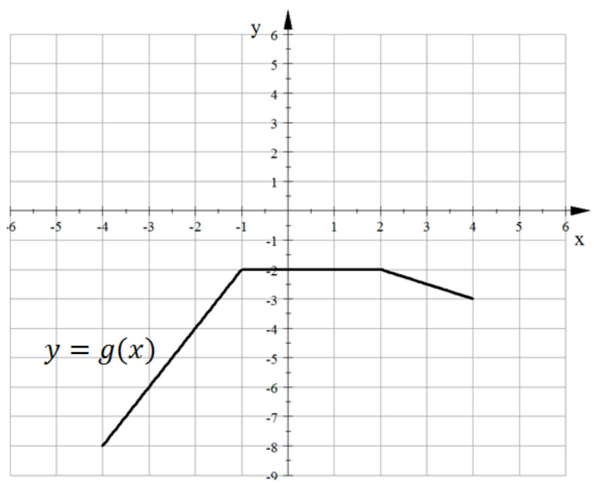
f.



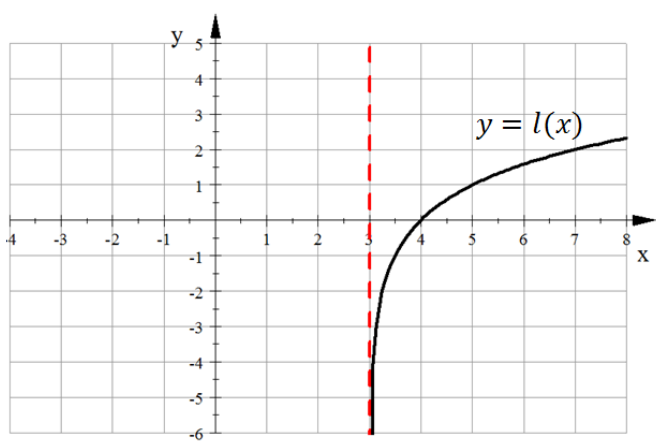
5. For each original graph $y = f(x)$ please graph the transformations, and label all relevant parts of the new graph appropriately.



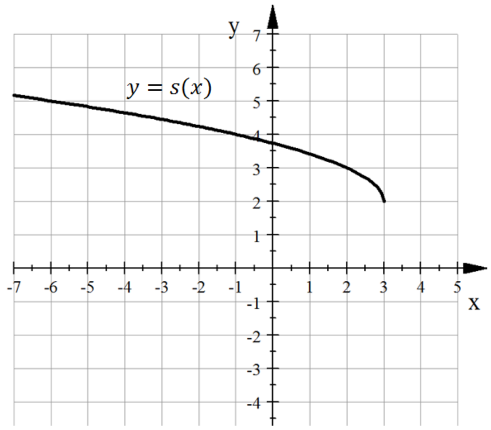
a. $y = f(-x)$
b. $y = -f(x)$
c. $y = f(x+3)+2$
d. $y = f(-x+2)-3$
e. $y = -\frac{1}{2}f(x)$



a. $y = g(x+1)+3$
b. $y = -g(-x)$
c. $y = \frac{1}{2}g(x)$
d. $y - 2 = g(-x-1)$

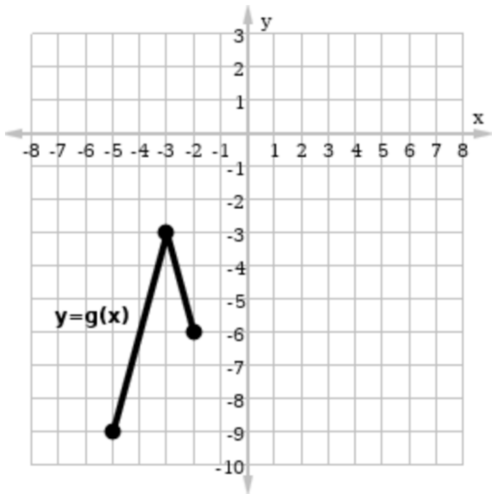


a. $y = l(-x-3)+1$
b. $y = -l(x)$
c. $y = -l(x)$
d. $y = 2(l(x)-1)$

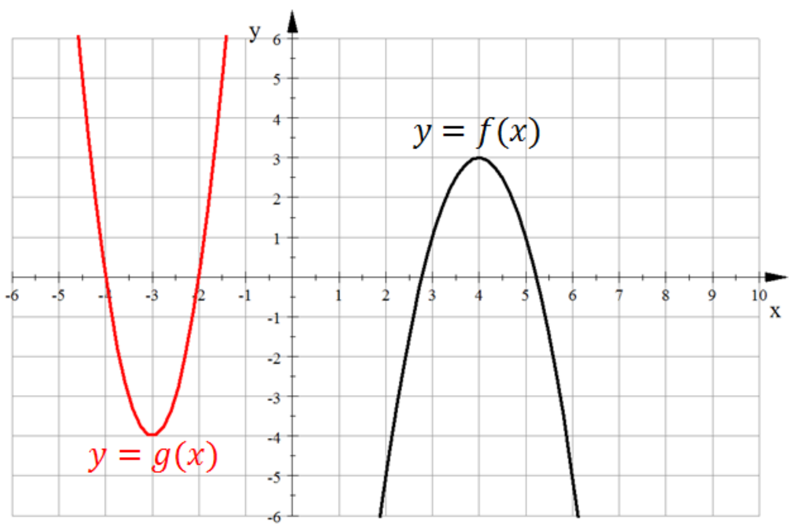
	a. $y = s(-x) - 3$ b. $y = -s(x) + 2$ c. $y = s(x-1)$ d. $y = s(x) + 2$

6. Sketch the graph of the function below. Please show all your work and clearly show relevant points.

   $y = g(x)$ has the graph below, use that to find the graph of $y = \frac{1}{3}g(-x)$.

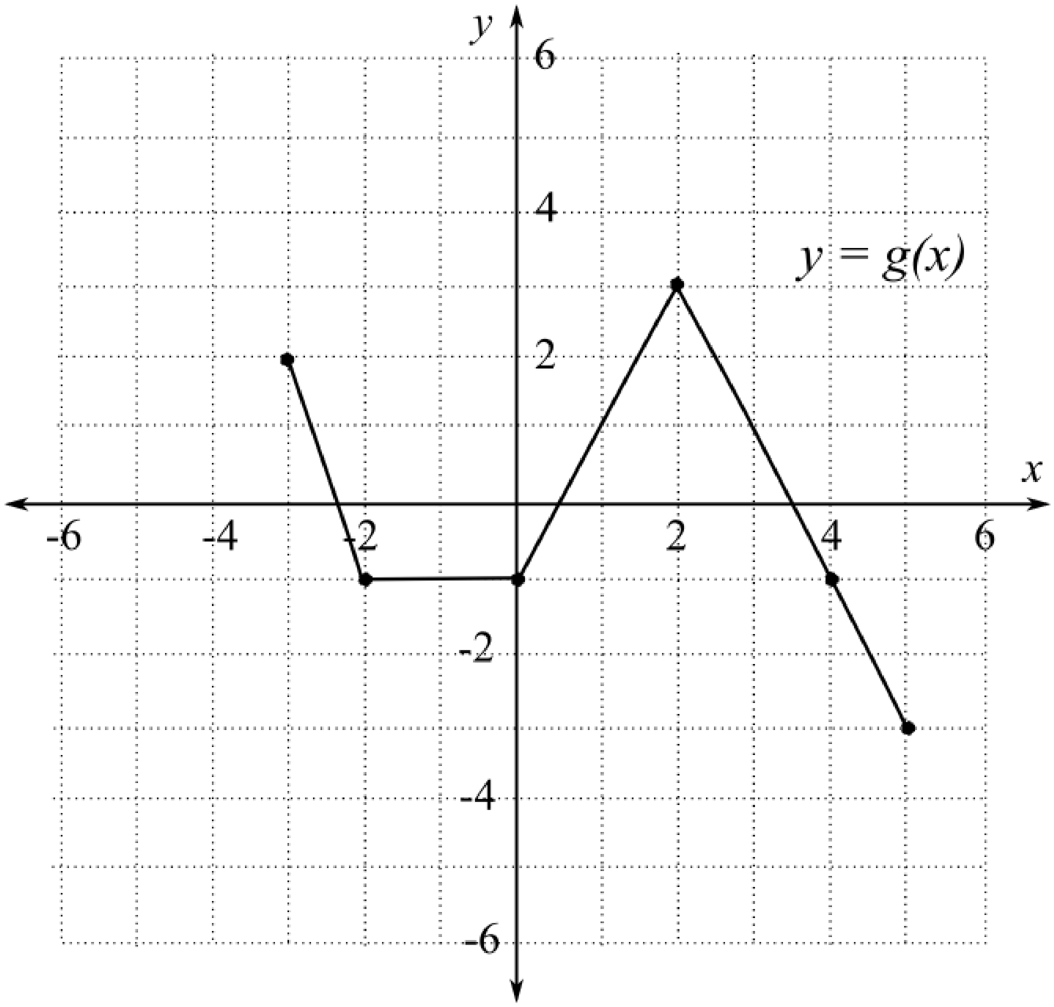


7. Given the two graphs below, find a algebraic relationship between the two so that one function is the result of a transformation of the other.



In other words, write a formula connecting $f(x)$ and $g(x)$.

8. The graph of a function g is shown. Use it to sketch the graph of $y = -g(x+1) - 2$ on the same axes. Show all your intermediary transformations using colored pens and label all the transformations.

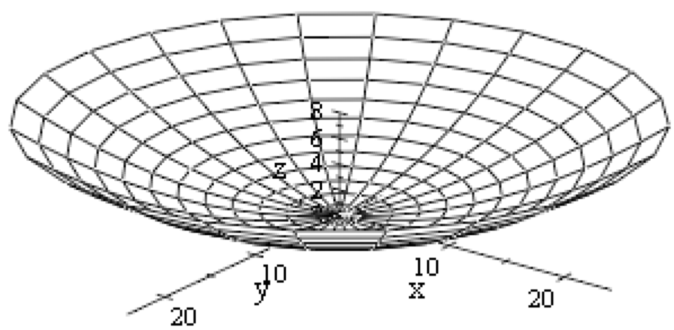
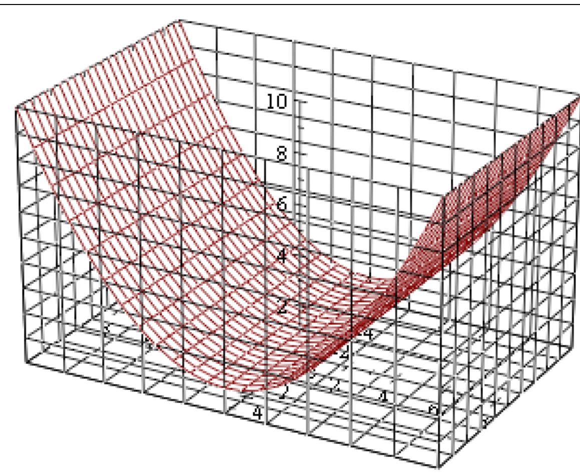


9. Investigate the difference between the graphs of the following functions. What is the conclusion you draw from it? You can use a graphing utility to explore these relations.
   a. $y = x^2 + 2$
   b. $y = x^2 + 2.5$
   c. $y = x^2 + 2.7$
   d. $y = x^2 + 3$
   e. $y = x^2 + 5$
   f. $y = x^2 - 2$
   g. $y = x^2 - 2.5$
   h. $y = x^2 - 2.7$
   i. $y = x^2 - 3$
   j. $y = x^2 - 5$
   k. $y = x^2 + x$

Section2.2a Worksheet	Date:_______________ Name:____________________________
Concept	**Meaning in words and/or examples as required**
1. What is a parabola?	
2. What are all the different forms in which we can write equation of a parabola?	
3. What are the different kinds of parabolas?	
4. What equation forms are possible for a parabola with vertex at $(\mathbf{3}, -\mathbf{5})$. There is one parameter $\boldsymbol{a}$ that is not given.	$x =$ Or $y =$
5. What is the maximum value of a function that is quadratic?	
6. What is the minimum value of a quadratic function?	
Difficulties encountered in the section:	

Exercises 2.2a

1. Sketch the graphs of the following relations. Find the vertex of the parabola and other relevant information if asked.
   a. $y = 2(x-3)^2 - 5$
   b. $y = -2(x-3)^2 - 5$
   c. $x = 2(y-3)^2 - 5$
   d. $x = -2(y-3)^2 - 5$
   e. $y = 3(x+1)^2 + 2$
   f. $x = -3(y+1)^2 + 2$
   g. $x^2 - 4x + 1 = y$
   (Hint: Use completing the squares)
   h. $-x^2 - 4x + 1 = y$
   (Hint: Use completing the squares)
   i. $\frac{2}{3}x^2 - 4x + \frac{1}{3} = y$
   j. $-\frac{2}{3}y^2 - 4y - \frac{1}{3} = x$
   k. $y = 4x^2$
   l. $y = 4(x-1)^2 - 4$
   m. $y = 4x^2 - 8x$
   n. $x^2 + 8x + 3y + 22 = 0$
   o. $2y^2 - 16y - x + 29 = 0$
   p. $3x^2 - 5x + 4 - 2y = 10$
2. Graph and find an equation of the parabola with vertex $(4,3)$ and focus $(4,6)$.
3. Graph and find an equation of the parabola passing through $(-1,3)$, $(3,3)$ and $(1,2)$. What is the vertex of this parabola?
4. Graph and find an equation of the parabola with vertex $(-4,3)$ and focus $(4,3)$.

5. An old satellite dish is has a cross-section that is in the shape of a parabola. The dish is 48 inches across and 8 inches deep at its center. The vertex is at this lowest point in the parabola. Put the vertex at $(0,0)$ and find an equation for the parabola cross section. Also determine the value of p and state how high above the vertex the focus is.	
6. Create a parabola for a parabolic trough solar cooker that can be contained inside of a copier paper box. These boxes are 17 inches long and 10 inches deep. Make it so your parabola has vertex within one inch of the bottom of the box and so that the focus is within 3 inches of the bottom center of the box. Find an equation for this parabola when the origin is at the center line of the bottom of the box.	

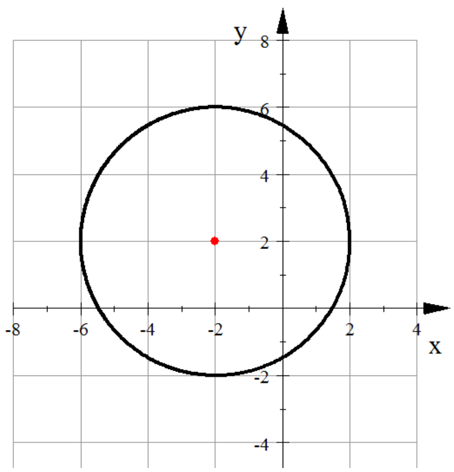
Section2.2b Worksheet	Date:_______________ Name:____________________________
Concept	**Meaning in words and/or examples as required**
1. What is a circle?	
2. What is an ellipse? How is it different than a circle?	
3. What are the major axis and minor axis of an ellipse?	
4. How do we find the center and the foci of an ellipse if we are given its equation?	
5. How do we find the center and radius of a circle from its equation?	
Difficulties encountered in the section:	

Exercises 2.2b

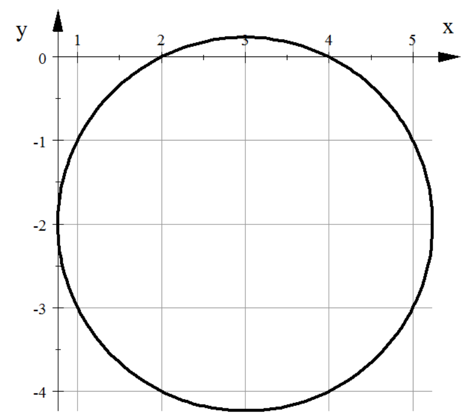
1. Sketch the graph of each circle and state its center and radius.
   a. $x^2 + y^2 = 36$
   b. $(x-3)^2 + (y-4)^2 = 25$
   c. $(x+2)^2 + (y-1)^2 = 9$
   d. $4x^2 + 8x + 4y^2 - 24y = 24$
   e. Plot the graph of $(x+5)^2 + (y-3)^2 = 9$.
   f. Plot the graph of $(x-2)^2 + (y+1)^2 = \frac{25}{4}$.
   g. $x^2 + 4x + y^2 - 6y = 12$
   h. $2x^2 + 8x + 2y^2 - 14y = -\frac{29}{2}$

2. Sketch graphs and write equations that represent each circle described below.
   a. The center is at $(2, 7)$ and the radius is $r = 9$.
   b. Two endpoints of a diameter are the points $(1, 2)$ and $(5,2)$.
   c. Find equation of the circle with center $(-3, 5)$ and radius 5.
   d. Find the equation of a circle with center $(2,5)$, and passing through $(-2,2)$.
   e. Find equation of the circle whose diameter has endpoints $(-1,4)$, and $(4,-2)$.
   f. Find an equation of the circle with center (-3, 5) that goes through the point (5, -1).

3. Write the equation in standard form for the graph of the circle shown below.



4. Write equation of the circle in standard form for the graph of the circle shown below.



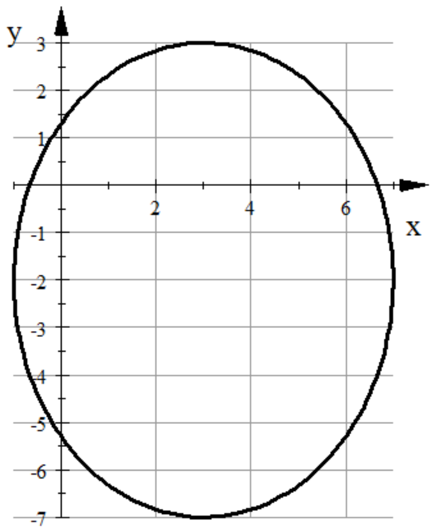
5. Plot the ellipses below and locate their center, vertices, foci, and the values of a and b.
   a. $\frac{(x+3)^2}{16} + \frac{(y-1)^2}{25} = 1$
   b. $16x^2 + y^2 - 16 = 0$
   c. $4x^2 - 16x + 9y^2 - 18y = 11$
   d. $\frac{(x-3)^2}{9} + (y+2)^2 = 1$

6. Find an equation of each ellipse described or plotted below and find the location of their foci.

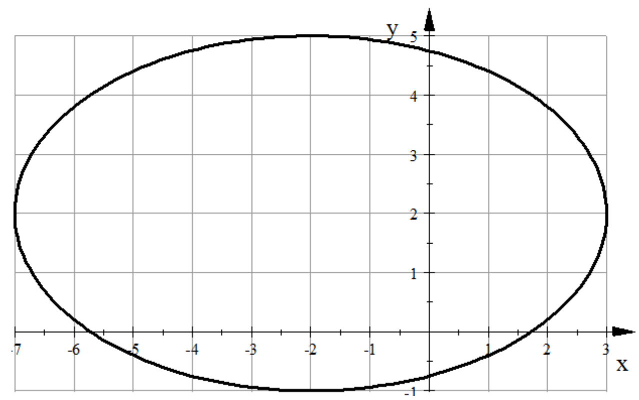
a. The ellipse is centered at $(3, 6)$ with a horizontal major diameter of 10 and minor diameter of 6.

b. The ellipse has vertices at $(-3, 6)$ and $(-3, -2)$ and $(-1, 2)$ is the endpoint of a minor diameter.

c. The ellipse plotted below:



d. The ellipse plotted below:



For extra credit, find out about the reflective property of hyperbolas and an application of the reflective property.

Section2.2c Worksheet	Date:_______________ Name:____________________________
Concept	**Meaning in words and/or examples as required**
1. What is a hyperbola?	
2. What are the different kinds of hyperbolas?	
3. How do you find center and foci of a hyperbola?	
4. How do we find the asymptotes of a hyperbola?	
Difficulties encountered in the section:	

Exercises 2.2c

1. In problems a through f, locate the center, vertices, foci, and the asymptotes. Then sketch the graph and show all the information you found in your graphs.

   a. $\frac{(y+3)^2}{9} - \frac{(x-1)^2}{16} = 1$

   b. $\frac{(x+3)^2}{16} - \frac{(y-1)^2}{9} = 1$

   c. $16x^2 - y^2 - 16 = 0$

   d. $4x^2 - 16x - 9y^2 + 18y = 29$

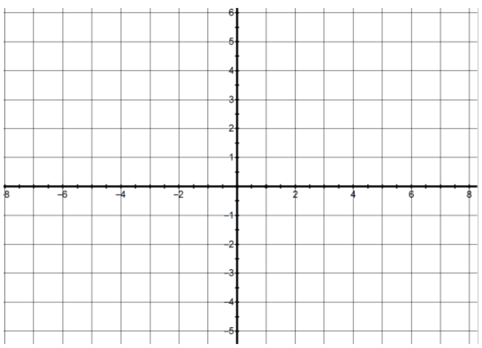
   e. $4y^2 - 16y - 25x^2 - 150x = 309$

   f. $-4x^2 - 16x + 9y^2 + 18y = 43$

2. For problems a through h, complete the square as needed to identify what conic section the equation represents, and locate the vertex(center), vertices, and focus(foci) and asymptotes as appropriate.

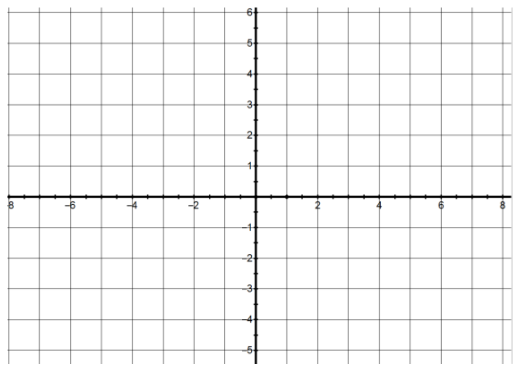
a. $y^2 + 6y - 2x + 5 = 0$

☐ Ellipse ☐ Hyperbola
☐ Circle ☐ Parabola



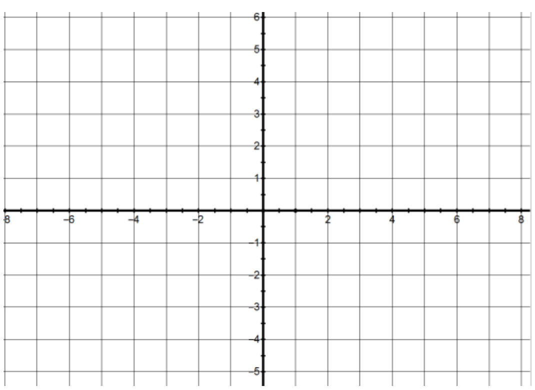
b. $x^2 + 6x - 8y - 31 = 0$

☐ Ellipse ☐ Hyperbola
☐ Circle ☐ Parabola



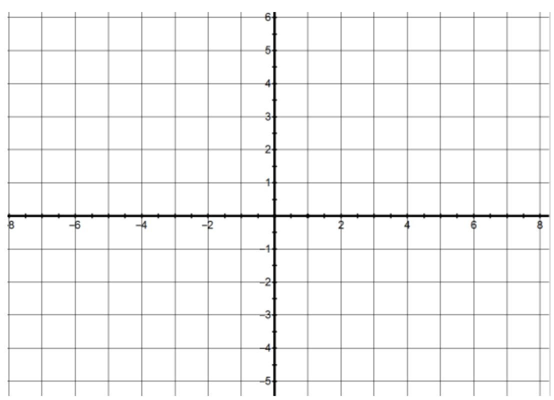
c. $16x^2 - 64x + 9y^2 + 108y = -244$

☐ Ellipse ☐ Hyperbola
☐ Circle ☐ Parabola



d. $16y^2 - 96y - 9x^2 - 18x = 9$

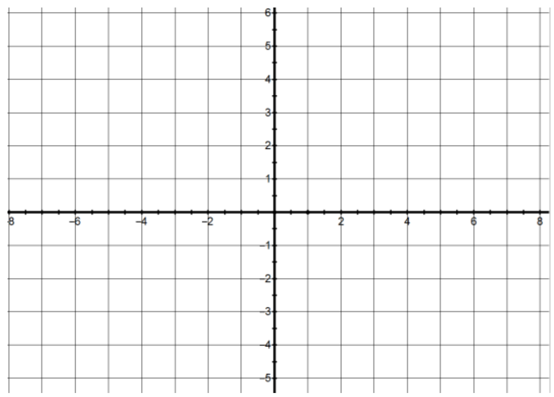
☐ Ellipse ☐ Hyperbola
☐ Circle ☐ Parabola



e. $y^2 = 2x - x^2$

☐ Ellipse ☐ Hyperbola

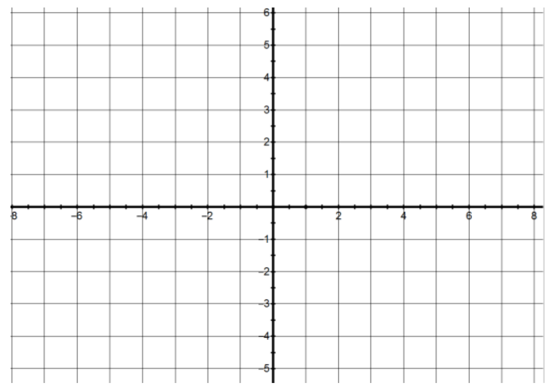
☐ Circle ☐ Parabola



f. $x^2 + y^2 - 2x + 4y = 4$

☐ Ellipse ☐ Hyperbola

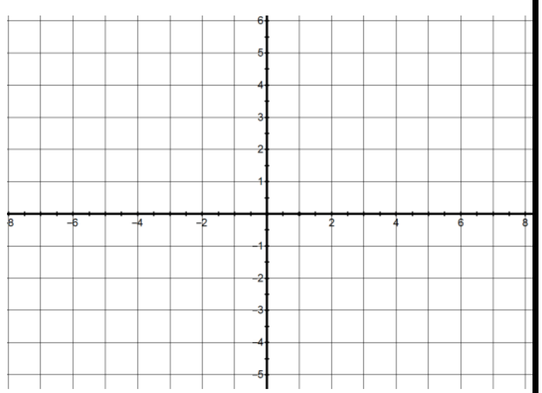
☐ Circle ☐ Parabola



g. $3y^2 - 4x + 6y = 4$

☐ Ellipse ☐ Hyperbola

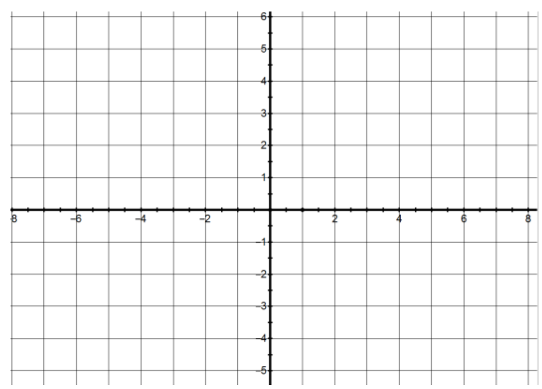
☐ Circle ☐ Parabola



h. $3y^2 + 4x + 6y = 4$

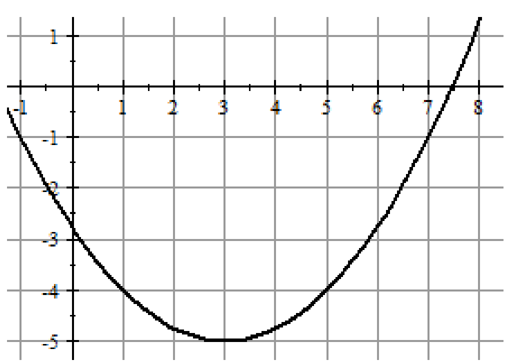
☐ Ellipse ☐ Hyperbola

☐ Circle ☐ Parabola

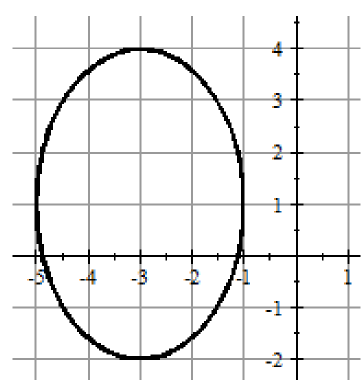


3. In problems a through d find an equation of the conic section for the given graphs

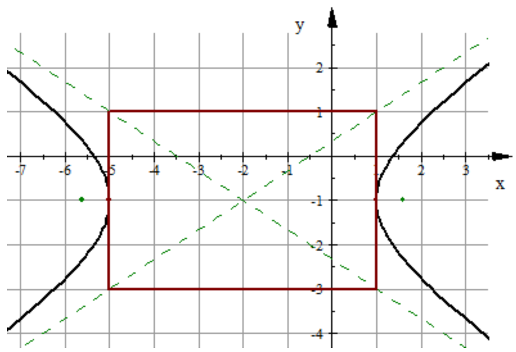
a.



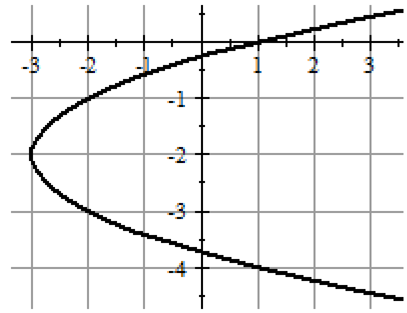
b.



c.



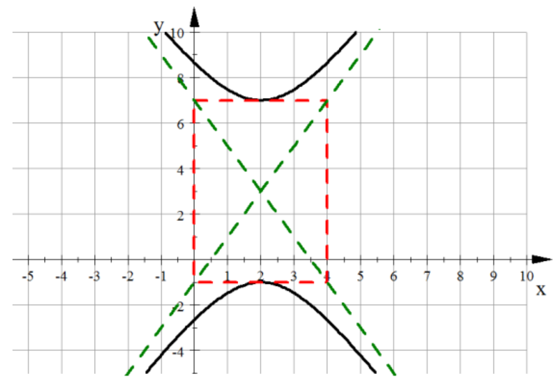
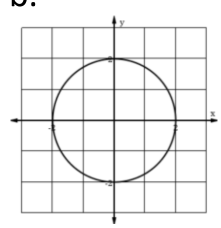
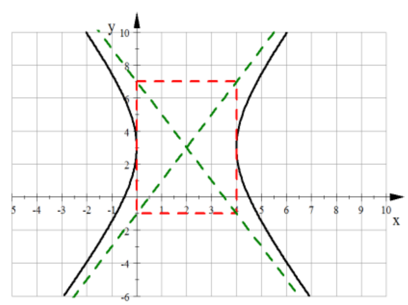
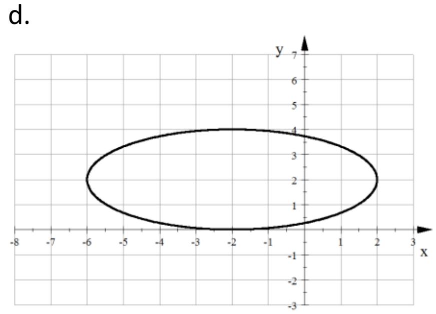
d.



4. Two friends live 10 miles from away from each other along an east-west highway. One day as they are chatting on their cell-phones, the eastern friend hears a loud clap of thunder over the cell phone and 40 seconds later hears the sound of the same thunder reaching her house directly. Since the sound of thunder travels a mile in 5 seconds, this means that the thunder traveled an additional 40 seconds to reach the eastern friend. Thus the distance from the lightning strike to the eastern friend was 8 miles longer than the distance to the western friend. Use the locus definition of a hyperbola to locate the possible locations of the lightning strike. (Hint: Draw a graph with the friends located on the x-axis at ± 5 and then draw an appropriate hyperbola.)

   Also, if the western friend notices that the flash of lightning that produced this thunder was directly north of her, determine exactly where the strike was.

5. Match each relation to its appropriate graph. If there is no match, please state so.
Match all the quantities in Column B that are equivalent to quantities in Column A. Some of the column B quantities may not have any corresponding items in column A, but all items in column A have at least one or more corresponding items in column B. Explain your reasoning for the choices you made. (3 pt each)

Column A	Answer	Column B
i. $\frac{(x+2)^2}{16} + \frac{(y-2)^2}{4} = 1$ Reasoning:		a. 
ii. $\frac{(x-2)^2}{4} - \frac{(y-3)^2}{16} = 1$ Reasoning:		b. 
iii. $-\frac{(x-2)^2}{4} + \frac{(y-3)^2}{16} = 1$ Reasoning:		c. 
iv. $x^2 + y^2 = 4$		d. 

Section2.3 Worksheet	Date:_______________ Name:____________________________
Concept	**Meaning in words and/or examples as required**
1. Degree of a polynomial	
2. **Leading coefficient of a polynomial**	
3. **Local extrema**	
4. x**-intercept**	
5. y**-intercept**	
6. Difference between solving an equation in one variable, and sketching the graph of a function in one variable	
7. Difference between solving an inequality in one variable, and sketching the graph of a function in one variable	
Difficulties encountered in the section:	

Exercises 2.3

1. Answer the questions below
   a. How does the degree of the polynomial affect the end behavior?
   b. What does the leading coefficient of a polynomial control in its graph ?
   c. What is the maximum number of local extrema you can expect in a polynomial of degree n?
   d. How does the exponent n in factors of the type $(ax+b)^n$ influence the shape of the graph near the x-intercept associated with this factor?
   e. How do you find all the x-intercepts of a polynomial function?
   f. How do you find all the y-intercepts of a polynomial function?
   g. How would you solve a an equation of the type
   $a(x-a_1)^{n_1}(x-a_2)^{n_2}(x-a_3)^{n_3}\dots.(x-a_k)^{n_k}=0$?
   h. What is the significance of the solutions to the equation in question 7 to the graph of the polynomial function $f(x)=a(x-a_1)^{n_1}(x-a_2)^{n_2}(x-a_3)^{n_3}\dots.(x-a_k)^{n_k}$?
   i. How would you solve an inequality of the type
   $a(x-a_1)^{n_1}(x-a_2)^{n_2}(x-a_3)^{n_3}\dots.(x-a_k)^{n_k}>0$
   Or
   $a(x-a_1)^{n_1}(x-a_2)^{n_2}(x-a_3)^{n_3}\dots.(x-a_k)^{n_k}<0$
   j. How are the solutions to the inequalities in question 9 related to the graph of the polynomial function $f(x)=a(x-a_1)^{n_1}(x-a_2)^{n_2}(x-a_3)^{n_3}\dots.(x-a_k)^{n_k}$?
   k. What is a local extrema of a polynomial function?
   l. Does every polynomial function have local extreme points? Explain your answer.

2. Sketch the graph of the functions below label all x-intercepts, y-intercepts, label all local extrema points, show end behavior.

a. $g(x)=x^2(x-1)(x+2)^3$ x-intercepts are _______________ y-intercept is _____________ End Behavior like ________________ Number of local extremum _______ Graph of the polynomial	b. $h(x)=x(x+1)(x+4)(x-2)(x-5)$ x-intercepts are _______________ y-intercept is _____________ End Behavior like ________________ Number of local extremum _______ Graph of the polynomial
c. $g(x)=(x+1)^3(x-1)(x-2)$	d. $h(x)=(x-1)(x+1)^2(x+2)^2(x-3)$

x-intercepts are _______________ y-intercept is _____________ End Behavior like ________________ Number of local extremum _______ Graph of the polynomial	x-intercepts are _______________ y-intercept is _____________ End Behavior like ________________ Number of local extremum _______ Graph of the polynomial

3. Determine the interval(s) on which the function is (strictly) increasing, or decreasing, or constant. Write your answer in interval notation.

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<tr>
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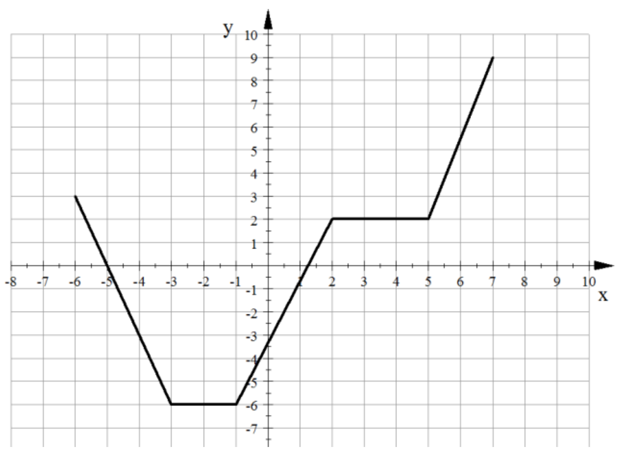
A.



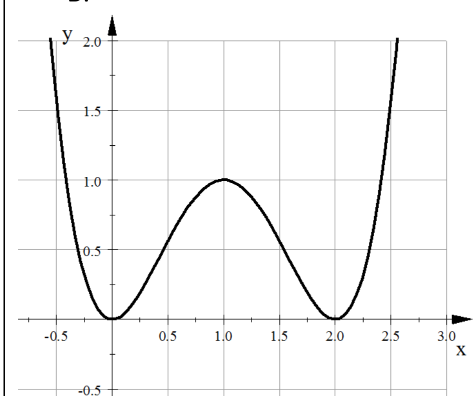
Increasing: ____________

Decreasing: ____________

Constant: ____________

</td>
<td>

B.



Increasing: ____________

Decreasing: ____________

Constant: ____________

</td>
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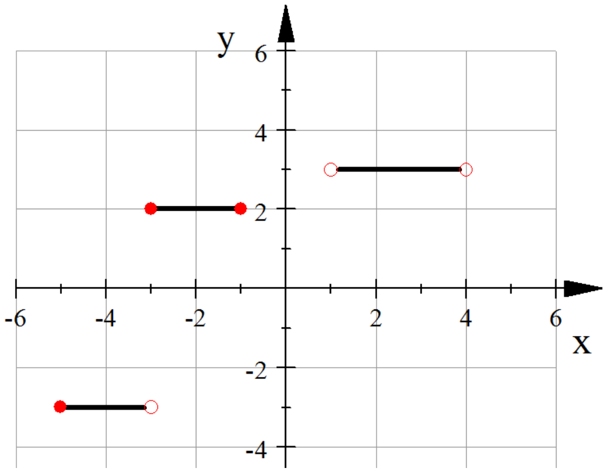
</table>

C.

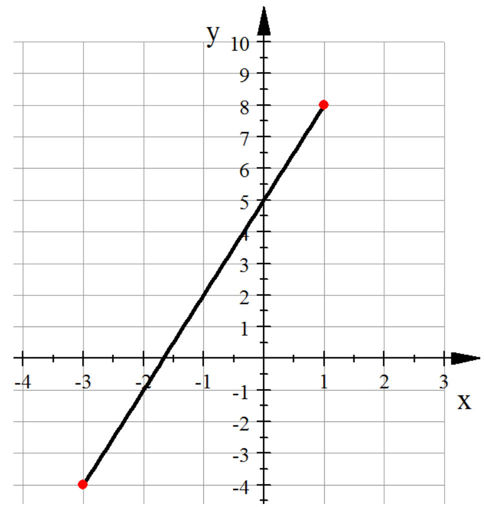


Increasing: ____________

Decreasing: ____________

Constant: ____________

D.



Increasing: ____________

Decreasing: ____________

Constant: ____________

4. Use the graph of the function f below to find (If there is more than one answer, separate them with commas)

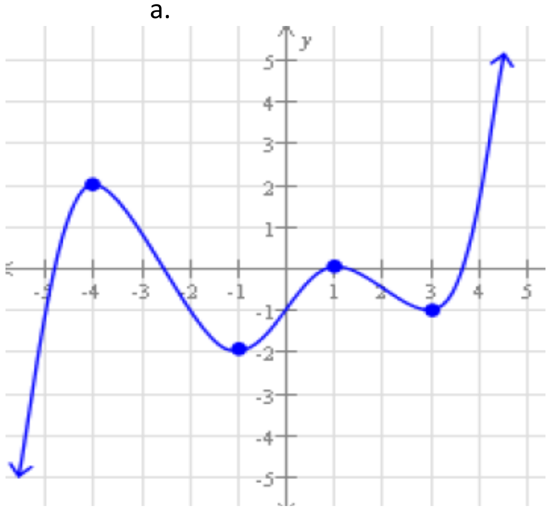
I. All values of x at which f has local minimum

II. All values of x at which f has local maximums

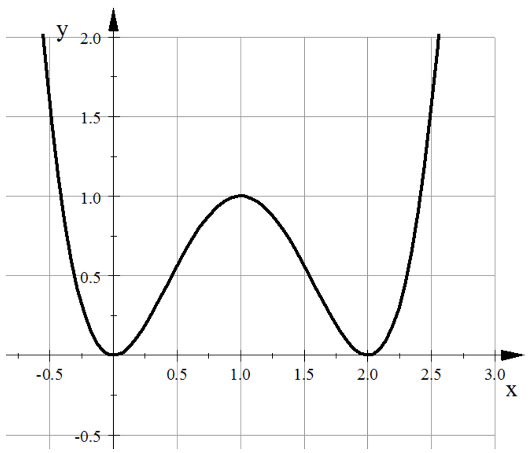
III. All local minimum values of f

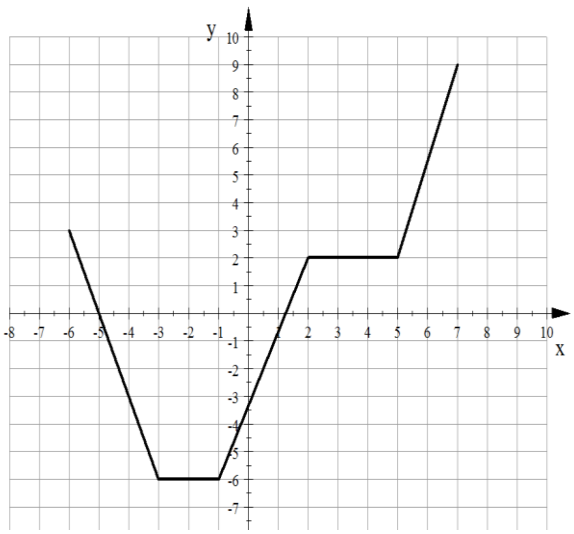
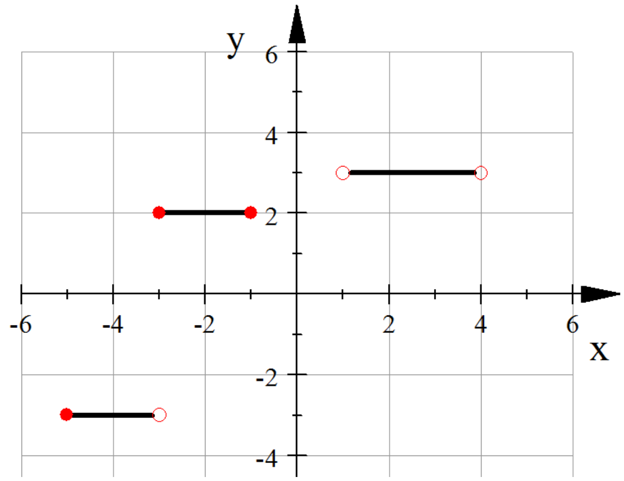
IV. All local maximum values of f.

a.



b.

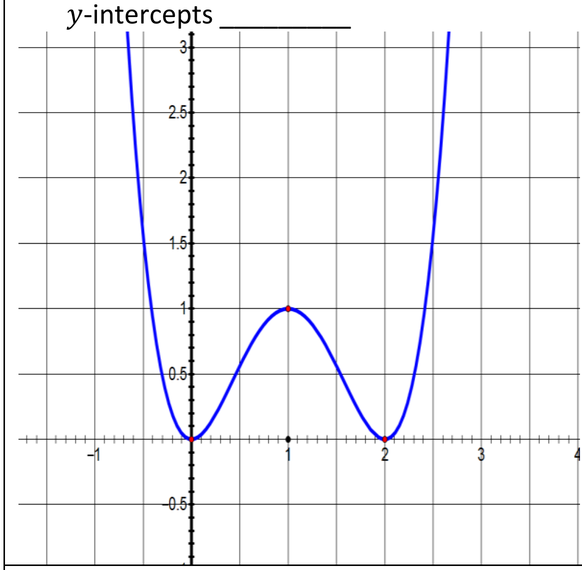


c.	d.
	

5. Find all the x-intercepts and y-intercepts of the functions below. If there is more than one answer, separate them with commas.

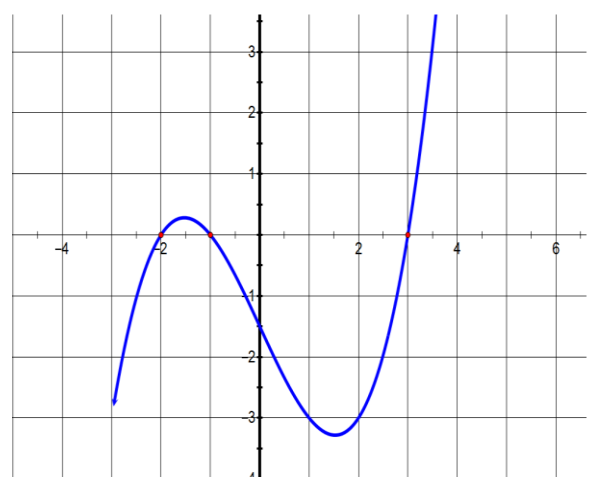
A. x-intercepts __________

y-intercepts __________



B. x-intercepts __________

y-intercepts __________



C. $f(x) = (x+3)(x-1)^2(x+5)$

x-intercepts __________ y-intercepts __________

D. (factor by grouping) $f(x) = 2x^3 + 2x^2 - 18x - 18$ x-intercepts __________ y-intercepts __________
E. $f(x) = x^4 - 4x^2 - x^3 + 4x$ x-intercepts __________ y-intercepts __________

6. Mark the end behavior of the graph of each polynomial function below.

A. $f(x) = (x+3)(x-1)^2(x+5)$

Left		Right	
☐ Falls	☐ Rises	☐ Falls	☐ Rises

B. $f(x) = -2x^3 + 2x^2 - 18x - 18$

Left		Right	
☐ Falls	☐ Rises	☐ Falls	☐ Rises

E. $f(x) = 5x^{11} - 4x^2 - x^3 - 2x$

Left		Right	
☐ Falls	☐ Rises	☐ Falls	☐ Rises

F. $f(x) = -4x^{32} - 5x^{20} + 4x - 1$

Left		Right	
☐ Falls	☐ Rises	☐ Falls	☐ Rises

7. Match the graphs below with the functions listed here.

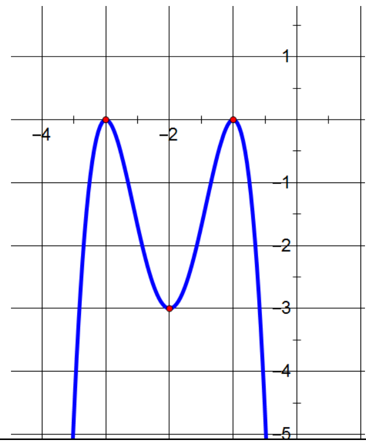
   I. $f(x) = -3(x+1)^2(x+3)^2$

   II. $h(x) = x^2(x-2)^3(x+1)$

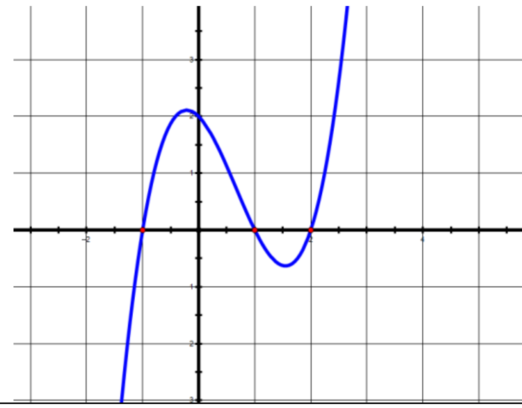
   III. $g(x) = \frac{1}{2}(x^3 - x^2 - 6x)$

   IV. $r(x) = (x-1)(x+1)(x-2)$

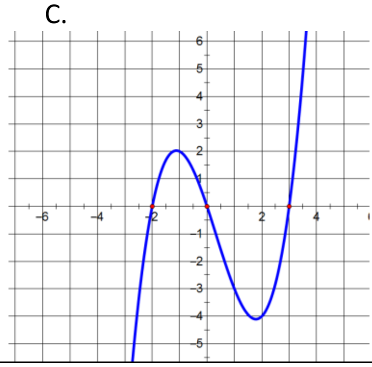
A.



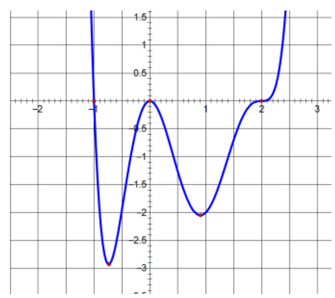
B.



C.

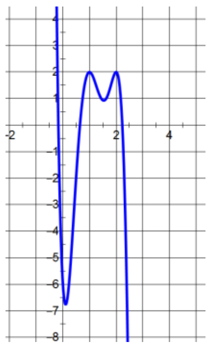


D.

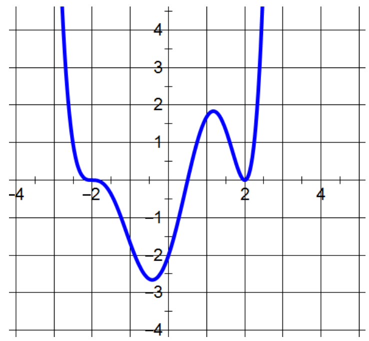


8. For the graphs below determine the following
    I. What is the sign of the leading coefficient of the polynomial function (positive, negative or not enough information?
    II. Which of the following is a possibility of the degree of the function? Choose all that apply $4, 5, 6, 7, 8, 9$.

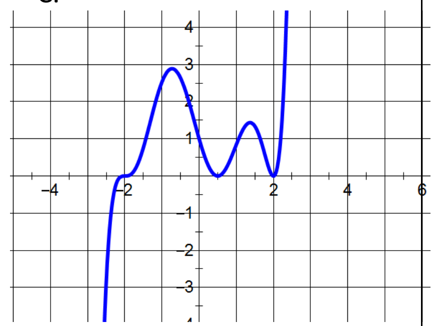
A.



B.

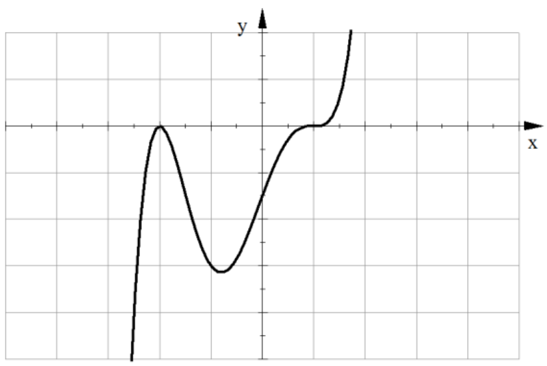


C.

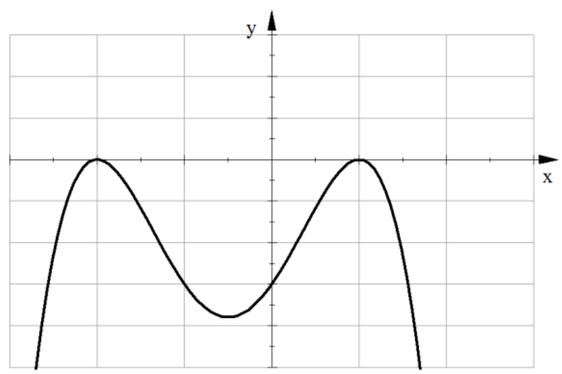


9. Find a possible equation that represents the graphs below.

a.



b.



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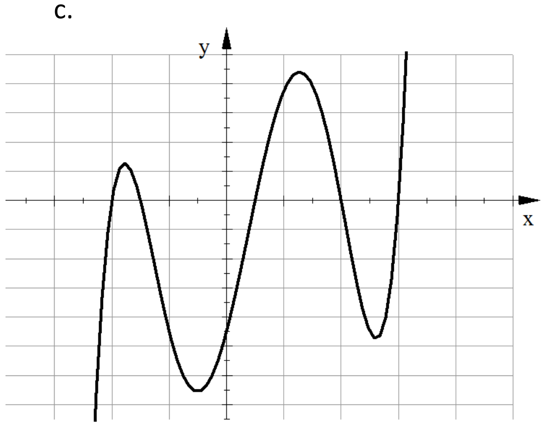

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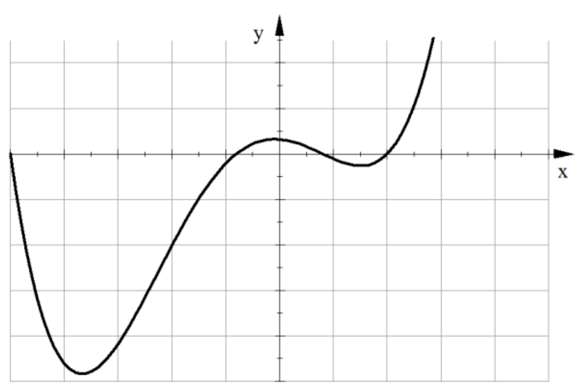


Section2.4 Worksheet	**Date:_______________ Name:____________________________**
Concept	**Meaning in words and/or examples as required**
1. Quotient Function	
2. Rational Function	
3. Domain of a rational function	
4. Range of rational function	
5. Vertical Asymptote	
6. Oblique or Horizontal Asymptote	
7. Points of intersection of a function with its horizontal, or oblique asymptote	
Difficulties encountered in the section:	

Exercises 2.4

1. In your own words describe what role the numerator function and the denominator function play in a quotient function. Give examples to demonstrate your ideas.

2. How do you locate vertical asymptotes? Why does your method work?

3. Can the graph of a rational function intersect a vertical asymptote? Why or why not?

4. How do you determine the horizontal or oblique asymptotes?

5. What would the graph of the function $f(x) = \frac{1}{x} + x^2$ look like?

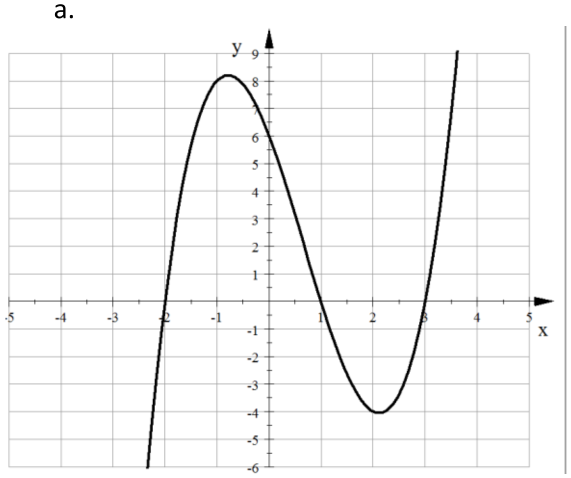
6. If the numerator and denominator have common factors in a rational function, how does that affect the graph of the rational function?

7. How do you find x-intercepts and y-intercepts of a rational function?

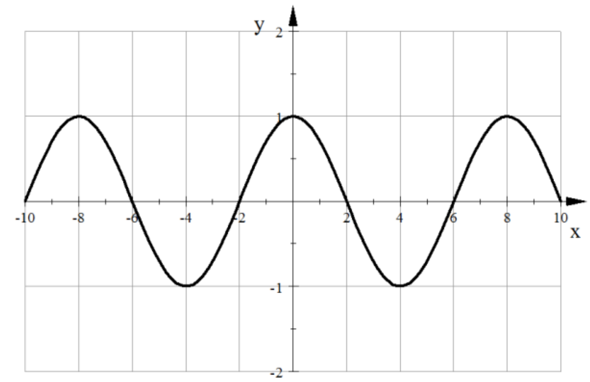
8. If you knew all the asymptotes of the rational function, would there be a unique function that corresponds to these asymptotes? What other information would you need?

9. For the functions $y = f(x)$ below, draw rough sketches their reciprocal function graphs $y = \frac{1}{f(x)}$ and clearly label any horizontal and vertical asymptotes.

a.

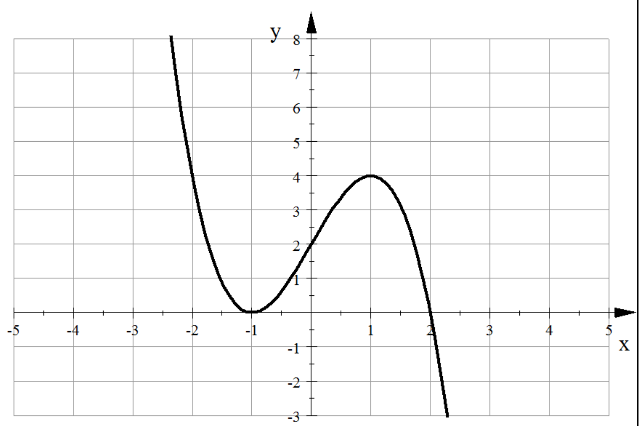


b.

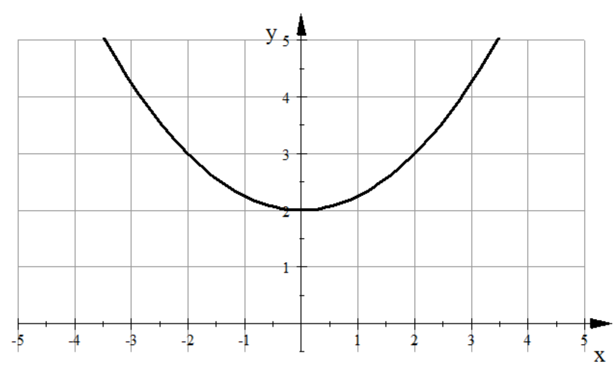


This function with a repeating pattern is called a "Periodic function" and is defined on $(-\infty, \infty)$.

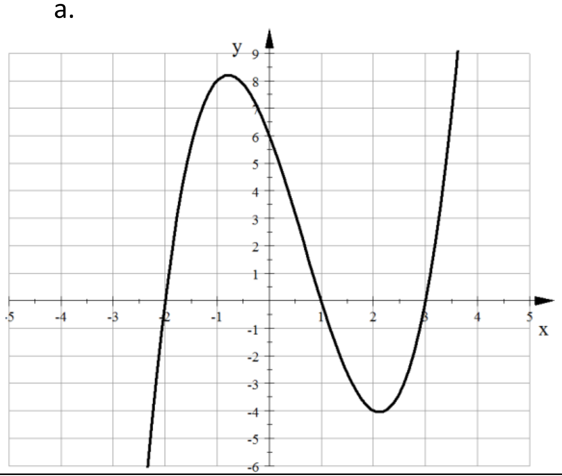
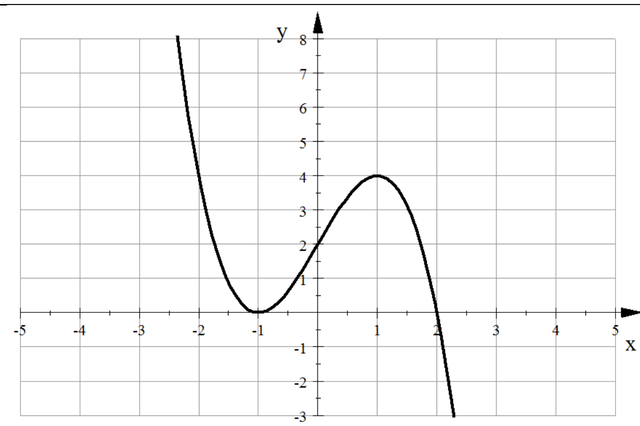
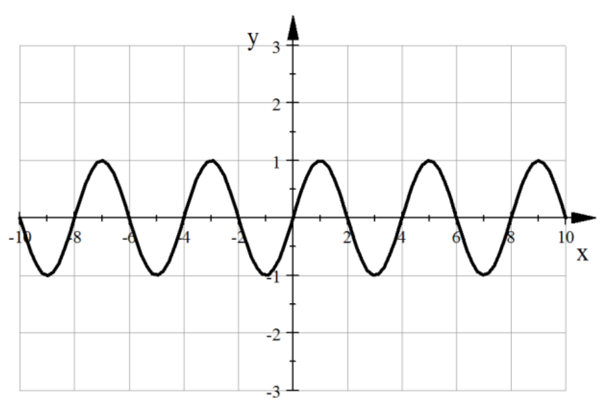
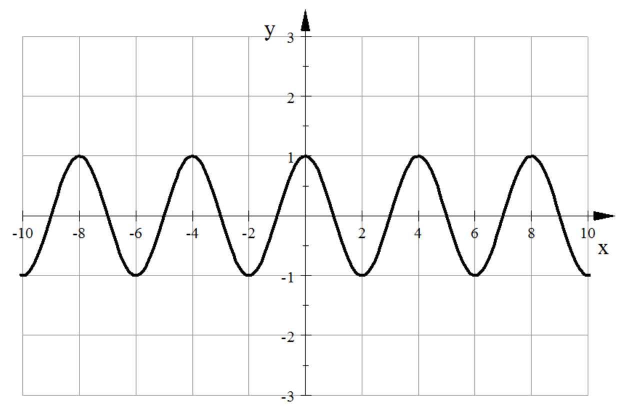
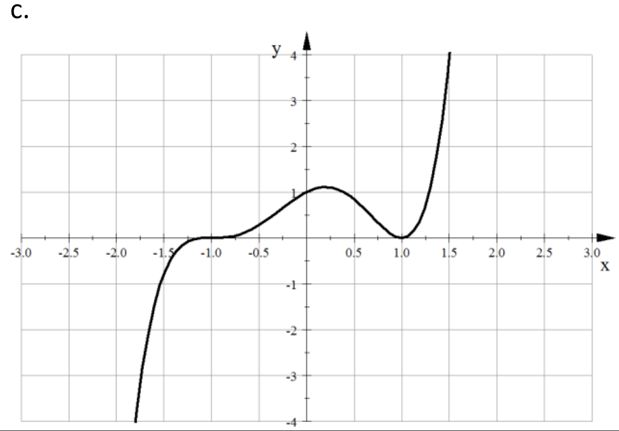
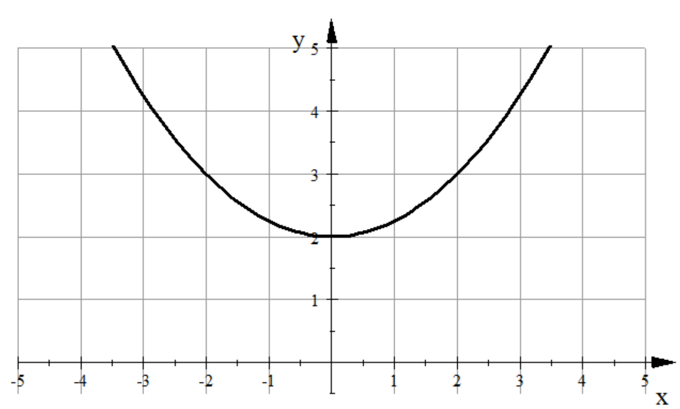
c.



d.



10. Draw a rough graph the function $= \frac{f(x)}{g(x)}$, given the graphs of $y = f(x)$, and $y = g(x)$. Locate all vertical asymptotes and x-intercepts.

$y = f(x)$	$y = g(x)$
a. 	
b.  This is not a polynomial but a periodic function defined on $(-\infty, \infty)$	 This is not a polynomial but a periodic function defined on $(-\infty, \infty)$
c. 	

11. For all the rational functions below find the requested information below and then graph the function. In your graph show all the relevant information and label all parts of the graph. Draw the asymptotes if any as dotted lines.

I. Perform long division and write the rational function $\frac{Numerator}{Denominator} = \frac{remainder}{Denominator} + Quotient$ to get the end-behavior.

II. Locate the vertical asymptotes.

III. Locate the x-intercepts and the y-intercept if possible

IV. Make a T-table to include a couple of points from each piece of the graph in between vertical asymptotes.

V. Find all points (if any) where the graph intersects the horizontal or oblique asymptotes.

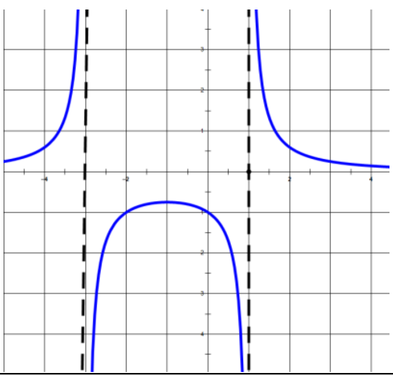
a) $g(x) = \frac{1}{x-3}$	n) $y = \frac{3x^2+4x+1}{x+2} = 3x - 2 + \frac{5}{x+2}$
b) $f(x) = \frac{1}{(x-3)^2}$	o) $f(x) = \frac{2x^2+1}{x-1} = 2x + 2 + \frac{3}{x-1}$
c) $f(x) = \frac{1}{(x-3)^3}$	p) $T(x) = \frac{3x^2+4x+1}{(x+2)^2} = 3 + \frac{-8x-11}{(x+2)^2}$
d) $g(x) = \frac{2}{x-3}$	q) $P(x) = \frac{3x^3+x^2+3x+1}{x^2-4} = 3x + 1 + \frac{15x+5}{x^2-4}$
e) $f(x) = \frac{2}{(x-3)^2}$	r) $S(x) = \frac{3x^3-3x-1}{(x+1)^2} = 3x - 6 + \frac{6x+5}{(x+1)^2}$
f) $f(x) = \frac{-2}{(x-3)^3}$	s) $r(x) = \frac{x+1}{x^2-4}$
g) $f(x) = \frac{x-1}{x-3}$	t) $f(x) = \frac{3x-2}{x^2-4}$
h) $f(x) = \frac{x-1}{(x-3)^2}$	u) $S(x) = \frac{x^2+2x+1}{x^2-1}$
i) $r(x) = \frac{1}{(x-3)(x+3)}$	v) $T(x) = \frac{x^2-1}{x^2+2x+1}$
j) $h(x) = \frac{x+1}{(x-3)(x+3)}$	w) $H(x) = \frac{x^2-1}{x+1}$
k) $R(x) = \frac{x+1}{(x+2)^2(x-1)}$	x) $Q(x) = \frac{x^3-x^2+4x}{x-1}$
l) $f(x) = \frac{1}{x^2-4}$	y) $R(x) = x^2 - \frac{1}{x}$
m) $g(x) = \frac{(x+2)^2(x-4)}{(x-1)}$	z) $Q(x) = \frac{e^x}{x}$

12. Choose the graphs below that match the graphs of the rational functions below.

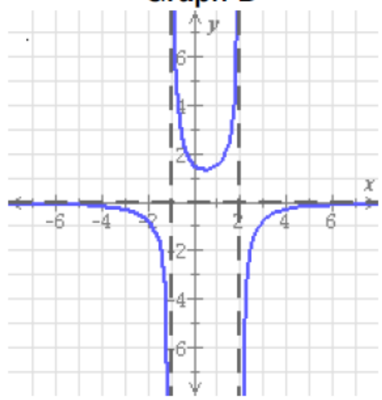
I. $g(x) = \frac{3x+3}{x^2+2x-3}$

II. $h(x) = \frac{3}{x^2+2x-3}$

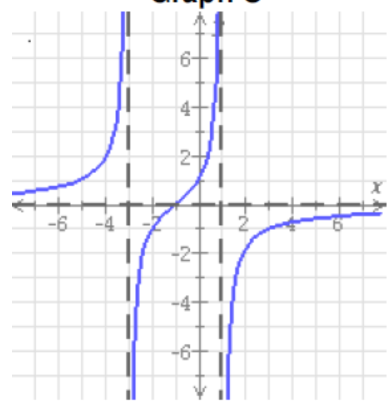
Graph A



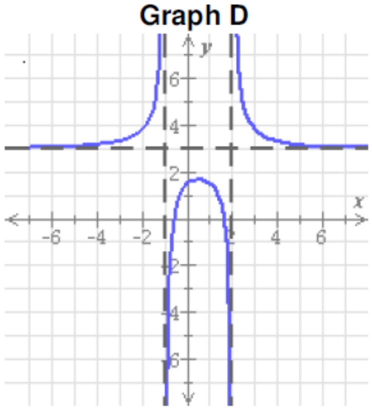
Graph B



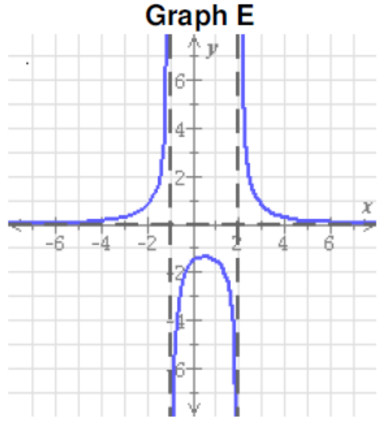
Graph C



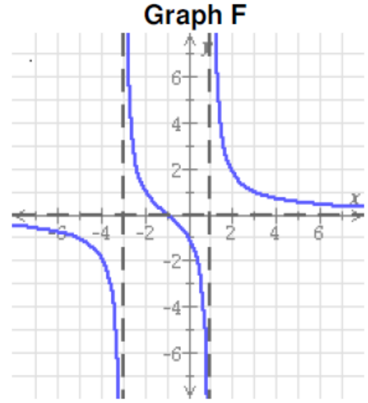
Graph D



Graph E

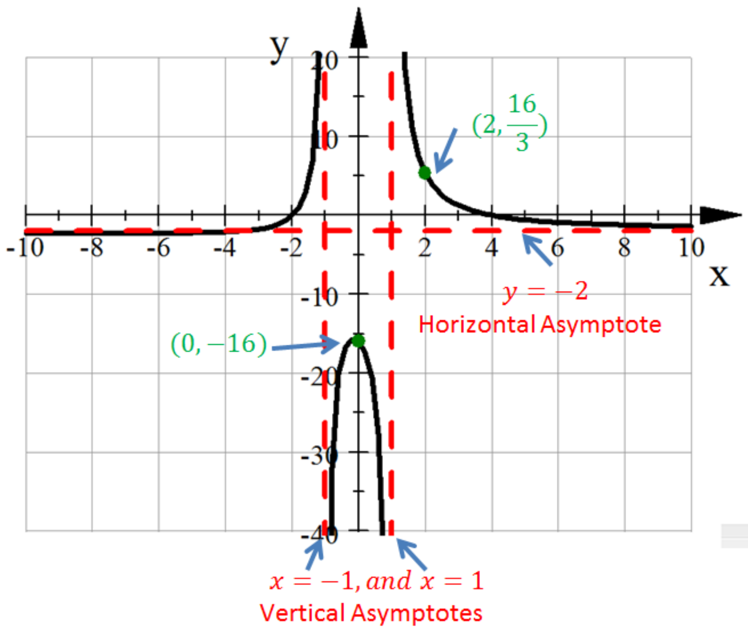


Graph F

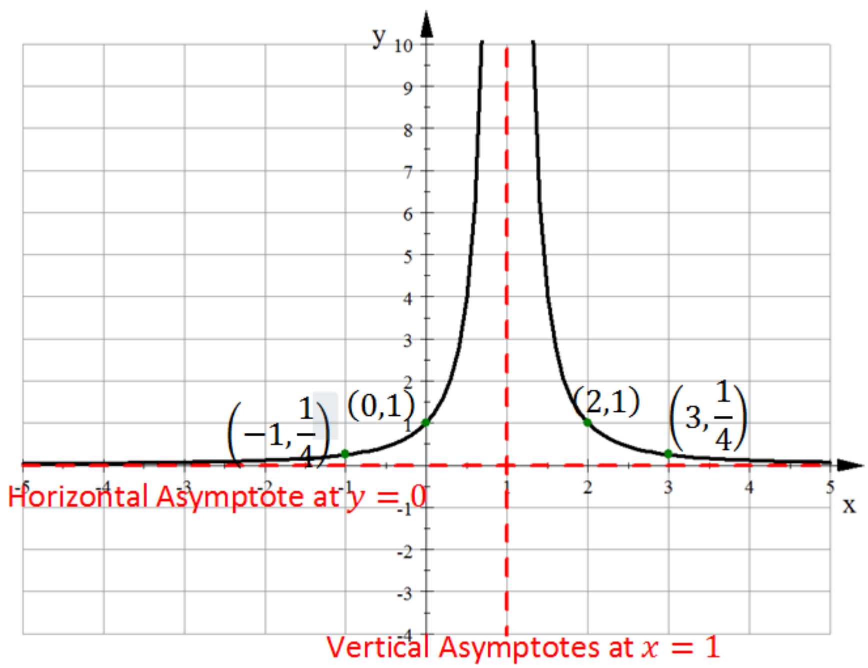


13. For all the graphs below find a rational function representing it. Use the lowest degree numerator and denominator to achieve the given vertical asymptotes and x-axis intercepts. Is there a unique formula for all of them, or can some of the graphs below could have more than one formula? Explain your reasoning clearly.

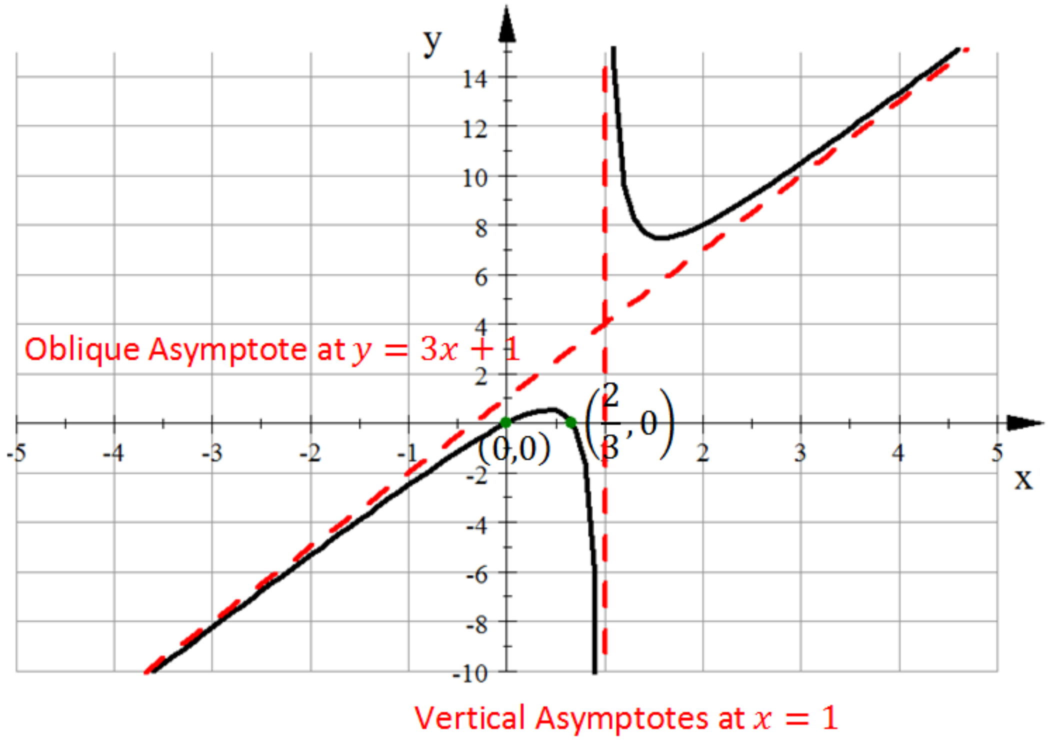
a.



b.



c.



14. For the functions below, please do the following.
    I. Find the domain of each function.
    II. Determine all the vertical asymptotes.
    III. Determine the end behavior of the function.
    IV. Use all the information found above to sketch the functions.

a. $y = \frac{\sqrt{x-1}}{x+1}$

b. $y = \sqrt{x-1} + \frac{1}{x+1}$

c. $y = 2x^2 - 4 + \frac{1}{x-1}$

d. $y = 2x^2 + 1 - \frac{1}{x^2-1}$

15. Create a rational function with vertical asymptotes at $x = -2$, $x = 1$ and slant asymptotes at $y = -3x + 5$.

16. Create a quotient of function $y = \frac{f(x)}{g(x)}$ with a vertical asymptote at $x = 2$, and so that as $x \to \pm\infty$, $y \to 3x^2 + 5x - 1$.

Section 3.1 Worksheet	**Date:_______________ Name:____________________________**
Concept	**Meaning in words and/or examples as required**
1. Equation	
2. Inequality	
3. Solutions, or zeros of an equation	
4. Solutions of an inequality	
5. Quadratic Formula	
6. Discriminant	
7. Complex numbers	
8. Maximum or Minimum Value of a quadratic function	
Difficulties encountered in the section:	

Exercises 3.1

1. What is the difference between x-intercepts, y-intercepts of a function, and solutions of an equation?

2. How would you find the maximum or minimum of a parabola?

3. What is the difference between solutions of an equation and that of an inequality?

4. Can you use your knowledge of solving quadratic equations, exponential and logarithmic functions to solve the following problems?
    a. $2^{2x} - 2^x - 2 = 0$
    b. $x^4 - 3x^2 - 4 = 0$

5. Simplify the following and write your answer in standard $a + bi$ form.

a. $\sqrt{-72}$

b. $\frac{\sqrt{-66}}{\sqrt{-6}}$

c. $\sqrt{-121}\sqrt{144}$

d. $(3 + 5i)(-5 + 6i)$

e. $\frac{4-2i}{-2-5i}$

f. i^{35}

6. Solve the following equations for the given variable. If there is more than one solution, separate them with commas.

a. $(5y + 4)(2y - 3) = 0$

b. $u^2 - 10u + 21 = 0$

c. $5w^2 = 17w - 6$

d. $x^2 - 10x + 10 = 0$
(by completing the square)
Form:

- $(x +$ ____ $)^2 =$ ______
- $(x -$ ____ $)^2 =$ ______

Solution

$x =$ ________

e. $2x^2 + 5x - 1 = 0$

f. $2x^2 - 3x + 6 = 0$

7. Find the discriminant and determine the number of real solutions of the quadratic equation. Then find the actual solutions (real or complex).

a. $4x^2 - 12x + 9 = 0$

Discriminant:

Number of solutions:

Actual solutions:

b. $-2x^2 - 6x + 8 = 0$

Discriminant:

Number of solutions:

Actual solutions:

c. $2x^2 - 5x + 8 = 0$

Discriminant:

Number of solutions:

Actual solutions:

8. Determine all the solutions to the equations below. If there is more than one solution, separate them with commas.

a. $x^4 - 5x^2 - 6 = 0$

b. $x^{4/3} - 5x^{2/3} - 6 = 0$

c. $y^6 - 5y^3 - 6 = 0$

d. $\frac{1}{x^2} - \frac{37}{x} + 36 = 0$

e. $(x-1)^2 - 37(x-1) + 36 = 0$

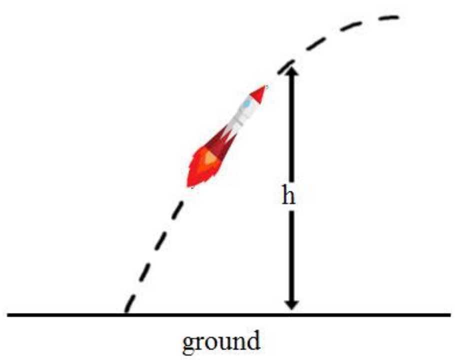
f. $u^4 + 2u^2 + 1 = 0$

g. $2x(x-1)^3(2x+3)^4(5-x)^2(x^2+4)=0$

Solve the following problems. If there is no solution, please state so.

9. The length of a rectangle is 5 yd less than twice the width, and the area of the rectangle is $33\ yd^2$. Find the dimensions of the rectangle.

10. A model rocket is launched with an initial velocity of 235 ft/s. The rocket's height h (in feet) t seconds after it is launched is given by the following.
$h = 235t - 16t^2$



a. Find all the values of t for which the rocket's height is 151 feet. Round your answer(s) to the nearest hundredth.
b. Also locate the time when the rocket is at its maximum height.
c. The rocket deploys a parachute just as it reaches maximum height and after that, it falls at a constant rate of $20\ \frac{ft}{s}$. Determine how long after the launch the rocket will hit the ground.

11. The cost C in (dollars) of manufacturing x wheels at Ravi's Bicycle Supply is given by the function $C(x) = 0.5x^2 - 170x + 25{,}850$. What is the minimum cost of manufacturing wheels? Do not round your answer.

12. Consider the parabola shaped graph of the quadratic function $f(x) = -2x^2 + 16x - 34$.

a. Complete the square to locate the vertex. Then find the maximum or minimum of the function $f(x)$ and indicate which it is a maximum, or a minimum.

b. Locate the focus and x- and y-axis intercepts and plot the graph of $y = f(x)$ with these features labled.

13. Find the intersection points of the graphs of functions below. Then sketch graph of both functions, and label the intersection points clearly.
$y = f(x) = x^2 - 6x + 4$ and $y = g(x) = 1 - 2x$

14. Find the intersection points of the graphs of both relations below. Then the sketch graph of both the relations, and label the intersection points clearly.
$y = f(x) = x^2 - 5$ and the circle given by $x^2 + y^2 = 5$.

Section 3.2 Worksheet	**Date:______________ Name:___________________________**
Concept	**Meaning in words and/or examples as required**
Rational Zeros Theorem	
Factor Theorem	
Remainder Theorem	
Complex Conjugates Zeros Theorem	
Real Zeros	
Complex Zeros	
Synthetic Division	
Fundamental Theorem of Algebra	
Difficulties encountered in the section:	

Exercises 3.2

1. Let $P(x)$ be a polynomial.
   a. What can you say about the non-real zeros of the equation $P(x) = 0$, if the coefficients of $P(x)$ are real?

   b. What is the maximum number of non-real zeros a polynomial of odd degree can have?

   c. Can a polynomial of odd degree have all non-real zeros?

   d. Why must an odd degree polynomial with real coefficients have at least one real zero?

   e. Can you have a polynomial with non-real coefficients of odd degree with all non-real zeros?

2. Suppose $R(x)$ is a polynomial of degree 13 whose coefficients are real numbers. Also, suppose that $R(x)$ has the following zeros: $7, -8\,, 5i, -2-4i$.

a) Find another zero of $R(x)$.

b) What is the maximum number of real zeros that $R(x)$ can have?

c) What is the maximum number of non-real zeros that $R(x)$ can have?

d) If the leading coefficient of the polynomial is -3, list three possible formulas for $R(x)$ with the degree and zeros given above.

3. Find a polynomial $f(x)$ of degree 4 that has the following zeros $-2,\ 1,\ -6,\ and\ 0$. Leave your answer in factored form.

4. Find the real polynomial of degree 4 with zeros $a = 3, -\frac{1}{2},\ and\ 2-3i$ and leading coefficient

5. Perform the following division to find the quotient and the remainder. Write your answer in the Quotient $+\frac{Remainder}{x-4}$ form. When possible use synthetic division.

a. $(6x^2 + 37x + 39) \div (x + 5)$

b. $(3x^2 - x^3 + 6x - 8) \div (x - 4)$

c. $(24x^3 + 4x^2 + 14x + 3) \div (6x - 2)$

6. Use the rational zeros theorem to list all possible zeros of the following. Be sure that no value in your list appears more than once.

a. $g(x) = -5x^4 - x^3 - 3x^2 - 3x + 3$

b. $g(x) = -6x^4 - x^3 - 3x^2 - 3x + 9$

7. The function below has at least one rational zero. Use this fact to find all the zeros of the function.
$g(x) = 4x^3 + 12x^2 - x - 3$

   If there is more than one zero, separate them with commas. Write exact values, not decimal approximations.

8. The function below has a rational zero. Use this fact to find all the zeros of the function.

   $f(x) = 2x^3 - 3x^2 + x - 6$

9. The function below has at least one rational zero. Use this fact to find all the zeros of the function. Write exact values, not decimal approximations.
$g(x) = 7x^4 + 20x^3 + 10x^2 - 5x - 2$

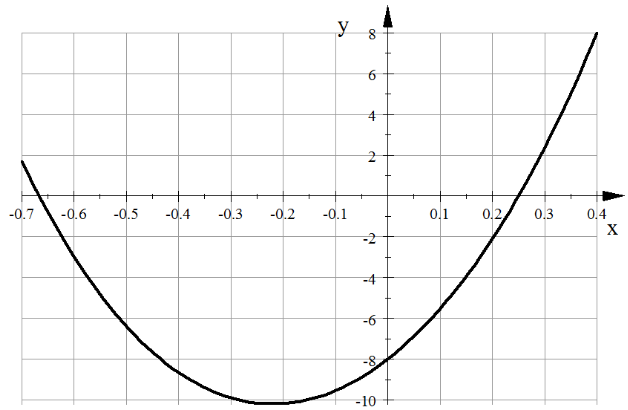
10. For the polynomial below -2 is a zero. Express $h(x)$ as a product of linear factors.

$h(x) = x^3 + 8x^2 + 30x + 36.$

11. The function below has at least one rational zero. Use this fact to find all the zeros of the function.
$h(x) = 5x^4 - 29x^3 - 40x^2 - 13x - 7$

12. Find all the other zeros of $P(x)$ given that $3-3i$ is a zero. (Hint: find a quadratic factor with real zeros, and then use long division to obtain a second quadratic factor.)
$P(x) = x^4 - 7x^3 + 22x^2 - 6x - 36$

13. Find all the zeros of $f(x) = 12x^4 + 5x^3 + 46x^2 + 20x - 8$ whose graph is given below.



14. Find the exact values of the points of intersection of the two functions below.
$f(x) = 3x^5 - 2x^4 - 4x^3 - 5x^2 + 6x - 10$ and $g(x) = x^5 + x^4 - 2x^3 - 8x^2 + 10x - 16$

15. Create two polynomial functions that intersect each other at $x = 2$, $x = -3$, $x = 2\sqrt{3}$, and $x = -2\sqrt{3}$.

Section 3.3 Worksheet	**Date:_______________ Name:____________________________**
Concept	**Meaning in words and/or examples as required**
List different methods of solving exponential equations	
List different methods of solving exponential equations	
Difficulties encountered in the section:	

Exercises 3.3

1. How are the product, quotient and power properties of logarithmic functions used to solve logarithmic or exponential equations?

2. List at least three applications of being able to solve exponential and logarithmic equations not listed in the section.

Find all the solutions to the following equations.

3. $log_3(x+1) = 2$

4. $log_3(x+1) - log_3(x) = 2$

5. $log_3(2x-1) + log_3(x+1) = 2$

6. $log_4(-1-2x)=-1$

7. $4+\log(2x-1)=5$

8. $log_5(x-3)=1-log_5(x-7)$

9. $\ln(x+4)-\ln 18=\ln 5$

10. $125 = 25^{-x-2}$

11. $2^{x^2-61x} = 64^{3-9x}$

12. $15^{-8y} = 6$

(Round your answer to the nearest hundredth. Do not round any intermediate computations.)

13. $e^{-8u} = 6$

(Round your answer to the nearest hundredth. Do not round any intermediate computations.)

14. $17^{-x-3} = 16^{-8x}$

(Write the exact answer using base-10 logarithms)

15. $3^{x-1} = 5^{2x-1}$

(Write the exact answer using natural logarithms)

16. $200e^{0.05t} = 50$

17. $3e^{2x} - 5e^{x} + 2 = 0$ (Hint a substitution $u = e^{x}$ will be useful.)

18. $9^{x} - 3^{x} - 2 = 0$ (Hint a substitution $u = 3^{x}$ will be useful

19. $1500\left(1 + \frac{0.05}{4}\right)^{4t} = 3000$

20. Write a word problem that would give rise to the exponential equation below and solve the equation
$2000(1.02)^{4t} = 5000$

21. Write a word problem that would give rise to the exponential equation below and solve the equation.

$$20 = 250\left(\frac{1}{2}\right)^{\frac{t}{5730}}$$

22. A car is purchased for $\$28{,}500$. After each year the resale value decreased by 35%. What will be the resale value be after 4 years? Also determine when the resale value will be less than \$5000. Round your answers to the nearest whole number.

23. A loan of $\$39{,}000$ is made at 5% interest, compounded annually. Assuming no repayment is made, after how many years will the amount due reach $\$63{,}000$ or more? (Use a calculator if necessary.) Write the smallest possible whole number answer.

24. The number of bacteria in a certain population increases according to a continuous exponential growth model, with a growth rate parameter of 4.1% per hour. How many hours will it take for the sample to double?

Note: This is a continuous growth model. $(A(t) = A_0 e^{rt})$

Do not round any intermediate computations, and round your answer to the nearest hundredth of an hour.

25. An initial amount of $\$1800$ is invested in an account at an interest rate of 2% per year compounded continuously. Find the amount in the account after 6 years. Round your answer to nearest cent. Also how many years are required for the value to reach $10,000.

26. The light intensity in Lake Superior decreases due to absorption by 12% for each meter of depth. If the intensity just below surface is at 150$\frac{w}{m^2}$. Write a formula for the intensity at dept x meters. Also determine how deep one must go for the light intensity to be 0.05$\frac{W}{m^2}$.

27. A model for world population in billions t years after the year 2000 is given by $(t) = \frac{24}{1+3e^{-0.04t}}$.
Determine in what year the population is projected to be 12 billion by this model.
Also, what does the model predict as t gets very large?

28. The value of an exponential growth stock fund increased from \$10,000 to \$18,000 over a 12 year period.

a. Find the continuous growth rate and model $A(t) = Pe^{rt}$ that describes this growth.

b. Find the quarterly compounding growth rate and model $A(t) = P\left(1 + \frac{r}{4}\right)^{rt}$ that describes this growth.

29. A sample of radioactive iodine decreased from 50 micrograms to 10 micrograms over a 19 day period.

a. Find a decay model $A(t) = A_0e^{rt}$ that models this decay.

b. Determine the half-life of this radioactive iodine.

30. Extra Credit: You want to buy a house that might be in range of \$180,000 to \$200,000 in 10 years. You know you will need to save at least a 20% down payment cost. How much should you invest per month starting now in an account that pays 5% interest so that in 10 years you will have enough for your down payment at that time? Please explain carefully how you computed this amount. You might start with \$100/month and see how that does and adjust this amount accordingly.

Section 3.4 Worksheet	**Date:______________ Name:___________________________**
Concept	**Meaning in words and/or examples as required**
1. System of Equations	
2. System of Inequalities	
3. Solutions to system of equations	
4. Solutions to system of inequalities	
5. Graphing Method	
6. Elimination Method	
7. Substitution Method	
Difficulties encountered in the section:	

Exercises 3.4

1. What is a system of equations?

2. When would you use the graphing method?

3. When would you use the elimination method?

4. When would you use the substitution method?

5. What is the difference between solving system of equations versus system of inequalities?

6. What is most number of solutions you expect from of a system of equations where both equations are second degree polynomials?

7. What is least number of solutions you expect from of a system of equations where both equations are second degree polynomials?

8. Find the solutions to system of equations and inequalities below.

a. $\begin{cases} 5x - y = 4 \\ x - 2y = 3 \end{cases}$

b. $\begin{cases} \frac{1}{2}x - y = 4 \\ x - 2y = 3 \end{cases}$

c. $\begin{cases} x - 2y = 4 \\ -4x + 16 = -8y \end{cases}$

d. $\begin{cases} 1.5x - 2y = 1 \\ x - 5y = -3 \end{cases}$

e. $\begin{cases} 2y = 1 - x \\ 4x - 5y = -3 \end{cases}$

f. $\begin{cases} y = 1 - 3x \\ 6x + 2y = 5 \end{cases}$

g. $\begin{cases} x = 4 - 3y \\ 2x + 8 = 5y \end{cases}$

h. $\begin{cases} \frac{2}{3}y - 3x = \frac{1}{2} \\ \frac{1}{3}x - \frac{2}{5}y = \frac{2}{5} \end{cases}$

i. $\begin{cases} 0.55x - 1.01y = 4.13 \\ 1.2 + 3.24y = -7.32 \end{cases}$

j. $\begin{cases} x^2 + y^2 = 4 \\ x - y^2 = -2 \end{cases}$

k. $\begin{cases} x^2 + y^2 = 4 \\ x^2 - y^2 = 1 \end{cases}$

l. $\begin{cases} x^2 + y^2 = 4 \\ x - y^2 = -6 \end{cases}$

m. $\begin{cases} 9x^2 + 4y^2 = 36 \\ x = 3 \end{cases}$

n. $\begin{cases} 9x^2 + 4y^2 = 36 \\ y = 3x - 1 \end{cases}$

o. $\begin{cases} 9x^2 + 4y^2 = 36 \\ 3x - y = -12 \end{cases}$

p. $\begin{cases} x - y < 4 \\ x - 2y \geq 3 \end{cases}$

q. $\begin{cases} x^2 + y^2 \leq 4 \\ x - y^2 < 1 \end{cases}$

r. $\begin{cases} x^2 + y^2 > 4 \\ x^2 - y^2 < 1 \end{cases}$

s. $\begin{cases} x^2 + y^2 \geq 4 \\ x - y^2 \geq -6 \end{cases}$

9. Use systems of equations or inequalities to solve the following.
   a. Paul invests \$100,000 in three stocks that pay dividends of 6%, 8%, and 10%. The amount invested in the 10% is twice the amount at 6% and the return on the whole investment is \$8600 per year. How much is invested in each stock type?
   b. Tickets at a play were sold for either \$5 for children, \$7 for college students and \$9 for general adult admission. The total number of tickets sold was 400 and the total ticket revenue was \$2500. How many tickets of each type were sold if the number of general adult tickets was 1/7th of the total of the child and student tickets sold.

Section 3.5 Worksheet	**Date:______________ Name:__________________________**
Concept	**Meaning in words and/or examples as required**
1. Matrix	
2. Elementary Row Operations	
3. Gauss Elimination Method	
Difficulties encountered in the section:	

Exercises 3.5

1. Please answer the following without looking at other resources.

a. Is it possible to add two different matrices? If yes, explain when and how.

b. Is it possible to subtract two different matrices? If yes, explain when and how.

c. Is it possible to multiply two different matrices? If yes, explain when and how.

d. Is it possible to divide two different matrices? If yes, explain when and how.

2. Why should we bother learning about the Gauss Elimination method?

3. What are the advantages of writing the system of equations using matrices?

4. Do you think we use matrices for a two by two system of linear equations?

5. What about using matrices for two by two nonlinear system of equations? If you think we can, explain how?

6. Solve the following systems using Gauss Elimination Method.

a. $\begin{cases} x - 2y + z = 4 \\ 3x - y + 6z = -1 \\ 4x + 3y - 7z = 5 \end{cases}$

b. $\begin{cases} 4x + 5y + 2z = -5 \\ x + y - 3z = -4 \\ -3x + 2y - z = -13 \end{cases}$

c. $\begin{cases} x + y + 2z = 4 \\ 2x + 2y + 4z = -4 \\ -3x + 2y - z = -13 \end{cases}$

d. $\begin{cases} x - y + 2z = -5 \\ x + y - 3z = -4 \\ -2x + 2y - 4z = 10 \end{cases}$

e. $\begin{cases} \frac{1}{2}x - \frac{1}{3}y + \frac{1}{5}z = \frac{11}{30} \\ x - \frac{2}{3}y + z = \frac{4}{3} \\ 4x - y + \frac{1}{2}z = \frac{7}{2} \end{cases}$

f. $\begin{cases} 0.21x - 0.2y - 1.4z = 2.01 \\ x - 0.33y + 1.4z = 0.26 \\ -0.4x + 3.1y + 2.1z = -8.7 \end{cases}$

g. $\begin{cases} 3x - 2z = 7 \\ -x + 4y + 5z = -7 \\ -2x - 4y - 3z = 0 \end{cases}$

h. $\begin{cases} x + y + z + w = 1 \\ 2x - y + 3z - y = -9 \\ x - y - z + w = 1 \\ x - 2y + z - 2w = -8 \end{cases}$

i. $\begin{cases} x - 2y + 3z - w = 12 \\ -x + 3y - 2z - 3w = 1 \\ -4x + 6y + z - 3w = 1 \\ x - 2z + w = -6 \end{cases}$