Math 110 Final Exam Review Name: _____

Please show all your work to receive full credit.

1. Create a function $y = f(x)$ that satisfies the following criteria.
   a. Is One-to-one
   b. Has domain $(-5,1] \cup [3,10]$
   c. $f(-3) = 0$
   d. $f(3) = -1$
   e. $f(0) = 5$
   f. For at least one x in the domain of f, $f(x) = -2$

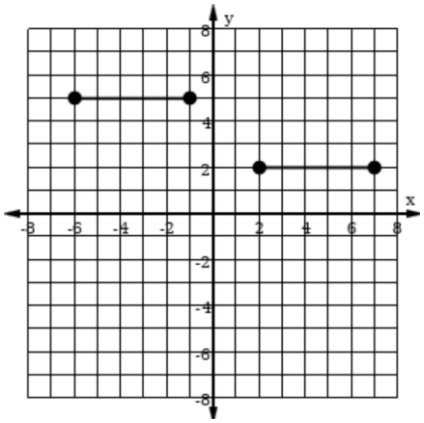
   Questions about your function-

   i) Do you think the function you came up with is unique? Explain why.
   ii) Find the range of your function.
   iii) What is the x-intercept?
   iv) What is the y-intercept?
   v) List all the intervals where your function is increasing, decreasing, or constant.
   vi) What is $f^{-1}(x)$?
   vii) Is your function even, odd or neither?

2. Create an exponential function so that $f(0) = 1500$ and then answer the following questions
   a. What is the inverse of this function?
   b. Find the domain and range of the function.
   c. Find the domain and range of its inverse.
   d. Find $f(5)$.
   e. Create a word problem in which the function you created is the mathematical representation of the situation.

3. Create a polynomial function $y = f(x)$ that has degree 8, leading coefficient of -2, zeros are as follows: $x = 1 + 2i, x = 3,\ x = 1$ of multiplicity 2, $x = -1$ of multiplicity 3. In addition, then
    a. What is a possible formula for the polynomial function?
    b. Is this formula unique? If not, what could the other possibilities be?
    c. What are all the x-intercept's?
    d. What is the y-intercept?
    e. Sketch the graph of the polynomial function $y = f(x)$ you found.
    f. Describe the end-behavior and show all relevant points.
    g. Solve the equation $f(x) = 0$ and find all real solutions.

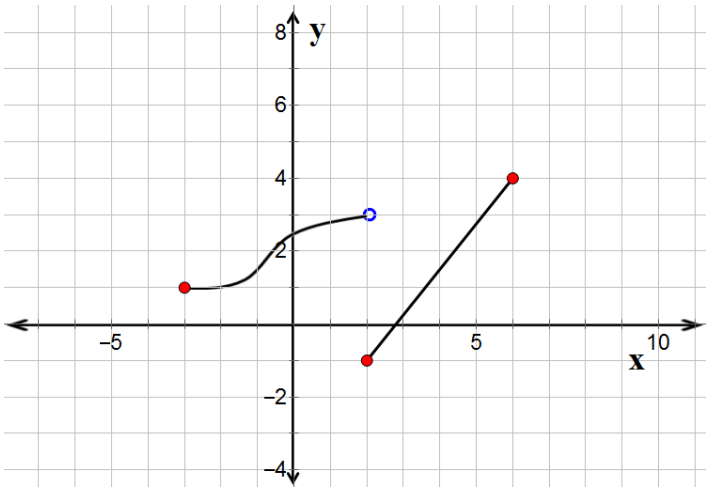
4. Create a system of equations in two variables that has the solutions of $(-2,2)$, and $(4,2)$.
5. Create a system of equations in three variables that has the solution of $(1, -2,3)$.
6. For the relations below, determine if they are functions or not. If they are functions are they one-to-one or not. Find the domain and range of the relations.



○ Function	○ One-to-One
○ Not a Function	○ Not One-to-One

Domain:

Range:

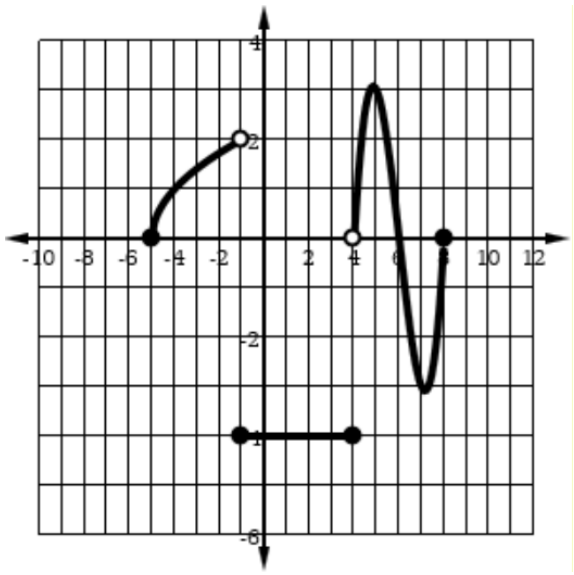


○ Function	○ One-to-One
○ Not a Function	○ Not One-to-One

Domain:

Range:

7. Use the graph of the function $y = f(x)$ shown to the right fill in the blanks below.



a. $f(2) =$ _____

b. $f($_____$) = 0$

c. Domain of $f =$_____

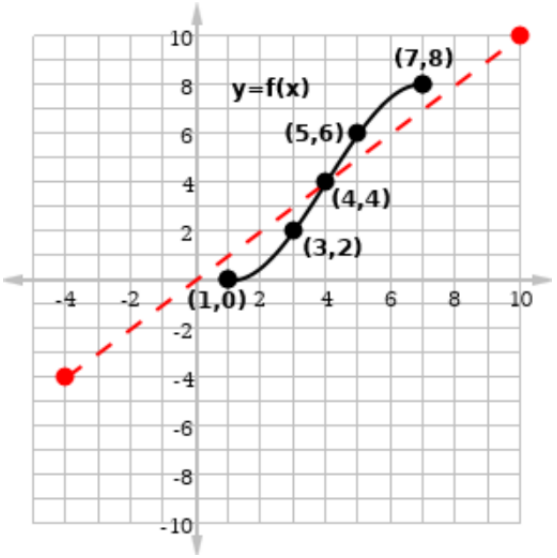
d. Range of $f =$_____

e. Is $f(x)$ a one-to-one function?

☐ Yes ☐ NO

8. Given the one-to-one $y = f(x)$ function below.

a. Sketch the graph of the inverse function on the same coordinate axes as $y = f(x)$.



b. Find the following

Domain of $f(x) =$_____

Range of $f(x) =$_____

Domain of $f^{-1}(x) =$_____

Range of $f^{-1}(x) =$_____

c. Use the graph to determine the value of

$f^{-1}(6) =$_____

9. The functions f us defined as follows: $f(x) = \begin{cases} 3\sqrt{x} & if\ x > 4 \\ 2x - 1 & if\ x < -4 \end{cases}$

Find the following. a) $f(4)$

b) $f(-1)$

10. Evaluate Let $f(x) = \sqrt{x + 11}$ and $g(x) = 2x^2 - 3$. Find and simplify each of the following.

(a) $(f - g)(0)$

(b) $(g \circ f)(x)$

(c) $(g \circ f)(-2)$

11. Sketch the graphs of the relations below. Show all relevant information like intercepts, asymptotes of any, center, vertex/vertices, focus, directrix, as necessary based on the relation. If it your relation is a conic section please identify which conic section it represents.

a. $y = \sqrt{x-2} + 5$

b. $y = (x-2)^2 - 3$

c. $y = e^{x+3} - 1$

d. $y = log_2(x+2) - 4$

e. $y = -3x^2(x-1)^3(x+1)^2$

f. $y = 2x^6 + x^5 - 5x^4 - 2x^3 + 4x^2 + x - 1$ (hint: Use the fact that $x = -1$ is a zero of multiplicity 3)

g. $y = \frac{1}{x-3}$

h. $y = \frac{2x-5}{3x-2}$

i. $y = \frac{2x^2-5x-3}{x^2-2x+1}$

j. $y = \frac{2x^2-5x-3}{x-1}$

k. $y = \frac{(x-1)^3(x+1)^2}{(x-2)(x+2)}$

l. $(x-2)^2 + (y+3)^2 = 4$

m. $\frac{(x-2)^2}{4} + \frac{(y+3)^2}{9} = 1$

n. $-\frac{(x-2)^2}{9} + \frac{(y+3)^2}{4} = 1$

o. $4x^2 - 9y^2 + 8x + 36y = 68$

p. $3y^2 - 6y - 5 = x$

q. $x = -2(y-1)^2 - 5$

r. $x^2 + y^2 - 4x + 8y = 5$

12. Find all intervals where the following inequalities are true. You may use graphs of functions from part 11 to help with your answers when approrpriate.

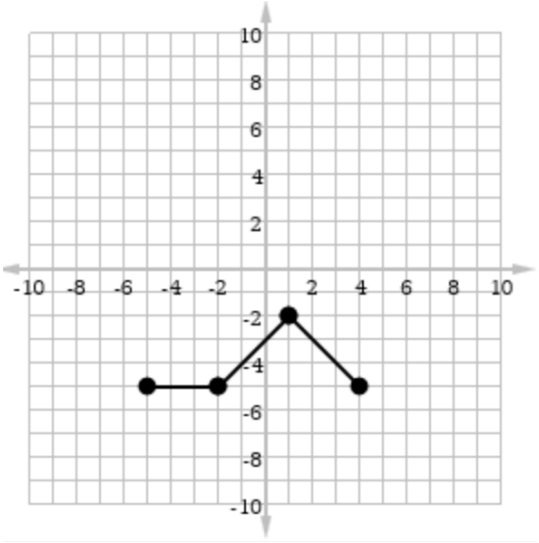
a. $-3x^2(x-1)^3(x+1)^2 > 0$

b. $2x^6 + x^5 - 5x^4 - 2x^3 + 4x^2 + x - 1 \leq 0$

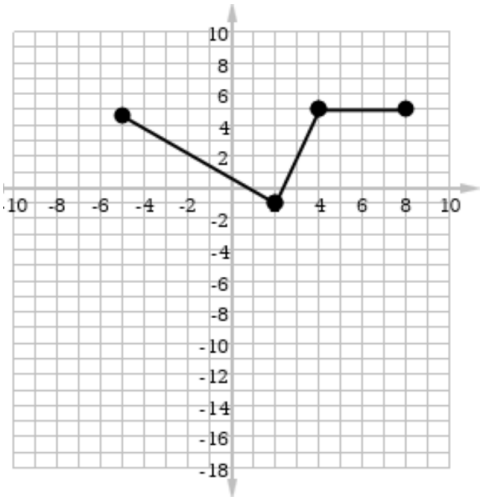
c. $\frac{2}{x-3} < \frac{1}{x-1}$

13. Sketch the graphs of the transformation of function $y = g(x)$ on the same axes.

a) Sketch the graph of $y = -g(x-2)+3$



b) Sketch the graph of $y = g(x-2)-3$



14. Solve the following equations and find all the solutions.

a. $3x^2 - 4x + 1 = 0$

b. $3(x-1)^2 - 4(x-1) + 1 = 0$

c. $3(2)^{2x} - 4(2)^x + 1 = 0$

d. $log_3(2x-1) + log_3(x+1) = 2$

e. $4 + \log(2x - 1) = 5$

f. $3x^2 - 5x + 2 = 3x - 1$

g. $2^{x^2-61x} = 64^{3-9x}$

h. $17^{-x-3} = 16^{-8x}$

i. $500e^{0.03t} = 2000$

15. Sketch solutions to systems of inequalities below. List at least two distinct solutions

a. $\begin{cases} 3x - 2y < 3 \\ x + y \geq 1 \end{cases}$

b. $\begin{cases} x - 4y \geq 1 \\ 3x + 4y < 3 \\ 2x + 4y > 2 \end{cases}$

c. $\begin{cases} y < 3x^2 - 3 \\ -5x + y \geq 5 \end{cases}$

d. $\begin{cases} y < 3^x \\ y \geq 5 \end{cases}$

16. Find the solutions to the system of equations below.

a. $\begin{cases} 3x - 2y = 3 \\ x + y = 1 \end{cases}$

b. $\begin{cases} x - 4y = 1 \\ 3x + 4y = 3 \\ 2x + 4y = 2 \end{cases}$

c. $\begin{cases} y = 3x^2 - 3 \\ -5x + y = 5 \end{cases}$

d. $\begin{cases} 9x^2 + 4y^2 = 36 \\ x = 3 \end{cases}$

e. $\begin{cases} 9x^2 + 4y^2 = 36 \\ 3x - 1 = y \end{cases}$

f. $\begin{cases} y = 3^x \\ y = 5 \end{cases}$

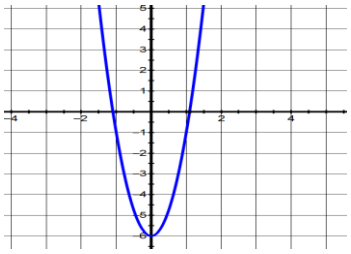
g. $\begin{cases} x + y + z = 2 \\ 2x - 3y + z = 11 \\ x - y - 3z = -6 \end{cases}$

h. $\begin{cases} x + y + 2z = 2 \\ 2x - y + 5z = 1 \\ 3x - 2y + z = -1 \end{cases}$

17. If Adam invested $5000 in an account paying 4% interest rate, compounded quarterly. How much money will he have in the account after 10 years?

18. Find the difference quotient $\frac{f(x+h)-f(x)}{h}$ where $h \neq 0$ for the function below. Explain what this quotient represents. Simplify your answer as much as possible.

$$f(x) = 5x^2 - 6$$



19. Two functions g and f are defined in the figure below. Find the domain and range of the compositions $(f \circ g)(x) = f(g(x))$, and $(g \circ f)(x) = g(f(x))$. Then evaluate the function values below.
$g(x) = \frac{x+6}{x-5}$, and $f(x) = 2x - 7$

Domain of f Domain of g		Range of f Range of g	
Domain of $(f \circ g)(x) = f(g(x))$ Domain of $(g \circ f)(x) = g(f(x))$		Range of $(f \circ g)(x) = f(g(x))$ Range of $(g \circ f)(x) = g(f(x))$	
a. $f \circ g(x)$	b. $g \circ f(6)$	c. $f \circ g(6)$	d. $g \circ f(0)$

20. Find the inverses of the following one-to-one functions. Then find the domains and ranges of the functions and their inverses.

a) $f(x) = \frac{7x+1}{2x-1}$

Domain of f:
Range of f:

Domain of f^{-1}:
Range of f^{-1}:

b) $g(x) = 2^x$

Domain of f: Domain of f^{-1}:

Range of f: Range of f^{-1}:

21. Rewrite the exponential equations in logarithmic form and logarithmic equations in exponential form. If possible, simplify your answers.

Exponential Equation	Logarithmic Equation
$e^x = 5$	
$2^{x+1} = 8$	
	$log_2(x) = -1$
	$\log(x+1) = 2$
	$\ln(x+1) = 3$
$5^{1-x} = 3$	
	$log_{\frac{1}{2}}(x) = -3$

22. Expand the following. Each logarithm in your answer should involve only one variable. Assume that all variables are positive.

a) $\log(x^3y^2) =$_____

b) $\log_2\left(\frac{x^3y^2}{\sqrt{z}}\right) =$_____

c) $\log\left(\frac{x^3}{\sqrt{z^5y}}\right) =$_____

23. Write the following as one term.

a) $4log_2x + 2log_2y =$ _____

b) $\frac{1}{3}logx - 2logy + 3logz =$ _____

24. Evaluate the following for the functions defined below.
$f(x) = 3x - 1$ and $g(x) = x^2 + 2$

a) $(f + g)(x) =$ _____

b) Domain of $(f + g)$

c) $(f + g)(3) =$ _____

d) $(fg)(x) =$ _____

e) Domain of $(fg) =$ _____

f) $(fg)(0) =$ _____

25. Solve the following

a) The length of a rectangle is 5 yd less than twice the width, and the area of the rectangle is 33 yd^2. Find the dimensions of the rectangle.

b) A rocket model is launched with an initial velocity of 235 ft/s. The rocket's height h (in feet) after t seconds is given by the following.

a. $h = 235t - 16t^2$

Find all the values of t for which the rocket's height is 151 feet. Round your answers to the nearest hundredth. If there is more than one answer, use or to separate them.

c) The cost C in (dollars) of manufacturing x wheels at Ravi's Bicycle Supply is given by the function $C(x) = 0.5x^2 - 170x + 25{,}850$. What is the minimum cost of manufacturing wheels? Do not round your answer.

d) A car is purchased for $\$28{,}500$. After each year the resale value decreased by 35%. What will be the resale value be after 4 years? Round your answer to the nearest dollar. (Write your final answer in a sentence.)

e) A loan of $\$39{,}000$ is made at 5% interest, compounded annually. After how many years will the amount due reach $\$63{,}000$ or more? (Use a calculator if necessary.) Write the smallest possible whole number answer.

f) The number of bacteria in a certain population increases according to a continuous exponential growth model, with a growth rate parameter of 4.1% per hour. How many hours will it take for the sample to double?

Note: This is a continuous growth model.

Do not round any intermediate computations, and round your answer to the nearest whole hundredth.

g) An initial amount of $\$1800$ is invested in an account at an interest rate of 2% per year compounded continuously. Find the amount in the account after 6 years. Round your answer to nearest cent.