Final Exam Review Sheet Mat 103 and Mat 105

NAME:____

Review pages 1-7 are due Dec 12. Completion of final exam review is the criterion for eligibility for final exam. The first 7 pages of the review also act as your last quiz. Please show all your work to receive full credit.

HONOR CODE: ***The work on this review represents my own. I understand that I cannot discuss or talk about anything on the review with any other human being (that includes spouses, children, classmates, tutors, other teachers, ...) until after I have receive graded work back. I also understand that I can use notes from class and/or the textbook. I further understand that in the event I don't follow this agreement I could receive a zero on this quiz and have further academic action taken against me.***

Signature: ____

When you review for the final, please make sure you can do the problems from all the previous reviews. A sampling of these problems is included here for your convenience, and you are required to attempt them all.

The most important aspect of reviewing for the final is to synthesize what you have learned so far, and to see what new directions this material can take you, to get a head start for the next course. Think about the mathematical principles and concepts you have learned. See how these concepts connect to each other, so you are not just memorizing procedures but also doing problems from a deeper understanding. This will also help you with long-term retention.

To accomplish these goals, solve the following problems using the tools you have learned all semester. Some of these problems you may have never seen before, but you only have to use the material you have learned so far. You should be confident you can do these (don't forget mindful breathing and awareness when you get stressed). Be creative in your solutions, and write them as if you are explaining it to someone who has not yet taken this course. Write the solutions neatly, and for every problem please list all the mathematical principles and concepts that you used.

Reasoning Assessment

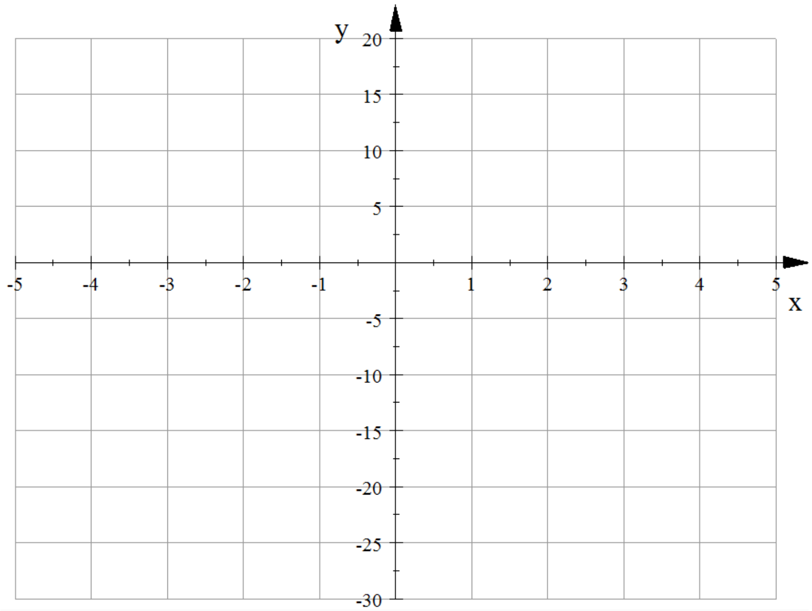
1. Consider the equation $y = (x-1)^3(x+1)^2(x-3)$. This equation has a dependent variable y and an independent variable x. For every real number input of x there is a unique real number output y.
   This makes y a function of x. In other words, $y = f(x) = (x-1)^3(x+1)^2(x-3)$.
   There are many aspects of this function that you have actually seen without necessarily knowing it. Answering the questions below will help you to identify these aspects of the function $f(x)$.
   a. Solve the equation $(x-1)^3(x+1)^2(x-3) = 0$. (3 pts)

   b. What information do the solutions of the equation in part "a." give regarding the graph of the function $y = (x-1)^3(x+1)^2(x-3)$? (3 pts)

   c. Solve the two inequalities below. Please write your solutions in interval notation and graphical notation. (HINT: Make a sign chart using test points…) (6 pts)
      i. $(x-1)^3(x+1)^2(x-3) > 0$
      ii. $(x-1)^3(x+1)^2(x-3) < 0$

   d. What information do the solutions of the inequalities in part "c." give regarding the graph of the function $y = (x-1)^3(x+1)^2(x-3)$? (2 pts)

e. Using all the information you obtained in parts a-d, sketch of the graph of the function $y = (x-1)^3(x+1)^2(x-3)$. Label all relevant information. (6 pts)



2. Let $f(x) = \sqrt{x^2 - 1}$. Find the following (parts a and b are each 2 pts, and part c is 6 pts).

   a. $f(2)$

   b. $f(-2)$

   c. Domain of $f(x)$

   (Recall: Domain of a function is all real numbers x so that $f(x)$ is a well-defined real number. In other words, given that this is a square root function, think about what x values will produce a real number output [or what x values will produce an imaginary number output].)

3. Let $f(x) = \frac{1}{x-4}$. Find the following (parts a and b are each 2 pts, and part c is 6 pts)

   a. $f(2)$

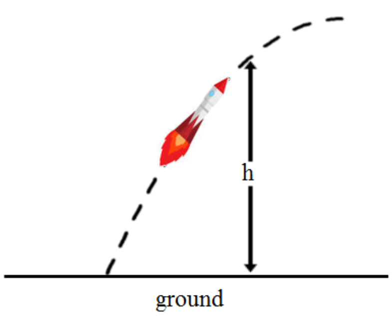
   b. $f(-2)$

   c. Domain of $f(x)$

   (Recall: Domain of a function is all real numbers x so that $f(x)$ is a well-defined real number. In other words, given that this is a rational function, think about what x values will produce a real number output [or what x values will produce undefined output].)

4. A model rocket is launched with an initial velocity of 235 ft/s. The rocket's height h (in feet) t seconds after it is launched is given by the following.

$$h = 235t - 16t^2$$



   a. Find all the values of t for which the rocket's height is 151 feet. Round your answer(s) to the nearest hundredth. (5 pts)

   b. Determine how long after the launch the rocket will hit the ground. (5 pts)

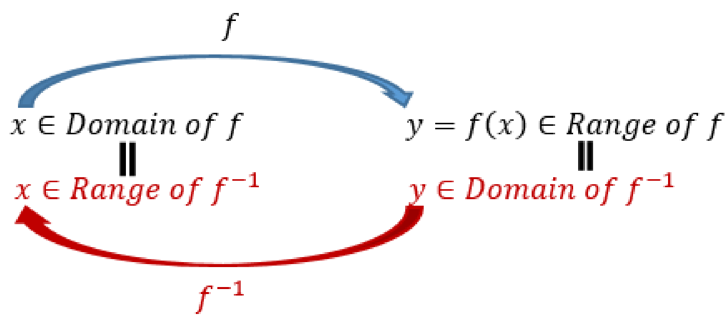
For this next set of questions process the definitions and examples below and then attempt the questions

One-to-One Function: A function in which each output y comes from only one input x is called a one-to-one function.

We define the inverse of a function to be the same pairing as the original function, but with the direction reversed.

Inverse Function: If $y = f(x)$ is a one-to-one function, then the function denoted by f^{-1} is called the inverse of f function and for every y in the range of f, f^{-1} takes this y as an input and gives the one x-value in the domain of f as its output. Thus when $y = f(x)$, we also have $x = f^{-1}(y)$. In other words the inverse function simply maps y back to x. Also f being one-to-one is exactly what is required for f^{-1} to be a function, i.e., only one x-value to go back to.

Note: The notation f^{-1} is the name of the inverse function and the -1 is not an exponent.



f^{-1} reverses the direction of the f relation. The roles of the domain and range for f^{-1} are reversed from that for f. Thus the range of f is the domain of f^{-1} and the domain of f is the range of f^{-1}.

The example below illustrate these properties of f and f^{-1} and the one-to-one necessity of f for f^{-1} to be a function.

i. $x \xrightarrow{f} y$

Domain	Range
-2	-7
-1	-5
0	-3
1	-1
2	1

$y \xrightarrow{f^{-1}} x$

Domain	Range
-7	-2
-5	-1
-3	0
-1	1
1	2

Domain of $f = \{-2, -1, 0, 1, 2\} =$ Range of f^{-1} , Range of $f = \{-7, -5, -3, -1, 1\} =$ Domain of f^{-1}

ii. $x \xrightarrow{f} y$

Domain	Range
-1	$\frac{1}{2}$
0	1
1	2
2	4
3	8

$y \xrightarrow{f^{-1}} x$

Domain	Range
$\frac{1}{2}$	-1
1	0
2	1
4	2
8	3

Domain of $f = \{-1,0,1,2,3\}$ = Range of f^{-1} , Range of $f = \left\{\frac{1}{2},1,2,4,8\right\}$ = Domain of f^{-1}

As you can see the functions in the examples above are one-to-one functions. You can also see that the inverse function is also another function. This is the concept of the inverse function you will learn in more details in your next course. You can see in the inverse function the roles that x and y play are reversed. So given a one-to-one function you can find its inverse by interchanging the variables x, and y in the equation $y = f(x)$ and the solving this new equation for y.

If we let function $f(x) = 2^x$ then you might recognize that the domain values of $\{-1,0,1,2,3\}$ will produce the same range values as shown in example ii. in the table above. So exponential functions have inverses….

5. Below are some one-to-one functions. Please fill in the missing values for the inverse function. Each missing entry is 1 pt. Domain and range are 2 pts each.

a. $x \xrightarrow{f} y$

Domain	Range
-2	-1
-1	1
0	3
1	5
2	1

$y \xrightarrow{f^{-1}} x$

Domain	Range
-1	
1	-1
3	
	1
	2

Domain of $f = \{$ $\}$ = Range of f^{-1} ,

Range of $f = \{$ $\}$ = Domain of f^{-1}

b. $x \xrightarrow{f} y$

Domain	Range
-2	$\frac{1}{9}$
-1	$\frac{1}{3}$
0	1
1	3
2	9

$y \xrightarrow{f^{-1}} x$

Domain	Range
$\frac{1}{9}$	-2
$\frac{1}{3}$	
1	
	1
9	

Domain of $f = \{$ $\} =$ Range of f^{-1} ,

Range of $f = \{$ $\} =$ Domain of f^{-1}

6. For the one-to-one function $y = \frac{2x-3}{4x-5}$, please find its inverse function. (4 pts)
   To begin, you should interchange x and y as done for you below…
   $$x = \frac{2y-3}{4y-5}$$
   Now solve this equation for y. This new function is the inverse function.