Mat 103 Exam III/Final Exam Review Sheet

- ☐ Problem numbers1-10 act as your Quiz 7 Take home and as Exam III review. These questions will also act as part of your final exam review. To count as Quiz 7 your work must be completed on a separate sheet of paper by Thursday December 1 to be eligible to take Exam III on Thursday December 1 and stapled if you are using multiple pages. Please write the original problem and then a detailed solution. You must show all your work to get full credit.
- ☐ The remaining problems on this review must be completed by Thursday December 8 also on a separate sheet of paper. Please do not try to squeeze your answers on this sheet.

Other Information

- The purpose of this review sheet is to help you organize all the information relevant to the exam III/final exam and give you practice problems so that you are prepared for the exams.
- Doing well on the practice problems below will help you do well on the two remaining exams but does not guarantee it.
- Please review your other old exams and quizzes also. We suggest you first study the topics listed below by looking into your notes, old quizzes and exams you took during the semester.
- Only after reviewing your old exams and notes, attempt the problems on this review sheet and see which topics you have forgotten so you can study appropriately.
- Do not cram the night before your exams. Get plenty of sleep and eat appropriately to help your brain.
- Resting your brain before an exam is vital so you can have your higher level thinking processes functioning during the exam!

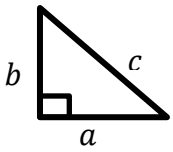
Start studying for the final exam NOW!!!

Topics you have learned during the semester.

1. Linear equations, absolute value equations and their applications, solving linear inequalities and absolute value inequalities.
2. Finding distance between two points and its applications. Finding equations of lines under different conditions and their applications. Finding domain and range of a functions and relations. Sketching graphs of linear inequalities.
3. Solving systems of linear equations, and applications.
4. Exponents (integer and rational), addition, subtraction, multiplication, division of polynomials.
5. Factoring a monomial from a polynomial and factoring by grouping, factoring trinomials, special factoring formulas (difference of squares and cubes, sum of cubes), completing the squares, solving equations using factoring, using factoring to solve for a variable in a formula or equation.
6. Reducing rational expressions, multiplication, division, addition, subtraction of rational expressions. Complex fractions, solving equations containing rational expressions and their applications.
7. Radicals, rational exponents, multiplying, dividing, addition, subtraction and simplifying radicals, solving radical equations.
8. Solving quadratic equations (quadratic formula).
9. The formulas listed below will not be provided to you on the final exam.

Formulas and notation you need to know that are not given to you on the final exam-

- Difference of Squares $a^2 - b^2 = (a-b)(a+b)$
- Difference of Cubes $a^3 - b^3 = (a-b)(a^2+ab+b^2)$
- Sum of Cubes $a^3 + b^3 = (a+b)(a^2-ab+b^2)$
- Quadratic Formula: Solutions to the quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$ are given by $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$
- Pythagorean Theorem: For any right triangle the sum of the square of the two shorter sides equals the square of the hypotenuse.



$a^2 + b^2 = c^2$

- Distance between two points (a,b) and (c,d) is given by $\sqrt{(a-c)^2 + (b-d)^2}$.
- Midpoint between two points (a,b) and (c,d) is given by $\left(\frac{a+c}{2}, \frac{b+d}{2}\right)$.
- Slope of a line passing through the two points (a,b) and (c,d) is given by $\frac{b-d}{a-c} = \frac{rise}{run}$.
- Equation of line in slope intercept form is given by $y = mx + b$, m = slope, and b =y-intercept.
- Equation of a line passing through the point (a,b) and having slope m, is given by $y = m(x-a) + b$.
- For a any real number and positive integer n, a^n represents multiplying a by itself n times.
- For $a \neq 0$ any real number and positive integer n, $a^{-n} = \frac{1}{a^n}$ represents one over a multiplied by itself n times.
- For a any positive real number and positive integer n, $\sqrt[n]{a} = a^{\frac{1}{n}}$, represents n^{th} root of a.
- For a any positive real number and positive integer n, $\sqrt[n]{a^m} = a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$, represents n^{th} root of a^m.
- For any odd integer n and any nonzero real number a, $\sqrt[n]{-a} = -\sqrt[n]{a}$
- For all non-zero real numbers a,b, and any integers n,m we have (if a,b are positive real numbers, the rules below would also apply to any real numbers n,m)

a. $(ab)^n = a^n b^n$	c. $a^n a^m = a^{n+m}$	e. $\frac{a^n}{a^m} = a^{n-m}$
b. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	d. $(a^n)^m = a^{mn}$	f. $a^0 = 1$

- Remember
  - Multiplication distributes over addition or subtraction.
  - Exponents distribute over multiplication and division.
  - Exponents **DO NOT** distribute over addition or subtraction.
  - You can **only divide multiplicative factors** and not additive factors.
  - Radicals (which are really fractional exponents) distribute over multiplication and division.
  - Radicals (which are really fractional exponents) **DO NOT** distribute over addition or subtraction
- When solving any equation is an "undoing process" so we can get the value of our variable, whereas simplifying expressions does not involve solving it just involves collecting like terms and rewriting of the original expression.
- During solving any equation or an inequality you can remember that

Operation	Undone by
Addition	Subtraction
Subtraction	Addition
Multiplication	Division
Division	Multiplication
Exponent of n	Nth root (remember $\pm$for even roots)
Nth root	Exponent of n

When dividing or multiplying by a negative number in an inequality remember to switch signs.

Practice Problems

1. Determine if the points a) $(-3,5)$, b) $\left(\frac{5}{3},-\frac{7}{4}\right)$, $c)$ $\left(-\frac{1}{3},\frac{3}{4}\right)$ on the line given by the equation $3x-4y=12$? Explain your answer.
2. Sketch the graph of the following lines. Show at least two of the points on the line. All lines must be drawn with the aid of a ruler.
   a. $y=5$
   b. $x=-5$
   c. $y=-\frac{2}{3}x+1$
   d. $x=-\frac{2}{3}y+1$ (Draw the line without converting it first into $y=mx+b$ form). Explain how the slope and intercepts of this line differ from the line in part c.
   e. The line goes through (2, -3) with slope $m=\frac{4}{3}$.
   f. Line passing through $(-1,-2)$ and $(3,5)$
   g. A line parallel to $2x-3y=6$. Do you think the line you drew is the only other line possible? If not, how would another solution differ from yours? Explain your answer.
   h. $-2x-5y=-5$
3. Find equations for the lines below.
   a. The line is parallel to $y=\frac{2}{3}x+4$ and goes through the point (3, -1) .
   b. The line is perpendicular to $y=\frac{2}{3}x+4$ and goes through the point (3, -1) .
   c. Line parallel to $x=-3y+4$ and passing through the point $(-1,2)$.
   d. Passing through $(-1,2)$ and $(4,-2)$
4. Sketch the graph of the circle $(x+2)^2+(y-1)^2=9$
5. Solve the system of equations below.

   a. $\begin{cases} y=x+5 \\ 3x+y=1 \end{cases}$ b. $\begin{cases} x-2y=1 \\ x+y=2 \end{cases}$ c. $\begin{cases} 3x-2y=3 \\ 2x+3y=2 \end{cases}$

6. Assume that relationship between the height of a candle related how long it has been burning is a linear relationship. After 1 hour the candle is 10 inches tall, and after 4 hours it is 6.5 inches tall.
   a. Determine the equation that describes this relationship. Use t =hours the candle has been burning for, and H =height of the candle at time t.
   b. What was the height of the candle at the beginning?
   c. What was the height of the candle after 10 hours?
   d. What is the maximum amount of time in hours and minutes that the candle will last if it was burnt completely?
7. Let the amount of water in a leaking bucket be given by $A=8-0.25t$, where A=amount of water in bucket in liters and t= time in minutes after it began leaking.
   a. How much water is in the bucket after 5 minutes?
   b. How much water was in the bucket at the beginning just when it started leaking?
   c. When will the bucket be completely empty?
   d. What is the t-intercept and the A-intercept? What is the significance of these intercepts?
   e. What is the slope of this line? What is the physical significance of the slope?
8. Let the amount of water in a leaking bucket be given by $A=8-0.25t$, where A=amount of water in bucket in liters and t= time in minutes after it began leaking.
   a. How much water is in the bucket after 5 minutes?
   b. How much water was in the bucket at the beginning just when it started leaking?
   c. When will the bucket be completely empty?
   d. What is the t-intercept and the A-intercept? What is the significance of these intercepts?
   e. What is the slope of this line? What is the physical significance of the slope?

9. Solve the equations below. (Some may have extraneous solutions! Some may require the quadratic formula.)

a. $\frac{2}{3}(x+1)-x=2(x+3)-5$

b. $\frac{2}{3x-1}=\frac{4}{x+1}$

c. $\frac{x}{x+1}+\frac{2}{2x-1}=\frac{2x}{2x-1}$

d. $\frac{3x}{x^2+3x-4}-\frac{2}{x+4}=\frac{5}{x-1}$

e. $\frac{1}{x-1}+\frac{1}{x+1}=\frac{-2x+4}{x^2-1}$

f. $3x^2-2x-5=0$

g. $5x^2-4x+2=0$

h. $3(4x-1)-2x(x-1)=3x^2-3x+3$

i. $3x^2+4x-5=0$

j. $x^2-2x=-5$

k. $3(x-1)-2x(x+1)=2x+5$

l. $\frac{5}{x+1}-\frac{6}{x-3}-7=0$

m. $\sqrt[3]{2x-3}=3$

n. $D=RT$, solve for R.

o. $ax+by=c$, solve for y.

p. $T=\frac{5}{9}(F-32)$, solve for F.

q. $(3-x)^{\frac{2}{3}}=6$

r. $(3x-1)^{\frac{3}{4}}=-8$

s. $\sqrt{2-x}=x+3$

t. $\sqrt{3x-5}+4=0$

u. $|2x-3|=7$

v. $|x+5|=-2$

10. Solve the inequalities and plot the solutions on a number line and also give the solution in interval notation.

a. $2x-3\leq 9$

b. $\frac{3}{4}-5x>4+2x$

c. $-5\leq 2x+3<15$

d. $|2x+5|\geq 11$

e. $|4-3x|<5$

f. $3-x>5$ and $3x-4\leq 2$

11. Factor the following completely if possible.

a. $2t^3-16$

b. p^3-27

c. a^3-27b^3

d. $8x^3+125y^3$

e. x^4-y^4

f. x^2-25

g. x^2+y^2

h. $10x^2-5x+6x-9$

i. $2x^2-5x+3$

j. x^2+2x-3

k. $8x^3-2x^2$

12. Perform the following operations and simplify your answer. Do not leave any negative exponents in your answer or radicals on the denominator. Write the answer in simplest form where necessary.

a. $\frac{2-4^2+25\div 5}{|3-5|}$

b. $6+4\div 2\times(-3)-2$

c. $2\sqrt{8}+\sqrt[5]{3}+3\sqrt{125}+7\sqrt{2}+6\sqrt[5]{3}-4\sqrt{5}$

d. $-3^2+4(2-3)$

e. $\left(\frac{-2}{3}\right)^{-3}$

f. $(-2x^{-2}y^{-4})(3x^{-5}y^{-2})$

g. $\left(\frac{-16a^{-4}b^3}{6a^5b^7}\right)^{-2}$

h. $\left(\frac{32a^{-4}b^3}{2b^7}\right)^{1/2}$

i. $\sqrt{\frac{18x^5y^2}{8x^2y^{-3}}}$

j. $\sqrt[3]{-48}$

k. $\sqrt{32}$

l. $(3-4i)-(5-2i)$

m. $\frac{2\sqrt{3}}{\sqrt{5}}$

n. $\frac{2}{\sqrt[3]{9}}$

o. $\sqrt{27x^6}-5\sqrt[3]{x^7y^6}+2\,x^2y^3\sqrt[3]{x}-7x^3\sqrt{3x}$

p. $\frac{4}{x+2}-\frac{x}{x^2-4}-\frac{2x}{x-2}$

q. $\left(2\sqrt{3}-4\right)^2$

r. $\left(2\sqrt{3}-5\right)\left(\sqrt{2}+3\sqrt{3}\right)$

s. $\frac{2+\sqrt{3}}{5\sqrt{2}-4}$

t. $\sqrt{3a}(t+p)+\sqrt{3a}(t+p)$

u. $(2x-3)^2$

v. $(2x^2-1)(3x^2+7x+2)$

w. $(2x^3-3x^2+5x-1)-(3x^2+7x+2)$

x. $(4x^3 - 16x^2 + 21x - 12) \div (2x - 3)$ (use long division to find quotient and remainder)

y. $\frac{9x^2+9x+2}{3x^2-7x-6} \times \frac{2x^2-7x+3}{6x^2-x-1}$

z. $\frac{9x^2+9x+2}{x^2-9} \div \frac{9x^2+3x-2}{3x^2+8x-3}$

aa. $\frac{4-\frac{1}{b}}{2-\frac{1}{b}}$

bb. $\frac{\frac{4}{x+1}-1}{2-\frac{1}{x+1}}$

13. Fill in the missing blanks.

a. $\sqrt{\square} = 3x^2$	b. $\left(\frac{\square}{\square}\right)^{-2} = \frac{x^4}{y^6}$
c. $x = 1$ is a solution to the equation $3x - 4 = \square x + 5$	d. In the equation of the line $x = \frac{4}{3}y + 6$, 6 is the ____-intercept and the slope of this line is given by _____.

14. For the problems below determine if there are any mistakes in the represented solutions. If yes, explain using mathematical terminology what the mistakes are.

a) Simplify -3^{-2}

Solution $-3^{-2} = 9$ since two negatives shown in the solution multiply to us a positive, and then $3^2 = 9$.

b) Simplify $x + x$

c) Solution: $x + x = x^2$

d) Simplify $\frac{3x^2+5x}{3x+2}$

Solution: $\frac{3x+5}{3x} = 5$

e) Simplify $3x(2)(5)$

Solution: $3x(2)(5) = 6x\ 15x = 90x^2$

f) Perform the following operation $(2x - 5)^2$

Solution $(2x + 5)^2 = 4x^2 + 25$

g) Solve the equation for x, $|x + 7| = 3$

Solution: $x = 3 + 7 = 10$ *or* $x = 3 - 7 = -4$

15. Answer true or false and justify your answers

a. $3 \times (2 \times 5) = (3 \times 2) \times (3 \times 5)$?

b. $(a+b)^2 = a^2 + b^2$

c. $\sqrt{a^2+b^2} = a + b$

d. $-\frac{2}{5} = \frac{-2}{-5}$

e. $2\frac{3}{4} = 2 \times \frac{3}{4}$

f. $x + x = x^2$

g. $a - b = b - a$

h. $\frac{\cancel{3}}{5} + \frac{2}{\cancel{3}} = \frac{2}{5}$

For the problems below, you can use any method you want as long as the logic is clearly stated using mathematical terms (strip diagrams or nonstandard methods)

1. A newsletter printing job needs to be done each week. Using an older copy machine the job took 2.5 hours to complete. Then a new machine was brought in and using both machines, the job could be completed in 1 hour. Determine how long it would take the new machine to do the job alone.
2. If price of a loaf of bread went down by 2 cents and the new price is 62 cents, what was the original price of the loaf of bread?
3. If Adam's salary went up by 5% and his new salary is $58000, what was his original salary before the raise?
4. A student has grades on her hour exams of 85%, 90%, and 60% where each exam counts for 20% of his grade and the final counts for 40% of the grade. In what range must his final % score be so that his overall course average comes out to be greater than or equal to 80%?
5. A bicyclist traveled 45 miles in 3 hours against the wind and 57 miles in the same time with the wind. Find the speed of the bicyclist in still air and the speed of the wind.
6. If the perimeter of a rectangle has to be 45 inches, what should the length and width be so that its area is 102 square feet.
7. Suppose you ordered 50 pounds of mulch for your garden. When the mulch was delivered you found out that they only sent you 30 pounds of mulch. What % of your order did you get? Solve using strip diagrams only.
8. Suppose that a truck's radiator is filled with 10 liters of 30% antifreeze solution. **How much** of the fluid must be drained and then replaced with pure antifreeze to get a 44% antifreeze solution?
9. The graph below shows the revenue from selling a commodity in millions of dollars with respect to the selling price in dollars of the commodity.

a. What is revenue when the price is set at 10$/unit?

b. What price should the company set to maximize the revenue?

c. What was the price/s of the commodity if the company made $800,000,000 in revenue?

d. What is the price range which will generate more than $800,000,000 in revenue?