2.3 Graphing Polynomial Functions

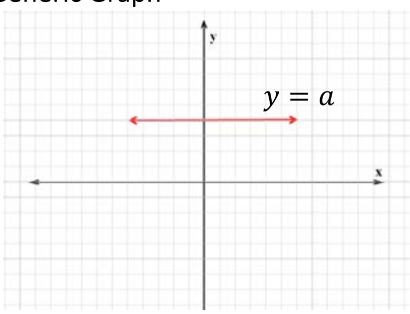
In chapter 1 we saw some examples of polynomial functions which is reviewed below.

1. **Polynomial function:** A function defined as $P(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots + a_nx^n$, where $a_0,\ a_1,\ a_2,\ a_{3,} \ldots a_n$are all real numbers and n≥ 0 is a whole number. Domain and range of these functions is all real numbers.

Example

i) **Constant Function:** A function defined as $f(x) = a$, where a is any real number.
A special case of the polynomial function of degree zero.

- Generic Graph

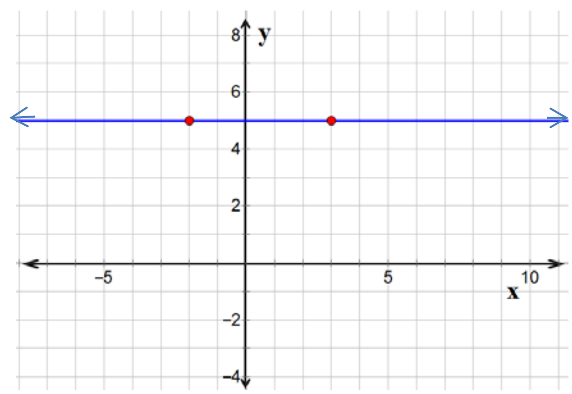


- Domain: All real numbers
- Range: $\{a\}$
- One-to-One: NO

Example

If $f(x) = 5$, then find

a. $f\left(\frac{2}{3}\right)$
b. $f(100)$
c. $f(-3456)$
d. $f(a + h)$
e. Sketch the graph



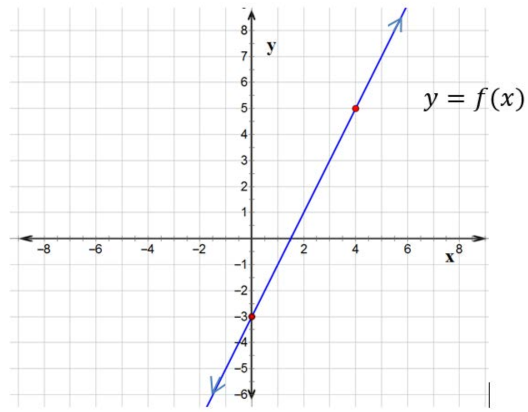
Solutions

a. $f\left(\frac{2}{3}\right) = 5$
b. $f(100) = 5$
c. $f(-3456) = 5$
d. $f(a + h) = 5$
e. As you can see from the parts a,b,and c, no matter what x-coordinate you plot the y-coordinate is always 5 so it is a horizontal line as shown above.

ii) Linear Function: A function defined as $(x) = mx + b$.
A special case of polynomial function with degree one. Remember m =slope of the line, $b = y$-intercept.

Example

1. If $f(x) = 2x - 3$, then
a. Find $f(-1)$
b. Find $f(0)$
c. Find $f(4)$
d. Find the inverse function
e. Sketch the graph of $y = f(x)$



Solution
a. $f(-1) = 2(-1) - 3 = -2 - 3 = -5$
b. $f(0) = -3$
c. $f(4) = 2(4) - 3 = 8 - 3 = 5$
d. Inverse function

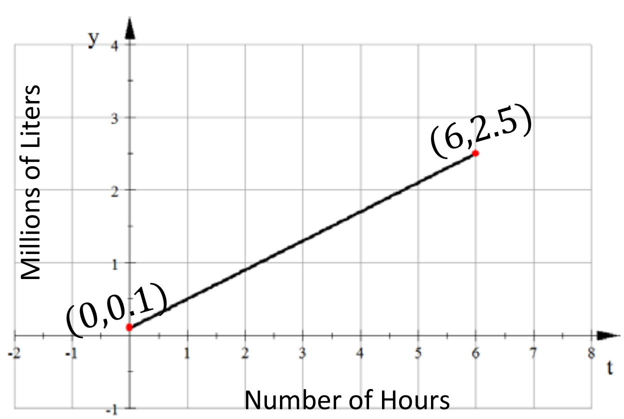
$$y = 2x - 3$$
$$x = 2y - 3$$
$$x + 3 = 2y$$
$$\frac{x+3}{2} = y$$

Inverse function is $f^{-1}(x) = \frac{x+3}{2}$

e. Graph of the function is to the right with slope of 2 and y-intercept of -3.

2. An Olympic size swimming pool holds 2,500,000 liters of water. If the pool currently holds 100,000 liters of water, and water is being pumped at 400,000 liters/hour into the tank.
a. Write a function that represents the amount of water in the tank at t hours.
b. Find the domain and range of this function.
c. Sketch the graph of this function.

Solution
a. $A(t) = 100000 + 400000t$ Liters, for $0 \le t \le 6$
The reason for the restriction is time is never negative and after 6 hours the tank will be full since it holds a maximum of 2,500,000 liters of water.
b. Domain of $A = [0,6]$ and Range of $A = [100000,2500000]$
c. Graph is to the right.
The scale is each tick mark represents 1 million liters on the y-axis and 1 hour on the t-axis.



iii) Square Function: A function defined as $(x) = ax^2 + bx + c$.
A special case of polynomial function with degree two

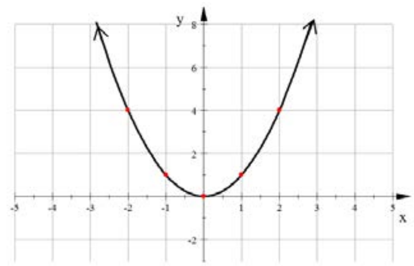
Example If $f(x) = x^2$, then

a. Find $f\left(\frac{2}{3}\right)$
b. Find $f(-2)$
c. Find $f(2)$
d. Find $f(a+h)$
e. Sketch the graph the function $y = x^2$
f. Is the function one-to-one?

Solution

a. $f\left(\frac{2}{3}\right) = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$
b. $f(-2) = (-2)^2 = 4$
c. $f(2) = (2)^2 = 4$
d. $f(a+h) = (a+h)^2 = a^2 + 2ah + h^2$
e. Plot a few points to sketch the graph of the function. See to the left
f. No. The function is not one-to-one as it does not pass the horizontal line test.

x	$y = x^2$
-2	$(-2)^2 = 4$
-1	$(-1)^2 = 1$
0	$(0)^2 = 0$
1	$(1)^2 = 1$
2	$(2)^2 = 4$



iv) Cube Function: A function defined as $f(x) = a + bx + cx^2 + dx^3$
A special case of polynomial function with degree three

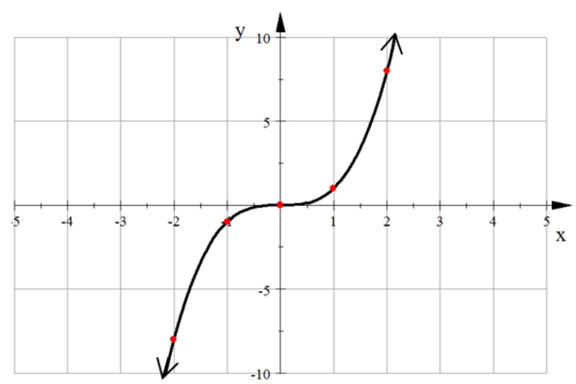
Example If $f(x) = x^3$, then

a. Find $f\left(\frac{2}{3}\right)$
b. Find $f(-2)$
c. Find $f(2)$
d. Sketch the graph the function $y = x^3$
e. Is the function one-to-one?

Solution

a. $f\left(\frac{2}{3}\right) = \left(\frac{2}{3}\right)^3 = \frac{8}{27}$
b. $f(-2) = (-2)^3 = -8$
c. $f(2) = (2)^3 = 8$
d. Plot a few points to sketch the graph of the function. See to the left
e. Yes. The function is one-to-one as it does pass the horizontal line test.
f. Finding inverse function write $y = x^3$, then we get $x = y^3$ solving for y we get that inverse function is $f^{-1}(x) = \sqrt[3]{x}$

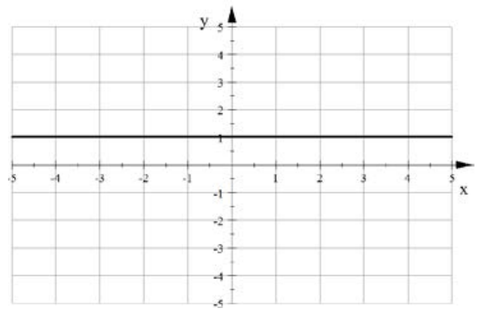
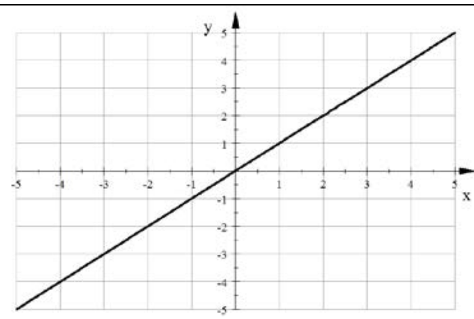
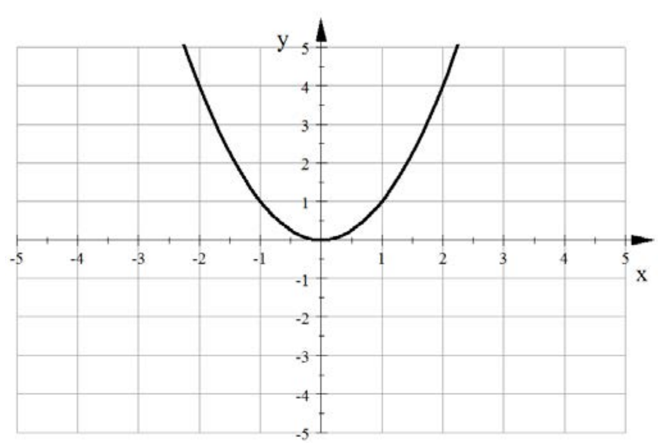
x	$y = x^3$
-2	$(-2)^3 = -8$
-1	$(-1)^3 = -1$
0	$(0)^3 = 0$
1	$(1)^3 = 1$
2	$(2)^3 = 8$

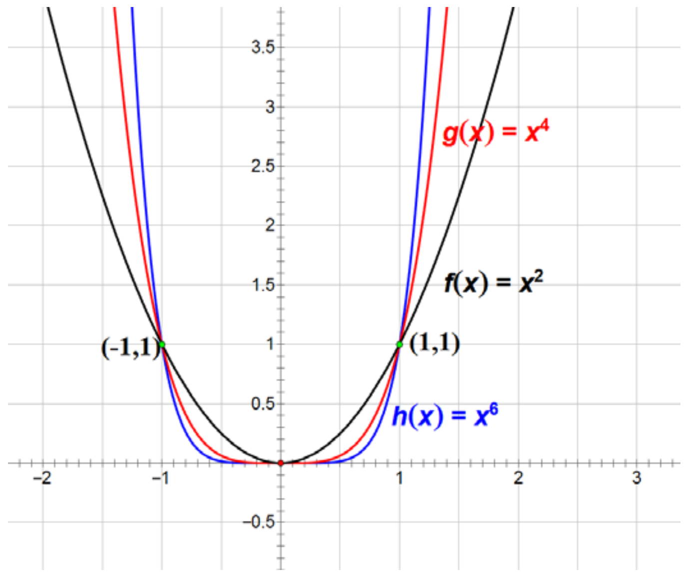
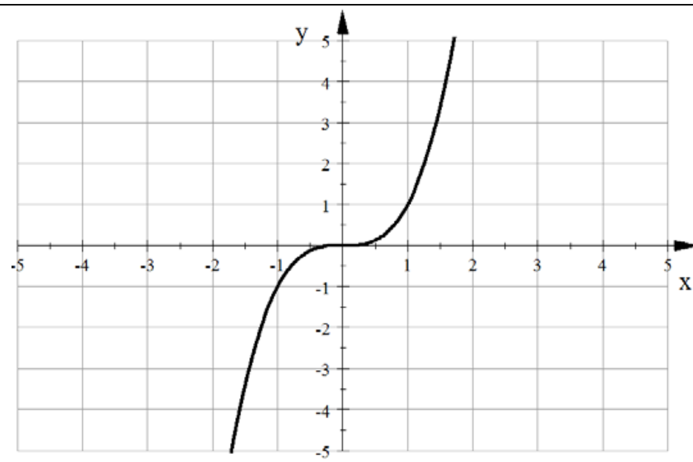


So we know how the graphs of constant functions, linear functions, quadratic functions, and simple basic cubic function defined as $(x) = x^3$. We know constant functions and linear function graphs are lines. In the previous section we saw that quadratic functions of the type $f(x) = ax^2 + bx + c$ is a parabola with vertex $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$ and faces up when $a > 0$ and faces down when $a < 0$.

We will use this information and the information we learnt about transformation of functions to explore how more complicated polynomial functions will look like.

Let us start with exploring functions of the type $f(x) = x^n$, where n is a whole number.

1	When $n = 0$ we have a constant function $f(x) = 1$ which is a horizontal straight line.	
2	When $n = 1$ we have $f(x) = x$ which is also a straight line passing through (0,0)	
3	When $n = 2$ we have a quadratic $f(x) = x^2$ which is a parabola with vertex (0,0) facing up.	

4	In general then when $n > 2$ and an even number we know it will behave similar to the graph of $f(x) = x^2$ in its generic shape. Plotting a few points we notice that a) When $-1 < x < 1$, $x^n < x^2$. For example, $0.1^4 = 0.0001 < 0.1^2 = 0.01$. b) When $-1 > x, and\ x > 1$, $x^n > x^2$. For example, $10^4 = 10000 > 10^2 = 100$ or $(-10)^4 = 10000 > (-10)^2 = 100$. See graphs of some of the functions of the type $f(x) = x^n$ in comparison to each other on the right.	
5	When $n = 3$ we have a cubic function $f(x) = x^3$ which is passes through (0,0) and is symmetric with respect to origin.	

6 In general then when $n > 3$ and an odd number we know it will behave similar to the graph of $f(x) = x^3$ in its generic shape. Plotting a few points we notice that

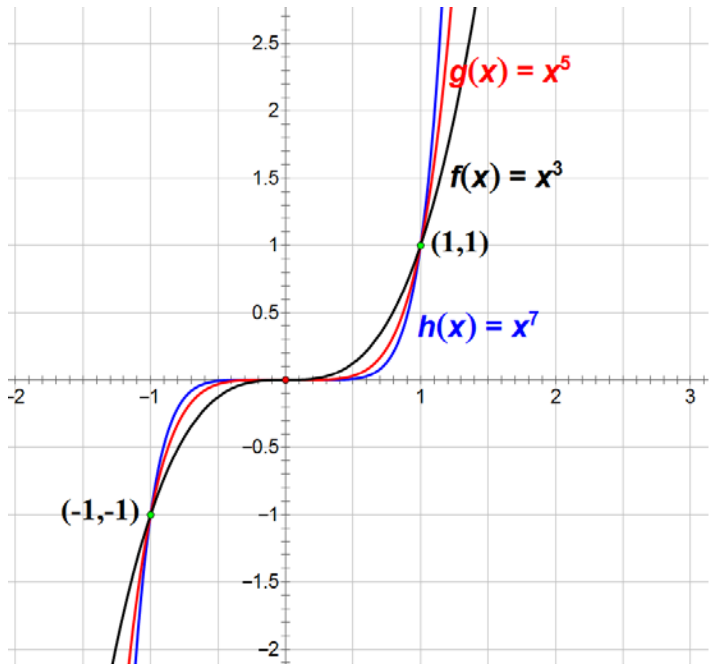
a) When $0 < x < 1,\ x^n < x^3$. For example, $0.1^5 = 0.00001 < 0.1^3 = 0.001$

b) When $-1 < x < 0,\ x^n > x^3$. For example, $(-0.1)^5 = -0.00001 > (-0.1)^3 = -0.001$

c) When $< -1, x^n < x^3$. For example, $(-10)^5 = -100000 < 10^3 = 1000$.

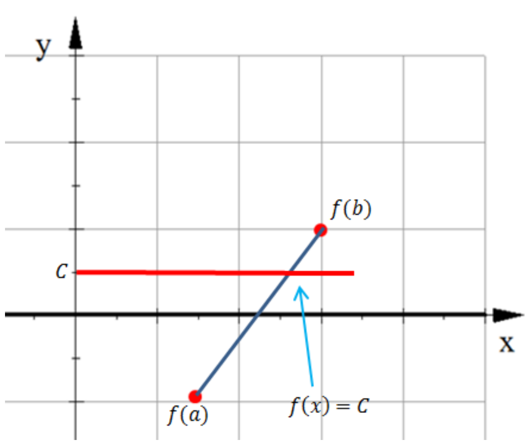
d) When $x > 1, x^n > x^3$. For example, $10^5 = 100000 > 10^3 = 1000$

See graphs of some of the functions of the type $f(x) = x^n$ in comparison to each other on the right.

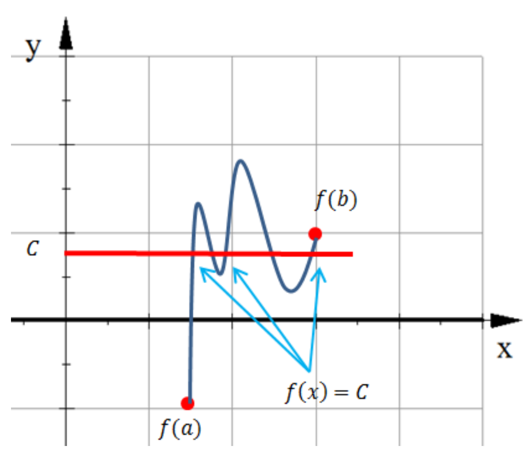


- All polynomial functions are continuous on the domain $(-\infty, \infty)$.
- Intermediate Value Theorem: For a polynomial function $f(x)$, if $f(a) < f(b)$, with $a < b$, then for every number C for which $f(a) \le C \le f(b)$, there exists a value of x with $a \le x \le b$ so that $f(x) = C$. That means f attains all values between $f(a)$, and $f(b)$ at some point x in the interval $[a, b]$.

How can we use this fact to our benefit? The Intermediate Value Theorem visually means what is below for a polynomial function.



In scenario at least once the function attains the value of C.



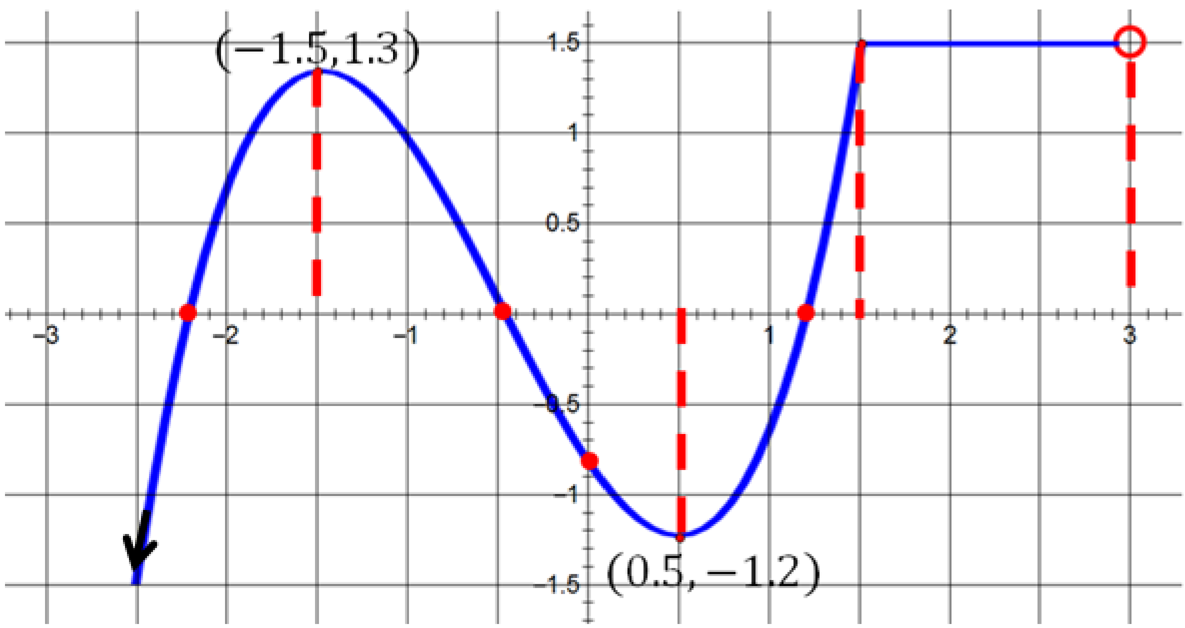
In scenario the function attains the value of C several times.

As you can see even from the graphs of the basic polynomial functions if you travel on the graph from left to right, sometimes the graph of the function does not rise or fall as in the case of the constant function, and other times it rises or falls like riding a roller coaster. The definitions below are for any function and not just polynomial functions.

- A function $f(x)$ is **constant** on an interval (a, b) iff for all $x_1 < x_2$ in the interval (a, b), $f(x_1) = f(x_2)$.
- A function $f(x)$ is **strictly increasing** on an interval (a, b) iff for all $x_1 < x_2$ in the interval (a, b), $f(x_1) < f(x_2)$.
- A function $f(x)$ is **strictly decreasing** on an interval (a, b) iff for all $x_1 < x_2$ in the interval (a, b), $f(x_1) > f(x_2)$.

Example

1. Determine all the intervals where the function below is increasing, decreasing and or constant.



The function is increasing on the interval $(-\infty, -1.5) \cup (0.5, 1.5)$
The function is decreasing on the interval $(-1.5, 0.5)$
The function is constant on the interval $(1.5, 3)$

Playing

See if you can come up with rough sketches of the graphs the following functions. Try using the knowledge you have accumulated so far (i.e., transformation of functions, arithmetic of functions). Do not peek after this page to see what the answers are. Cultivating your know intuition will aid in your deeper understanding of the material. You can always resort to plotting points that might be of interest to you.

Practice problems

1. $f(x) = (x-1)^2(x+1)^3$
$= x^5 + x^4 - 2x^3 - 2x^2 + x + 1$

2. $f(x) = (x-1)^2(x+1)^3 + 2$
$= x^5 + x^4 - 2x^3 - 2x^2 + x + 3$

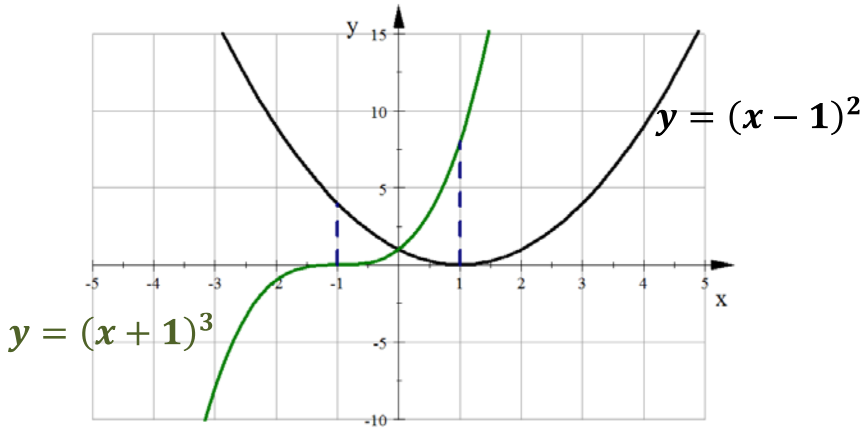
3. $f(x) = \frac{1}{10}(x-4)(x+4)(x-2)^3(x+2)^2$

4. $f(x) = -\frac{1}{10}(x-4)(x+4)(x-2)^3(x+2)^2$

Solutions to practice problems

1. $f(x) = (x-1)^2(x+1)^3 = x^5 + x^4 - 2x^3 - 2x^2 + x + 1$

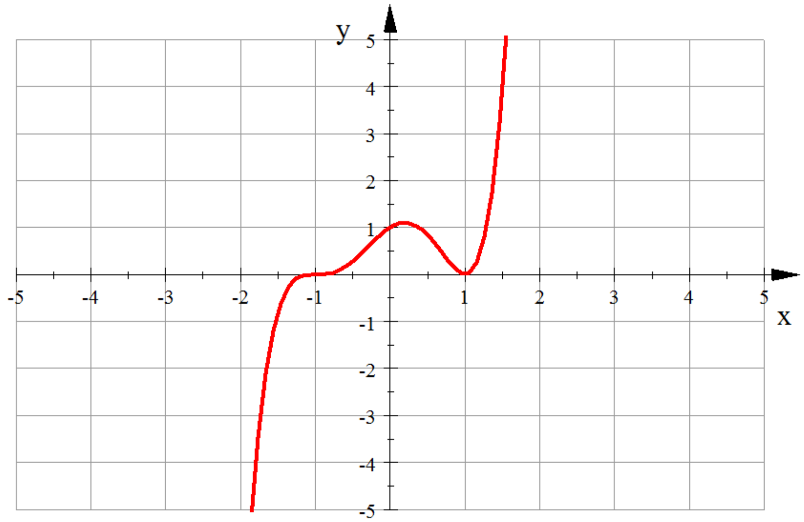
We know that the function $y = (x-1)^2$ is a parabola, and $y = (x+1)^3$ is a cubic function. These individual graphs are shown below. Plotting them on the same coordinate axis will give us insight into the graph of their product. Note y-intercept is $f(0) = (0-1)^2(0+1)^3 = 1$



To plot the graph of $f(x)$ we need to see how the product of the two functions we know behaves.

	$x < -1$	$-1 < x < 1$	$x > 1$
Test Point	-3	0	1.5
$f(x) = (x-1)^2(x+1)^3$	$(-3-1)^2(-3+1)^3 = -128 < 0$	$(0-1)^2(0+1)^3 = 1 > 0$	$(1.5-1)^2(1.5+1)^3$ $3.90625 > 0$
	Graph of $f(x)$ is below the x-axis	Graph of $f(x)$ is above the x-axis	Graph of $f(x)$ is above the x-axis

$f(1) = (1-1)^2(1+1)^3 = 0$ and $f(1) = (-1-1)^2(-1+1)^3 = 0$ that means $x = 1, -1$ are x-intercepts. So just knowing the sign of the function we can sketch a rough sketch. Domain of the function $f(x)$ is all real numbers. We can see the two functions $= (x-1)^2$, and $y = (x+1)^3$ have no breaks or holes in them and so that is what we would expect in the product function. You can plot several points to see what happens to convince yourself but we know that each of them will try to preserve their shapes. Our rough sketch would look as shown below. We do not exactly know how high it will rise before turning back down when $-1 < x < 1$ but we know it must to reach the point (1,0).

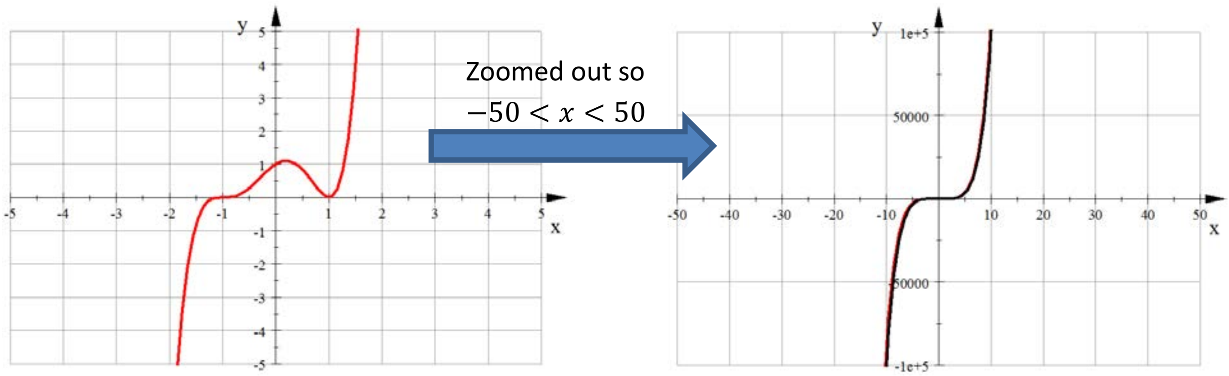


$f(x) = (x-1)^2(x+1)^3 \quad = x^5 + x^4 - 2x^3 - 2x^2 + x + 1$

We can see that for very large values of x and very small values of x the term with the highest degree in the polynomial will dictate how big or small the values will be or that this term will kind of take over the behavior of the function when $x \to \pm\infty$. This is called the end behavior. See table below.

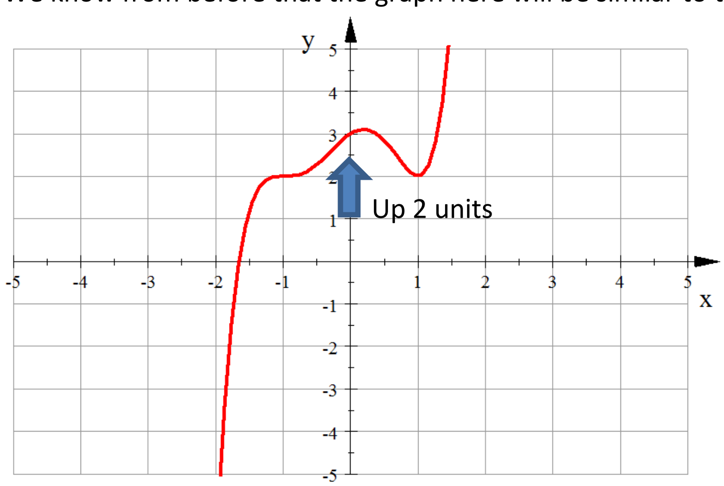
x	x^5	$f(x)$
1000	$1{,}000{,}000{,}000{,}000{,}000$	$1{,}000{,}997{,}998{,}001{,}001$
10000	$100{,}000{,}000{,}000{,}000{,}000{,}000$	$100{,}009{,}997{,}999{,}800{,}010{,}001$
100000	$10000000\ 000000000\ 000000000$	$10000099\ 997999980\ 000100001$
-1000	$-1{,}000{,}000{,}000{,}000{,}000$	-998,998 ,002,000,999
-10000	$-100{,}000{,}000{,}000{,}000{,}000{,}000$	-99 ,989,998,000, 200,009,999
-100000	$-10000000\ 000000000\ 000000000$	-9,999,899 ,998,000,020 ,000,099,999

What this means is that when $x \to \pm\infty$, $f(x)$ will behave very much like $y = x^5$.
See graph below when we zoom out

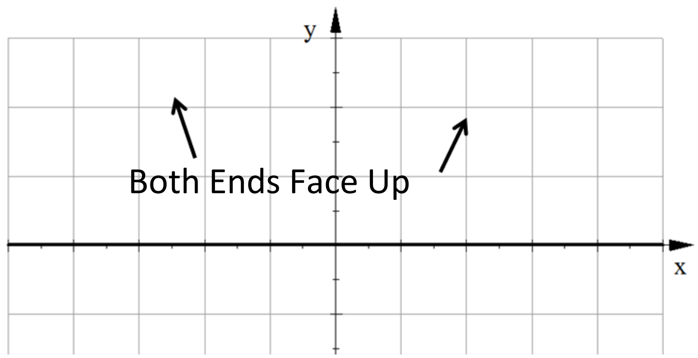
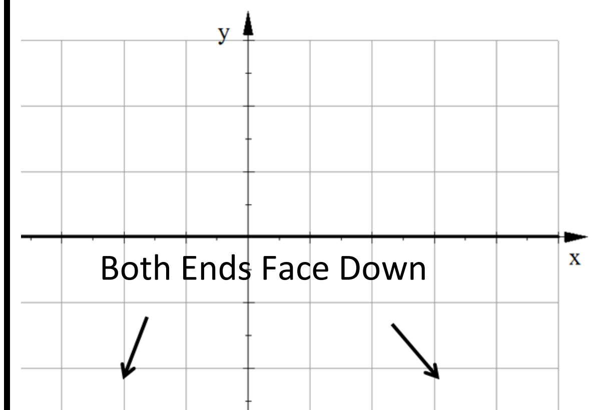
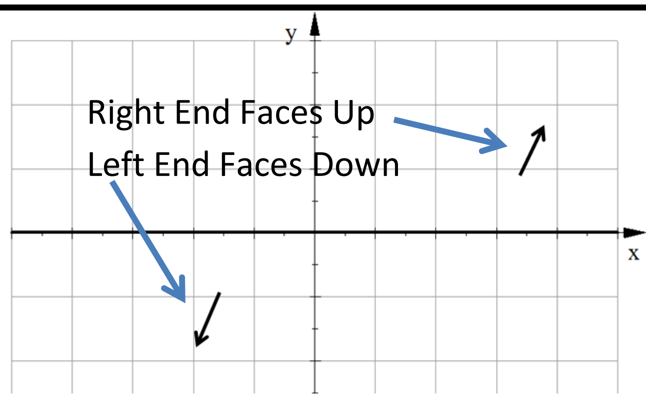
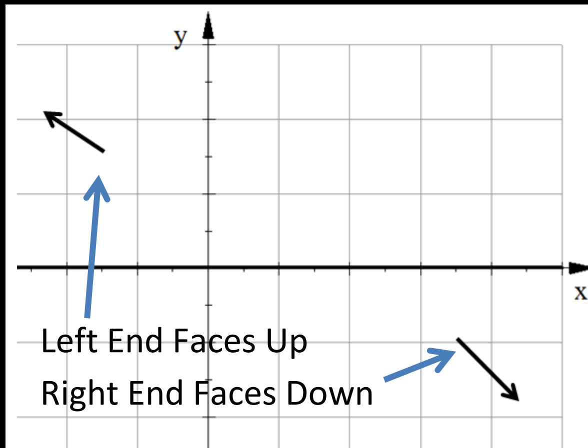


This is why it is important for us to explore the graphs and not just the graphing utilities like graphing calculators, Wolfram Alpha to graph our functions. We need to know where to look to see the true picture. If you are interested in these kinds of graphs you must take calculus to fully explore how more advanced mathematical tools allow us to find the exact shape of the graphs including the highest point it rises to in the interval $-1 < x < 1$.

2. $f(x) = (x-1)^2(x+1)^3 + 2 \quad = x^5 + x^4 - 2x^3 - 2x^2 + x + 3$
We know from before that the graph here will be similar to the one before just moved up 2 units.



End behavior of a polynomial function is governed by the its degree and the leading coefficient.

Degree of the Polynomial	**Leading Coefficient**	
	Positive	**Negative**
Even		
Odd		

3. $f(x) = \frac{1}{10}(x-4)(x+4)(x-2)^3(x+2)^2$

Step 1: x-intercepts

Setting $f(x) = 0$ we get we get $x = 4, -4, 2, -2$ will be the x-intercepts.

Step 2: y-intercept

Setting $x = 0$ in the function we get $f\ (0) = \frac{1}{10}(0-4)(0+4)(0-2)^3(0+2)^2 = \frac{256}{5} = 51.2$ will be the y-intercept.

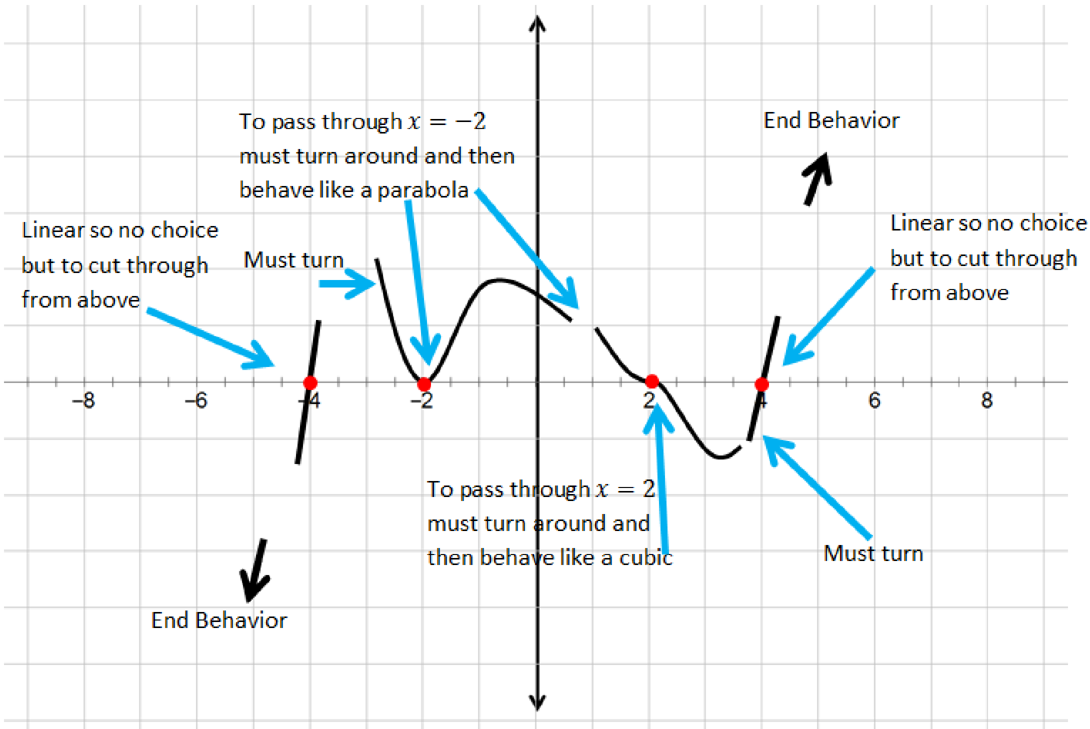
Step 3: End behavior

Note that if we were to expand the polynomial the degree of the polynomial would be 7 since we have two linear terms, a cubic term and a square term. So our polynomial will behave like x^7 on the ends.We do expect parabola behavior at $x = -2$, a cubic behavior at $x = 2$ and linear behavior at $x = 4, -4$.

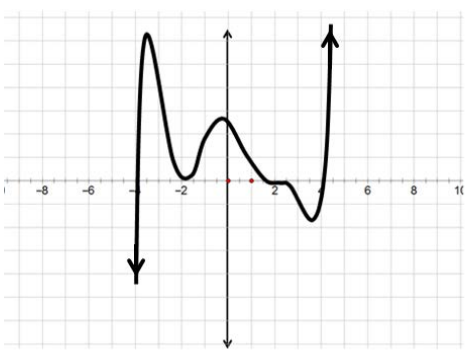
Step 4: Test points

Explore the signs of the function between different x-intercepts to get a rough sketch of the graph.

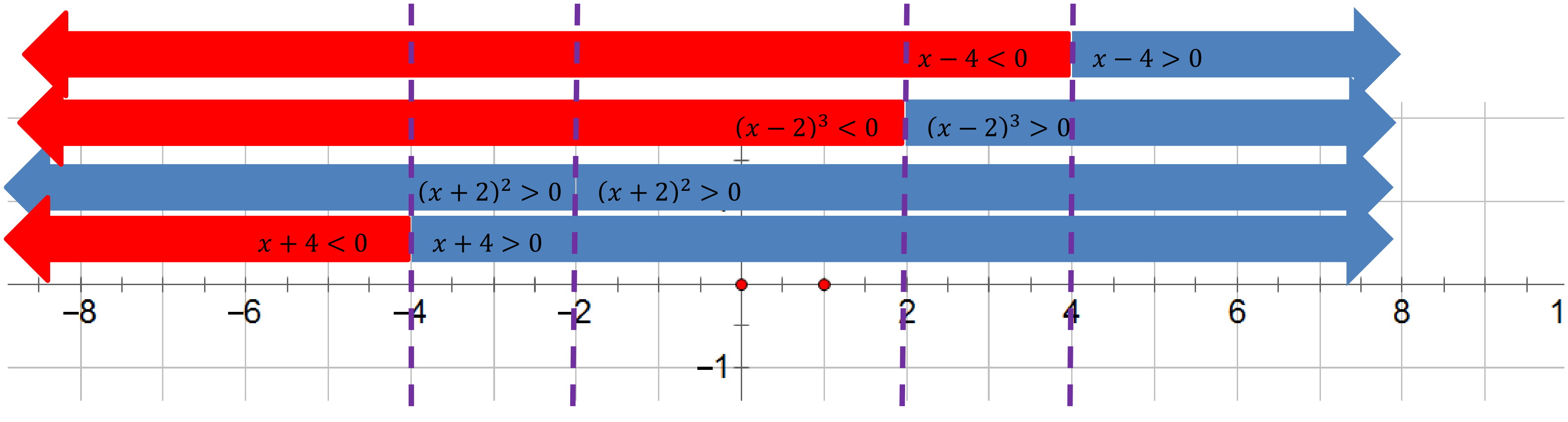
We could use our knowledge of the end behavior and intercepts finish the graph making use of our observation of where the function is linear, quadratic, or cubic. So the chart is really unnecessary for a rough sketch. See if you can come with the graph before looking on the next page.



Putting all pieces together we have our rough sketch below.



Another way to sketch is to investigate signs of the function at different x values.
We know that an odd number of negative numbers multiply to give us a negative number, and an even number of negative numbers multiply to give us a positive number. An odd number of positive numbers and an even number of positive numbers multiply to give a positive number.
We also know that a linear polynomial of the type $ax + b > 0$ for all real numbers $x > -\frac{b}{a}$, and $ax + b < 0$ for all real numbers $x < -\frac{b}{a}$. Using these two facts we can determine whether the function $f(x) > 0$ or $f(x) < 0$ for certain values of x.
See details below.



	$x < -4$	$-4 < x < -2$	$-2 < x < 2$	$2 < x < 4$	$x > 4$
Test Points	-5	-3	0	3	5
$x + 4$	$-$	$+$	$+$	$+$	$+$
$(x+2)^2$	$+$	$+$	$+$	$+$	$+$
$(x-2)^3$	$-$	$-$	$-$	$+$	$+$
$x - 4$	$-$	$-$	$-$	$-$	$+$
$f(x)$	$-$	$+$	$+$	$-$	$+$

$$f(x) = \frac{1}{10}(x-4)(x+4)(x-2)^3(x+2)^2$$

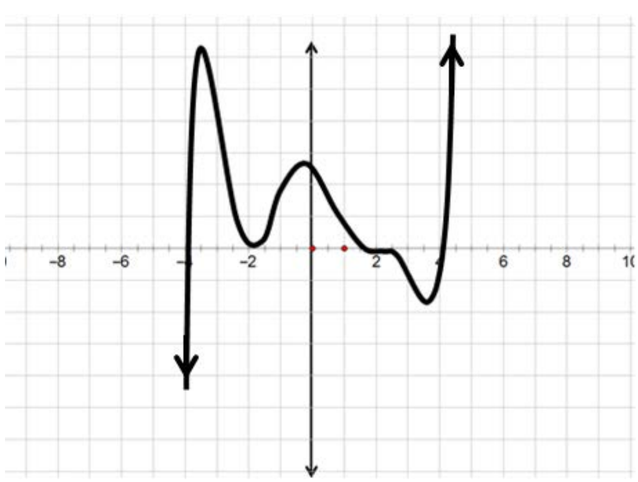
The chart we did is called a sign chart.

	$x < -4$	$-4 < x < -2$	$-2 < x < 2$	$2 < x < 4$	$x > 4$
Test Points	-5	-3	0	3	5
$x + 4$	$-$	$+$	$+$	$+$	$+$
$(x+2)^2$	$+$	$+$	$+$	$+$	$+$
$(x-2)^3$	$-$	$-$	$-$	$+$	$+$
$x - 4$	$-$	$-$	$-$	$-$	$+$
$f(x)$	$-$	$+$	$+$	$-$	$+$

So now we can draw a rough sketch of the graph remembering that the end behavior is like a 7th degree polynomial that means the two ends are as shown below.

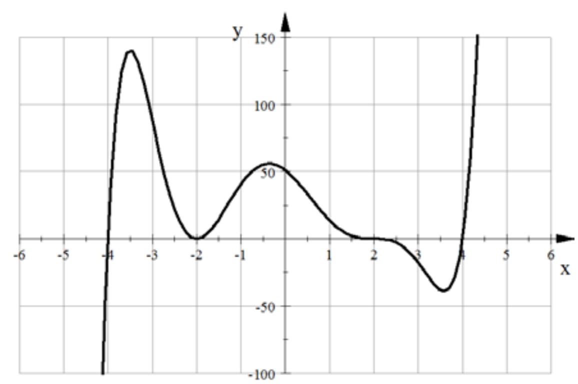
Using the chart above, the end behavior, and our expectation of a parabola behavior at $x = -2$, a cubic behavior at $x = 2$ and linear behavior at $x = 4, -4$ our $f(x)$ must have a shape like

Our attempt to get a rough sketch



When we draw a rough sketch we cannot tell how high or low the graph will dip.

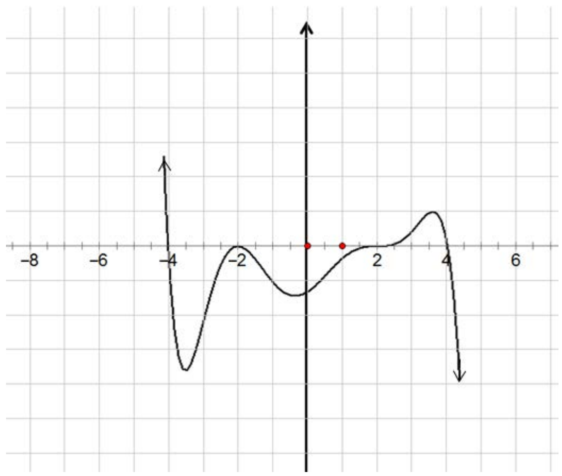
Using graphing utility



When we use a graphing utility we have to adjust the viewing window so we can see the relevant parts of the graph.

4. $f(x) = -\frac{1}{10}(x-4)(x+4)(x-2)^3(x+2)^2$

We know that putting a negative in front of the graph will just reflect the graph in previous example about the x-axis. So the graph will look like



The peaks and valleys we see have names. They are called local extrema or local maximum and minimum.

Local Maximum: In an interval (a, b) a point $(c, f(c))$ where $a < c < b$ is called a local maximum if $f(c) \geq f(x)$ for all x in the interval (a, b).

Local Minimum: In an interval (a, b) a point $(c, f(c))$ where $a < c < b$ is called a local minimum if $f(c) \leq f(x)$ for all x in the interval (a, b).

See if you can now attempt to graph the following functions.

5. $g(x) = (2x-9)(x+3)^2(x-1)^2$

x-intercepts are _______________

y-intercept is _____________

End Behavior like ________________

Number of local extremum _______

Graph of the polynomial

6. $h(x) = x(x+5)(x+2)(x-1)(x-4)$

x-intercepts are _______________

y-intercept is _____________

End Behavior like ________________

Number of local extremum _______

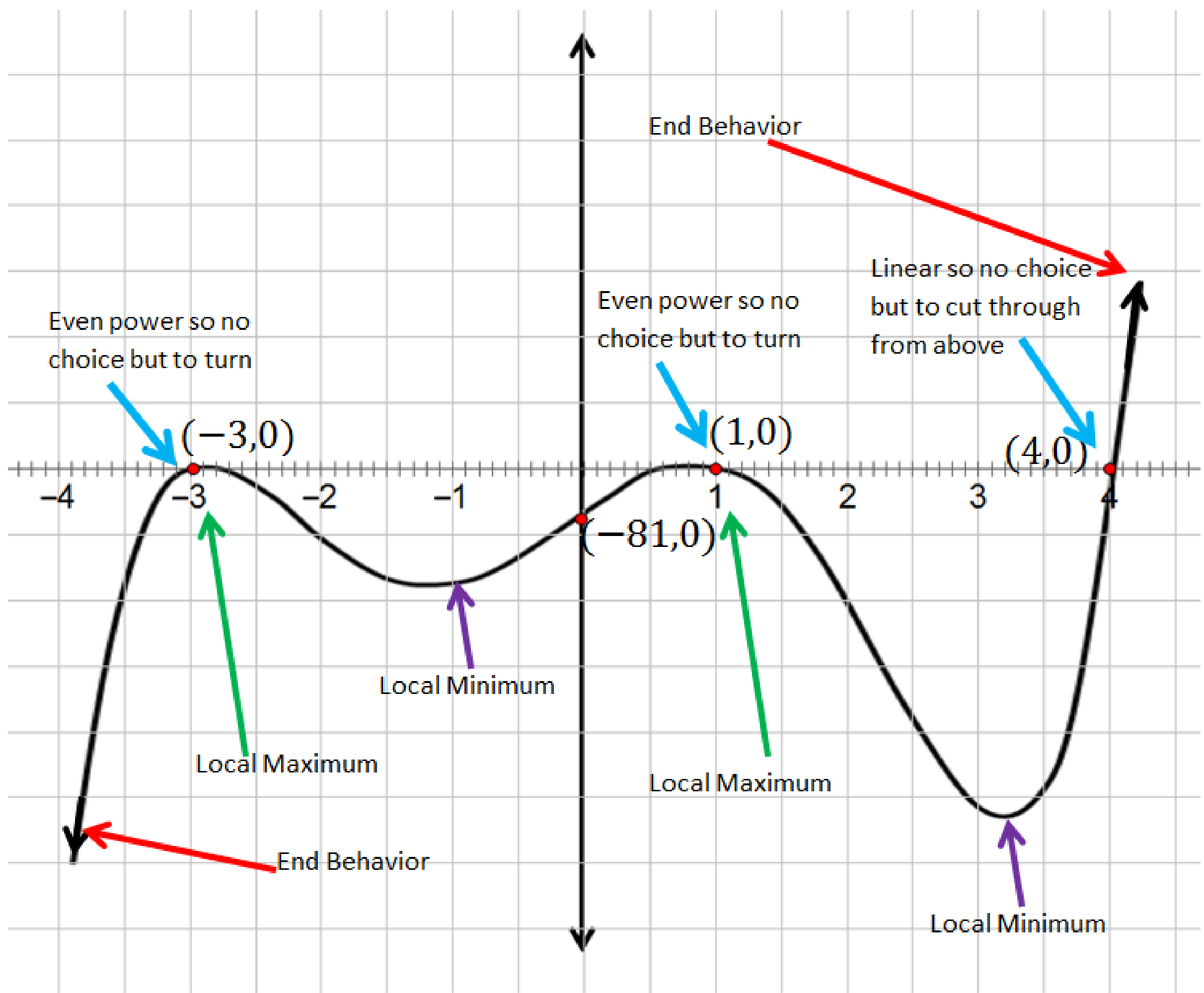
Graph of the polynomial

Solution

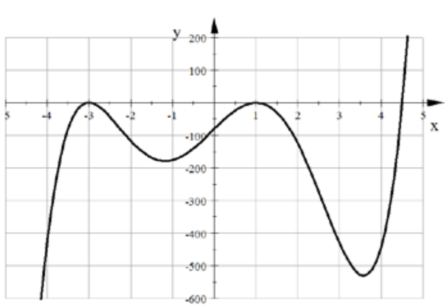
5. $g(x) = (2x - 9)(x + 3)^2(x - 1)^2$

x-intercepts are $x = \frac{9}{2} = 4.5, -3, 1$ y-intercept is $y = g(0) = (-9)(3)^2(1)^2 = -81$

End Behavior like $y = 2x(x^2)(x^2) = 2x^5$ Number of local maximums/minimums total 4



Using graphing utility we can see how low the function dips between $x = 1, x = 4$ there is now way for us to tell that now with the knowledge we have



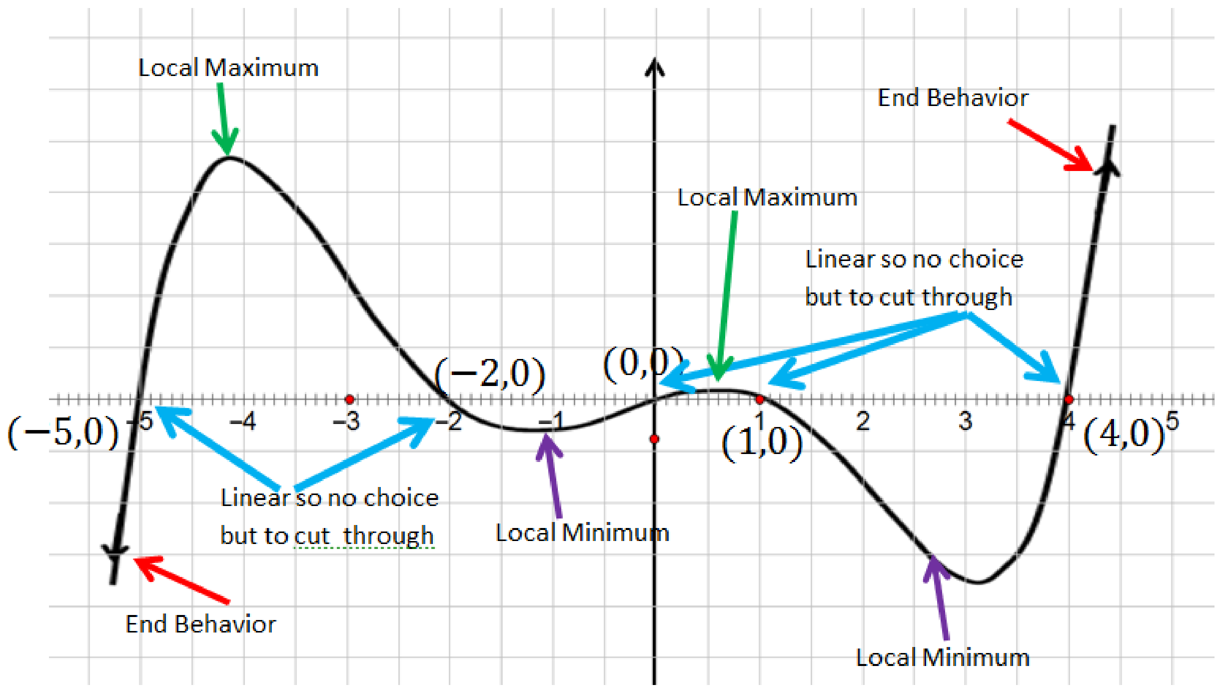
6. $h(x) = x(x+5)(x+2)(x-1)(x-4)$

x-intercepts are $x = 0, -5, -2, 1, 4$ y-intercept is $h(0) = 0(5)(-2)(1)(4)=0$

End Behavior like $x(x)(x)(x)(x) = x^5$

Number of local extrema 4

Graph of the polynomial



Observation:

- You can see from the examples we have done so far that a factor of the type $(ax+b)^n$ will either cross or touch (skim) the x-intercept $x = -\frac{b}{a}$ depending on whether n is an odd or an even number respectively.
- Maximum number of local extrema possible is one less than the degree of the polynomial

Polynomial function in factored form

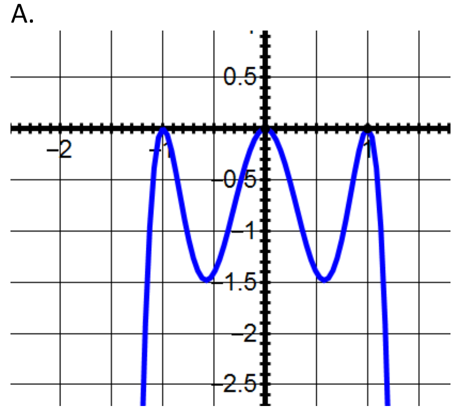
- Even power of a linear factor, graph touches the x-intercept.
- Odd power of a linear factor graph crosses the x-intercept.

As you can see a lot of information is stored in the polynomial function. However, in all the examples above we had the polynomial in factored form.

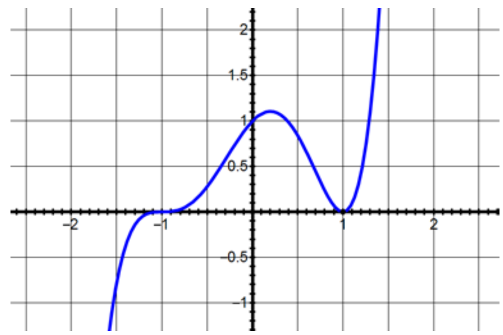
Match the functions on the right with their graphs on the left. Explain your asnwers.

1. $f(x) = (x-1)^2(x+1)^3$
2. $g(x) = (x-1)^2(x+1)^2$
3. $h(x) = -10x^2(x-1)^2(x+1)^2$
4. $R(x) = 10x^3(x-1)^2(x+1)^2$

A.

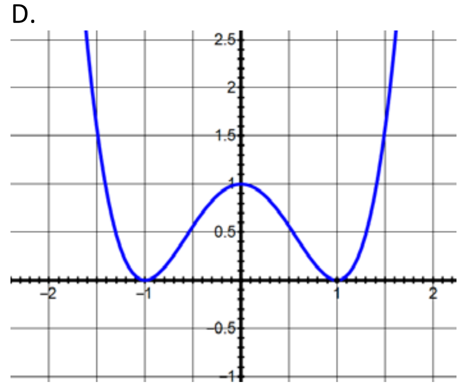


B.

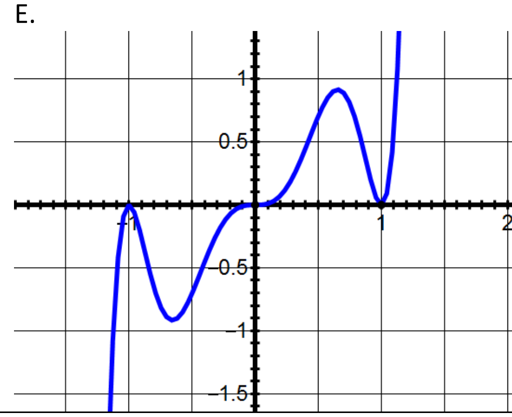


C.

D.



E.



Solution

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1. $f(x) = (x-1)^2(x+1)^3$
This graph matches B. Since the end behavior is like x^5, and at $x = 1$ we have a parabola shape, and a cubic shape at $x = -1$.

2. $g(x) = (x-1)^2(x+1)^2$
End behavior is like x^4, and parabola shapes at $x = 1, -1$, so must match graph C.

3. $h(x) = -10x^2(x-1)^2(x+1)^2$
Both ends down since leading coefficient is -10 and degree is 6. Parabola shapes at $x = 0, -1, 1$ so must match graph A.

4. $R(x) = 10x^3(x-1)^2(x+1)^2$
There is only one graph left D.

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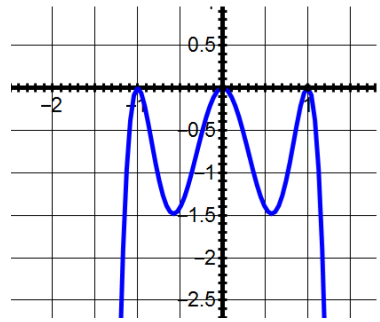
A.



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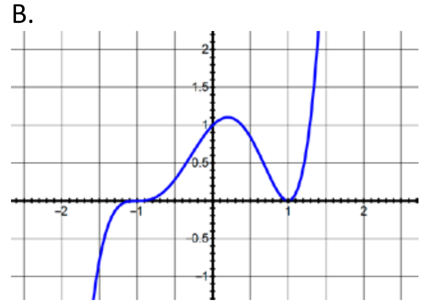
B.



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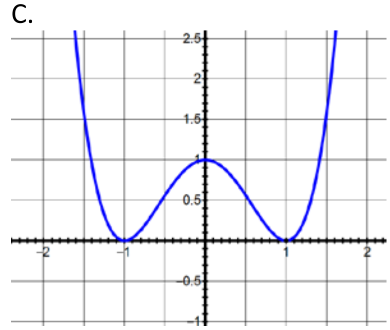
C.



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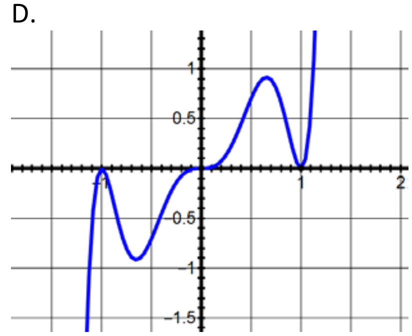
D.



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Playing

Question that you may want to start thinking about is can all polynomial functions be written in the factored form?

Cover Sheet for Sec 2.3 **Name: ____**

Please write your observations by filling the blanks below.

1. How does the degree of the polynomial affect the end behavior?

2. What does the leading coefficient of a polynomial control in its graph ?

3. What is the maximum number of local extrema you can expect in a polynomial of degree n?

4. How do each of the factors of the type $(ax + b)^n$ keep their identity in the graph of the entire polynomial function?

5. How do you find all the x-intercepts of a polynomial function?

6. How do you find all the y-intercepts of a polynomial function?

7. How would you solve a an equation of the type
   $a(x-a_1)^{n_1}(x-a_2)^{n_2}(x-a_3)^{n_3}\dots.(x-a_k)^{n_k}=0$?

8. What significance of the solutions to the equation in question 7 have to the graph of the polynomial function $f(x)=a(x-a_1)^{n_1}(x-a_2)^{n_2}(x-a_3)^{n_3}\dots.(x-a_k)^{n_k}$?

9. How would you solve a an inequality of the type
   $a(x-a_1)^{n_1}(x-a_2)^{n_2}(x-a_3)^{n_3}\dots.(x-a_k)^{n_k}>0$

Or

$a(x-a_1)^{n_1}(x-a_2)^{n_2}(x-a_3)^{n_3}\dots.(x-a_k)^{n_k}<0$

10. What significance of the solution to the inequalities in question 9 have to the graph of the polynomial function $f(x)=a(x-a_1)^{n_1}(x-a_2)^{n_2}(x-a_3)^{n_3}\dots.(x-a_k)^{n_k}$?

11. What is a local extrema of a polynomial function?

12. Does every polynomial function have a local extrema points? Explain your answer.

Video Log 2.3

Sketch the graph of the functions below label all x-intercepts, y-intercepts, label all local extrema points, show end behavior.

1. $g(x) = x^2(x-1)(x+2)^3$

x-intercepts are _______________

y-intercept is _____________

End Behavior like ________________

Number of local extremum _______

Graph of the polynomial

2. $h(x) = x(x+1)(x+4)(x-2)(x-5)$

x-intercepts are _______________

y-intercept is _____________

End Behavior like ________________

Number of local extremum _______

Graph of the polynomial

3. $g(x) = (x+1)^3(x-1)(x-2)$

x-intercepts are _______________

y-intercept is _____________

End Behavior like ________________

Number of local extremum _______

Graph of the polynomial

4. $h(x) = (x-1)(x+1)^2(x+2)^2(x-3)$

x-intercepts are _______________

y-intercept is _____________

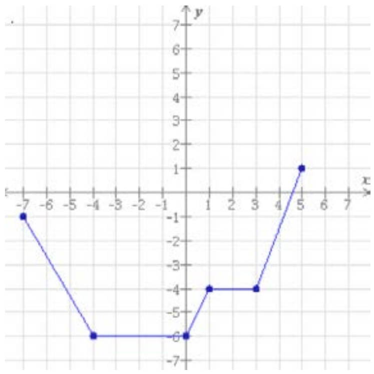
End Behavior like ________________

Number of local extremum _______

Graph of the polynomial

5. Determine the interval(s) on which the function is (strictly) increasing, or decreasing, or constant. Write your answer in interval notation.

A.

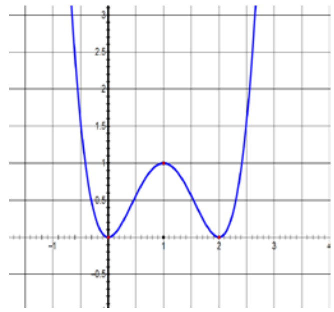


Increasing: ____________

Decreasing: ____________

Constant: ____________

B.

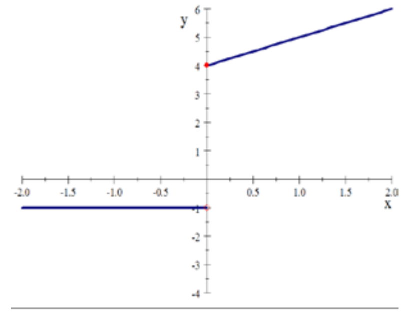


Increasing: ____________

Decreasing: ____________

Constant: ____________

C.



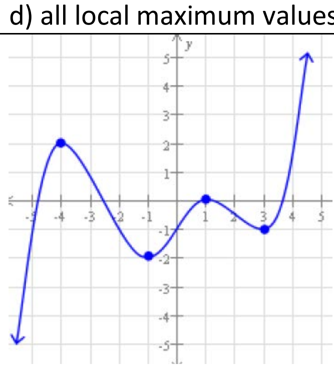
Increasing: ____________

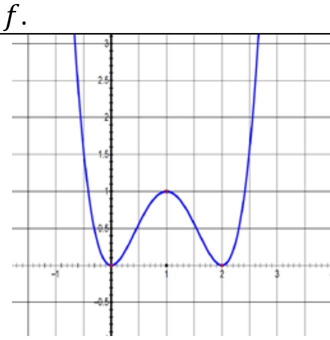
Decreasing: ____________

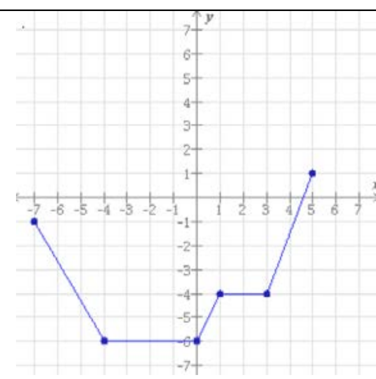
Constant: ____________

6. Use the graph of the function f below to find (If there is more than one answer, separate them with commas)

    a) All values at which f has local minimum
    b) All values at which f has local maximums
    c) all local minimum values of f
    d) all local maximum values of f.



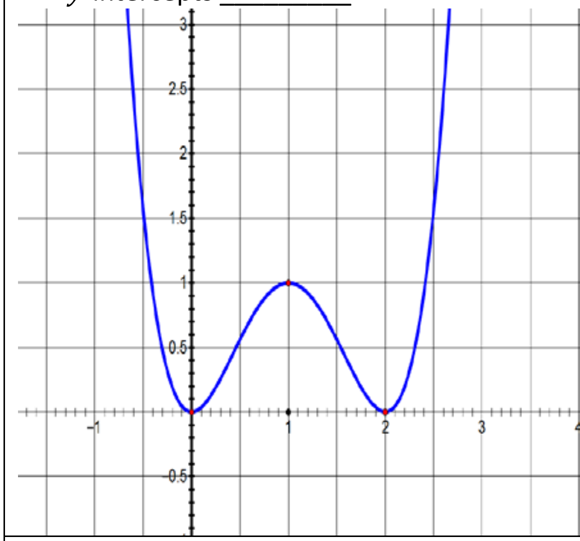




7. Find all the x-intercepts and y-intercepts of the functions below. If there is more than one answer, separate them with commas.

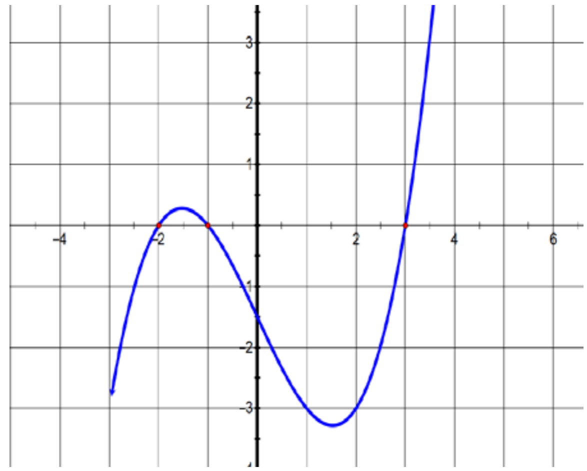
A. x-intercepts __________

y-intercepts __________



B. x-intercepts __________

y-intercepts __________



C.

$$f(x) = (x+3)(x-1)^2(x+5)$$

x-intercepts __________

y-intercepts __________

D.

$$f(x) = 2x^3 + 2x^2 - 18x - 18$$

x-intercepts __________

y-intercepts __________

E.

$$f(x) = x^4 - 4x^2 - x^3 + 4x$$

x-intercepts __________

y-intercepts __________

8. Choose the end behavior of the graph of each polynomial function below.

A. $f(x) = (x+3)(x-1)^2(x+5)$

Left		Right	
☐ Falls	☐ Rises	☐ Falls	☐ Rises

B. $f(x) = -2x^3 + 2x^2 - 18x - 18$

Left		Right	
☐ Falls	☐ Rises	☐ Falls	☐ Rises

D. $f(x) = 5x^{11} - 4x^2 - x^3 - 2x$

Left		Right	
☐ Falls	☐ Rises	☐ Falls	☐ Rises

E. $f(x) = -4x^{32} - 5x^{20} + 4x - 1$

Left		Right	
☐ Falls	☐ Rises	☐ Falls	☐ Rises

9. Match the graphs below with the functions listed here.

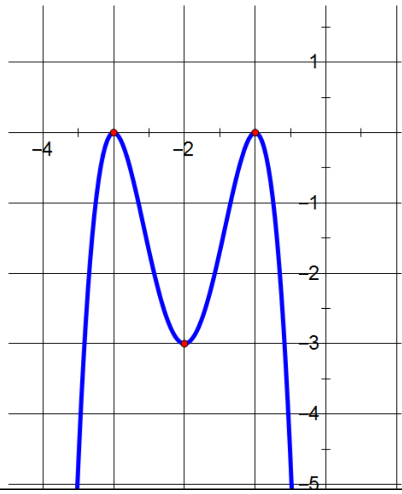
I. $f(x) = -3(x+1)^2(x+3)^2$

II. $h(x) = x^2(x-2)^3(x+1)$

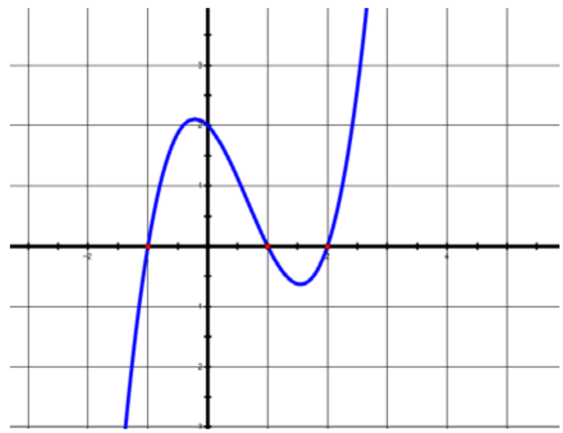
III. $g(x) = \frac{1}{2}(x^3 - x^2 - 6x)$

IV. $r(x) = (x-1)(x+1)(x-2)$

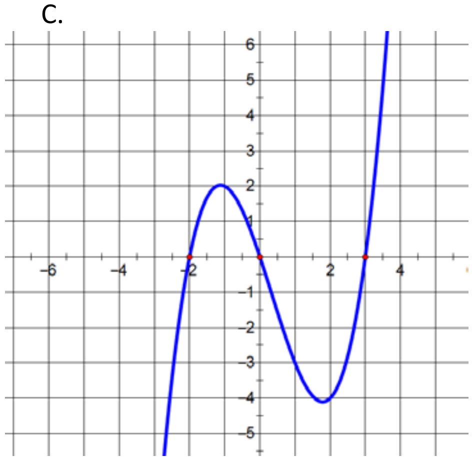
A.



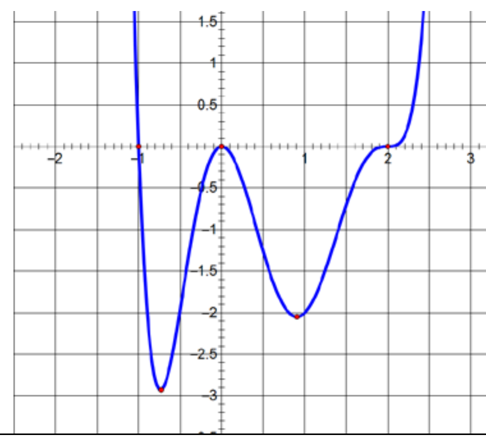
B.



C.

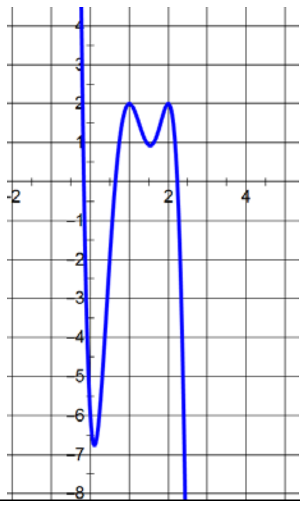


D.

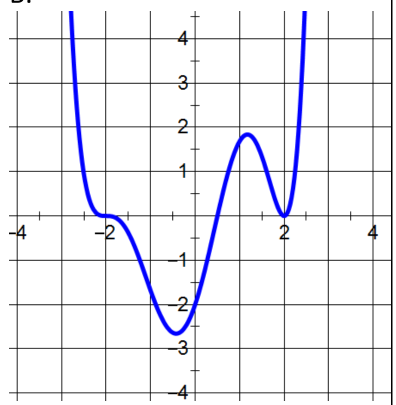


10. For the graphs below determine the following
    I. What is the sign of the leading coefficient of the polynomial functions below (positive, negative or not enough information?
    II. Which of the following is a possibility of the degree of the function? Choose all that apply 4,5,6,7,8,9.

A.



B.



C.