Chapter 2: Graphing Functions and Relations

2.1 Library of Functions

Lecture

- Graphing Part 1 (14 min) https://www.youtube.com/watch?v=oxILQDhM3yM
- Graphing Part 2 (15min) https://www.youtube.com/watch?v=zdO_bttea_E
- Graphing Part 3 (15 min) https://www.youtube.com/watch?v=4xwIHBko9RA

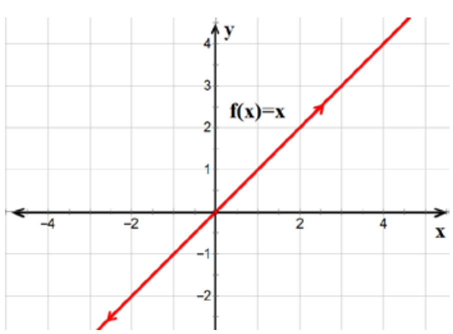
We will focus on the graphs of relations and functions in this chapter. We already have some basic understanding of specific functions from last chapter. You graphed versions of the basic polynomial, rational, square root, absolute value, exponential and logarithmic functions. One way to graph functions and relations is to plot enough points to give you a good sense of the main features of the graph. On the other hand as a mathematician we can gain deeper understanding of functions and relations by exploring their generic forms and then exploring how certain modifications of these parameters affect the basic form leading us to predictable changes in the graphs.

Before we begin let us create a library of graphs of some of the functions you may have encountered before now and see how these graphs can lead to understanding of more complex graphs.

Library of graphs functions

1. $f(x) = x$

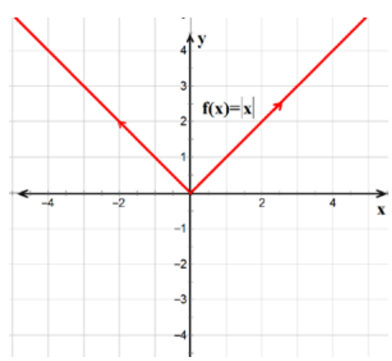
x	$y = x$
-2	-2
-1	-1
0	0
1	1
2	2



This is an odd function or symmetric with respect to the origin. With $(0,0)$ being the x-intercept and y-intercept.

2. $f(x) = |x|$

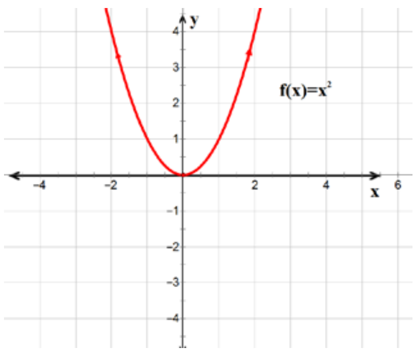
x	$y = \|x\|$
-2	2
-1	1
0	0
1	1
2	2



Notice how this graph differs from the graph of $y = x$ to the left. You can see that all the absolute value changed in the graph of $y = x$ is to invert bottom left part and make it positive. So not the graph of $y = |x|$ is an even function or symmetric with respect to the y-axis. And $(0,0)$ is the x-intercept and y-intercept.

3. $f(x) = x^2$

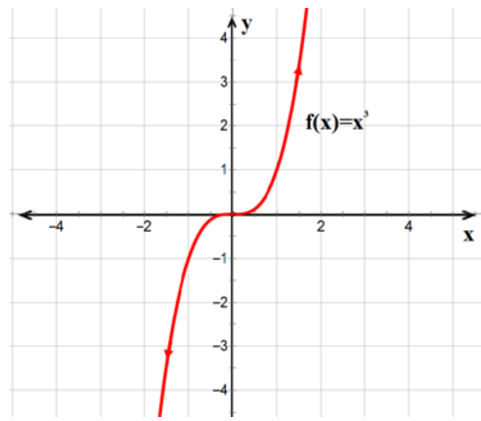
x	$y = x$
-2	4
-1	1
0	0
1	1
2	4



This is an even function or symmetric with respect to the y-axis. With $(0,0)$ being the x-intercept and y-intercept.

4. $f(x) = x^3$

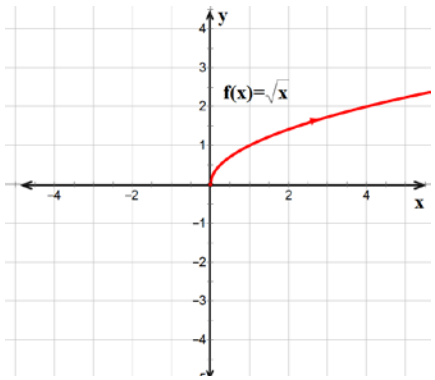
x	$y = x$
-2	-8
-1	-1
0	0
1	1
2	8



This is an odd function or symmetric with respect to the origin. With $(0,0)$ being the x-intercept and y-intercept.

5. $f(x) = \sqrt{x}$

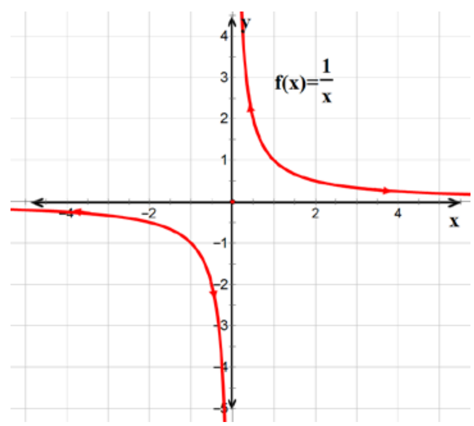
x	$y = x$
0	-2
1	-1
4	0
9	1



This function is neither odd nor even. With $(0,0)$ being the x-intercept and y-intercept.

6. $f(x) = \frac{1}{x}$

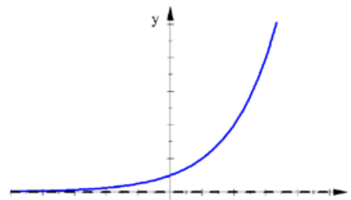
x	$y = \frac{1}{x}$		x	$y = \frac{1}{x}$
-0.1	$\frac{1}{-0.1} = -10$		-10	$\frac{1}{-10} = -0.1$
-0.01	$\frac{1}{-0.01} = -100$		-100	$\frac{1}{-100} = -0.01$
-0.001	$\frac{1}{-0.001} = -1000$		-1000	$\frac{1}{-1000} = -0..001$
0.1	$\frac{1}{0.1} = 10$		10	$\frac{1}{10} = 0.1$
0.01	$\frac{1}{0.01} = 100$		100	$\frac{1}{100} = 0.01$
0.001	$\frac{1}{0.001} = 1000$		1000	$\frac{1}{1000} = 0..001$



This is an odd function or symmetric with respect to the origin. There are no x or y intercepts. The lines $y = 0$ and $x = 0$ are horizontal and vertical asymptotes respectively.

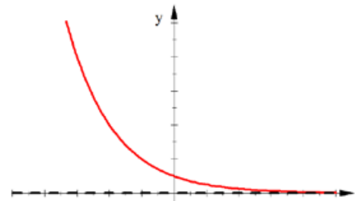
7. $f(x) = a^x$

Graph of $f(x) = a^x$, for $a > 1$



Horizontal Asymptote $y = 0$

Graph of $f(x) = a^x$, for $a < 1$

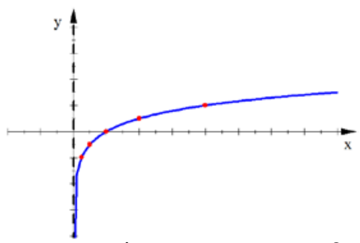


Horizontal Asymptote $y = 0$

The line $y = 0$ is a horizontal asymptote.

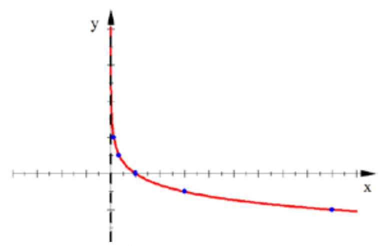
8. $f(x) = log_a x$

Graph of $f(x) = log_a x$, for $a > 1$



Vertical Asymptote $x = 0$

Graph of $f(x) = log_a x$, for $0 < a < 1$



Vertical Asymptote $x = 0$

The line $x = 0$ is a vertical asymptote.

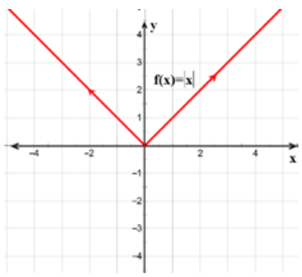
Playing

Once we have the basic graphs of a few functions we can use these to figure out how the graphs of these original functions change, when we change a few parameters. So if we know the graph of the function $y = f(x)$, can we find graph the function $y = af(x - h) + k$. In other words play with functions below to figure out what the number a, b, and c actually do the graph of $y = f(x)$.
Please fill in the blanks for the graphs below. Then write your observations. Do not peek ahead to see what happens. The work below will give you a good insight into the graphing of functions.

Please sketch the graphs of the functions below by filling the chart and then plotting the appropriate coordinates.

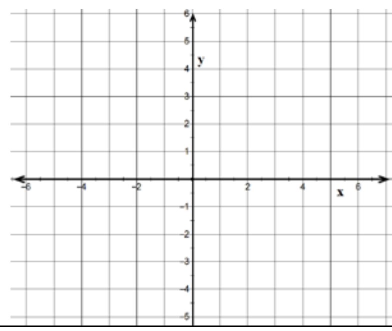
1. $f(x) = |x|$

x	$y = \lvert x \rvert$
-2	2
-1	1
0	0
1	1
2	2



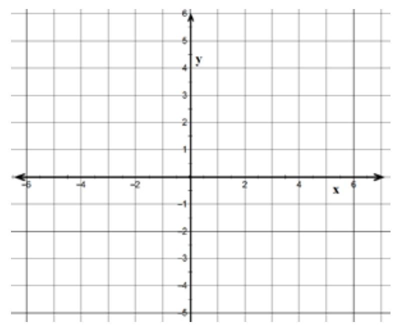
2. $f(x) = |x| + 2$

x	$y = \lvert x \rvert + 2$
-2	
-1	
0	
1	
2	



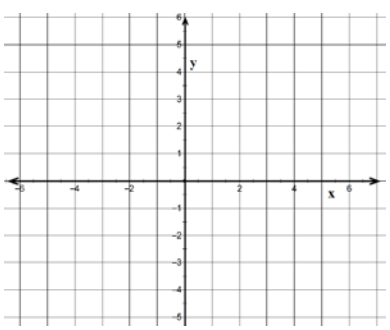
3. $f(x) = |x| - 2$

x	$y = \lvert x \rvert - 2$
-2	
-1	
0	
1	
2	



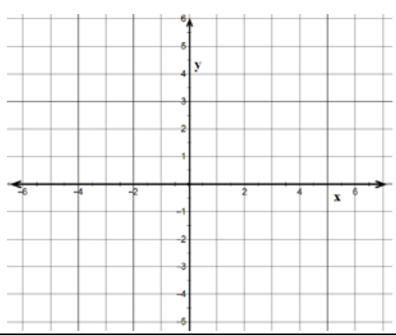
4. $f(x) = |x - 2|$

x	$y = \lvert x - 2 \rvert$
0	
1	
2	
3	
4	



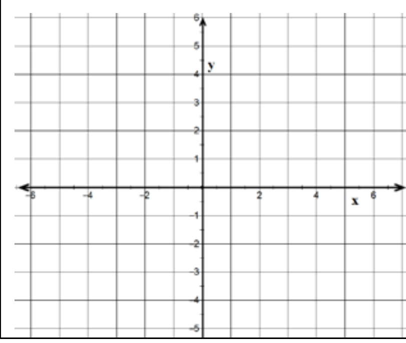
5. $f(x) = |x + 2|$

x	$y = \lvert x + 2 \rvert$
-4	
-3	
-2	
-1	
0	



6. $f(x) = |x - 2| + 2$

x	$y = \lvert x - 2 \rvert + 2$
-4	
-3	
-2	
-1	
0	

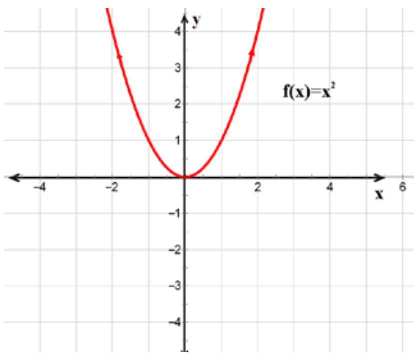


Observations

Graph of $f(x) = |x - h| + k$ is the same as the graph of $y = |x|$ but is shifted _______ units up/down, and shifted _________ units left/right.

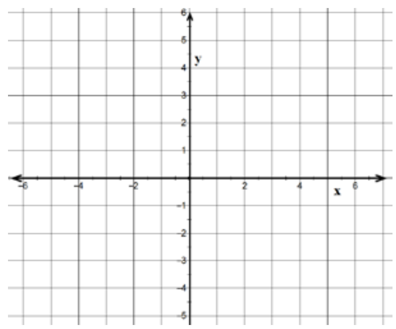
7. $f(x) = x^2$

x	$y = x^2$
-2	4
-1	1
0	0
1	1
2	4



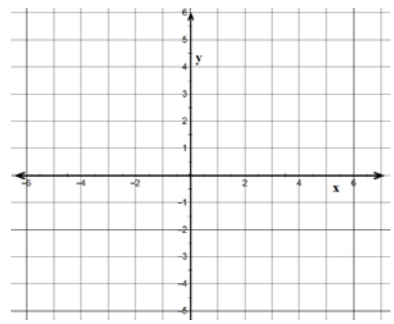
8. $f(x) = x^2 + 2$

x	$y = x^2 + 2$
-2	
-1	
0	
1	
2	



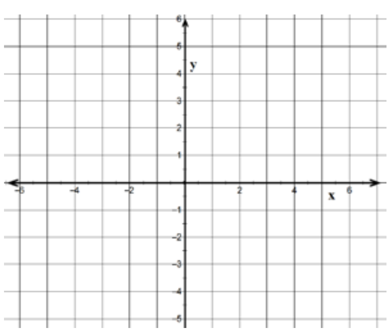
9. $f(x) = x^2 - 2$

x	$y = x^2 - 2$
-2	
-1	
0	
1	
2	



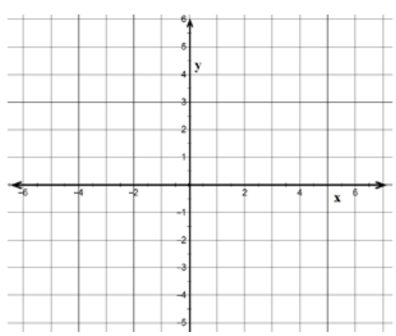
10. $f(x) = (x - 2)^2$

x	$y = (x - 2)^2$
0	
1	
2	
3	
4	



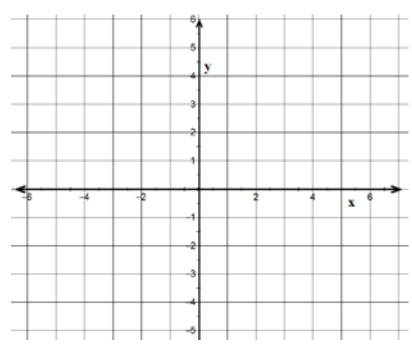
11. $f(x) = (x + 2)^2$

x	$y = (x + 2)^2$
-4	
-3	
-2	
-1	
0	



12. $f(x) = (x + 2)^2 + 2$

x	$y = (x + 2)^2 + 2$
-4	
-3	
-2	
-1	
0	

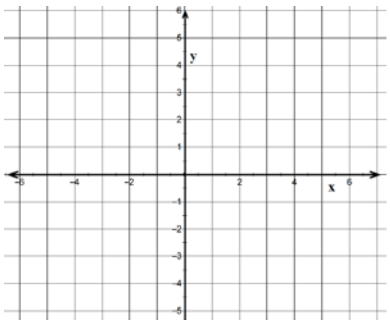


Observations

Graph of $f(x) = (x - h)^2 + k$ is the same as the graph of $y = x^2$ but is shifted _______ units up/down, and shifted _________ units left/right.

13. $f(x) = 3^x$

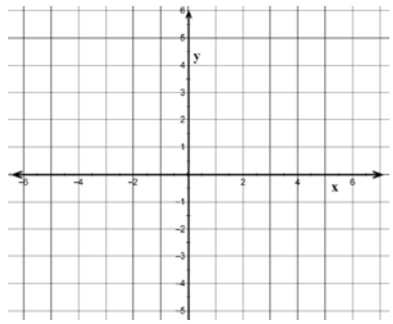
x	$y = 3^x$
-2	
-1	
0	
1	
2	



Horizontal Asymptote at
$y =$___________

14. $f(x) = 3^x + 2$

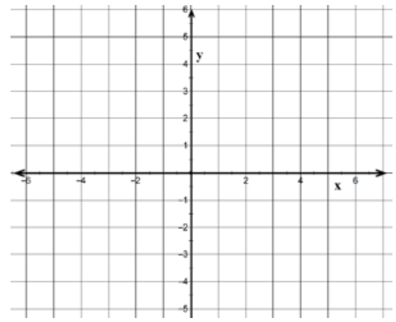
x	$y = 3^x + 2$
-2	
-1	
0	
1	
2	



Horizontal Asymptote at
$y =$___________

15. $f(x) = 3^x - 2$

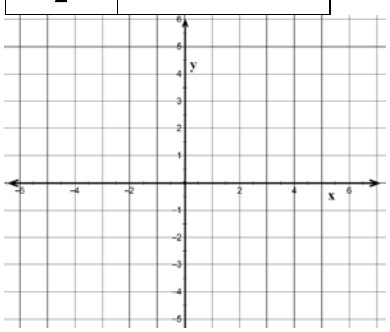
x	$y = 3^x - 2$
-2	
-1	
0	
1	
2	



Horizontal Asymptote at
$y =$___________

16. $f(x) = -3^x$

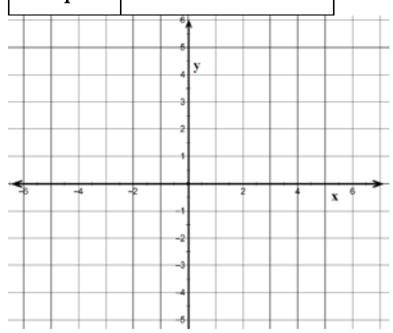
x	$y = -3^x$
-2	
-1	
0	
1	
2	



Horizontal Asymptote at
$y =$___________

17. $f(x) = 3^{x-2}$

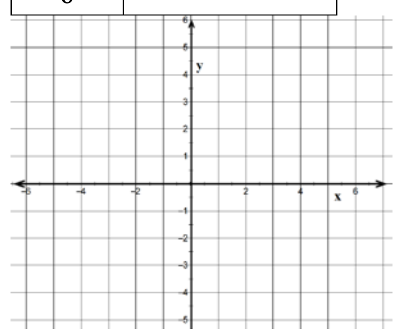
x	$y = 3^{x-2}$
0	
1	
2	
3	
4	



Horizontal Asymptote at
$y =$___________

18. $f(x) = 3^{x+2}$

x	$y = 3^{x+2}$
-4	
-3	
-2	
-1	
0	

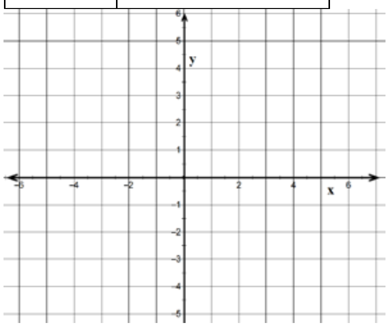


Horizontal Asymptote at
$y =$___________

Observations: Graph of $f(x) = a^{x-h} + k$ is the same as the graph of $y = a^x$ but is shifted _______ units up/down, and shifted _________ units left/right. The horizontal asymptote of the graph of $f(x) = a^{x-h} + k$ is given by $y =$______________.

19. $f(x) = log_3 x$

x	$y = log_3 x$
$\frac{1}{9}$	
$\frac{1}{3}$	
1	
3	
9	

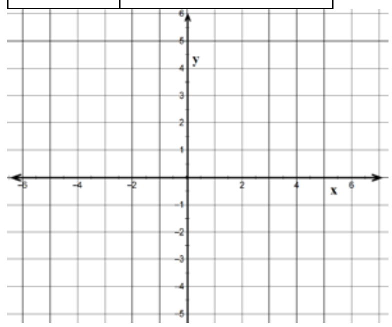


Vertical Asymptote at

$x =$___________

20. $f(x) = log_3 x + 2$

x	$y = log_3 x + 2$
$\frac{1}{9}$	
$\frac{1}{3}$	
1	
3	
9	

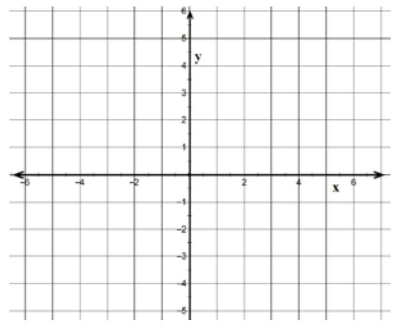


Vertical Asymptote at

$x =$___________

21. $f(x) = log_3 x - 2$

x	$y = log_3 x - 2$
$\frac{1}{9}$	
$\frac{1}{3}$	
1	
3	
9	

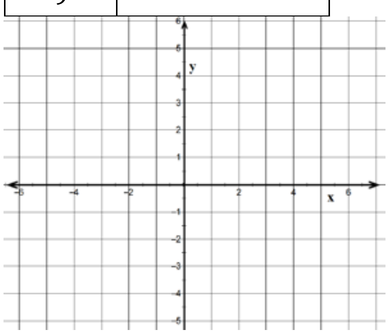


Vertical Asymptote at

$x =$___________

22. $f(x) = -log_3 x$

x	$y = -log_3 x$
$\frac{1}{9}$	
$\frac{1}{3}$	
1	
3	
9	

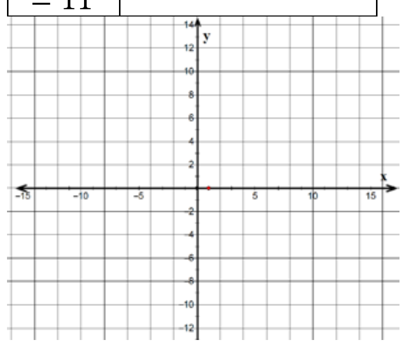


Vertical Asymptote at

$x =$___________

23. $f(x) = log_3(x - 2)$

x	$y = log_3(x - 2)$
$\frac{1}{9} + 2$	
$\frac{1}{3} + 2$	
$1 + 2 = 3$	
$3 + 2 = 5$	
$9 + 2 = 11$	

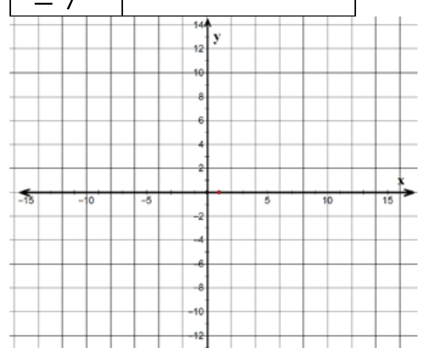


Vertical Asymptote at

$x =$___________

24. $f(x) = log_3(x + 2)$

x	$y = log_3(x + 2)$
$\frac{1}{9} - 2$	
$\frac{1}{3} - 2$	
$1 - 2 = -1$	
$3 - 2 = 1$	
$9 - 2 = 7$	



Vertical Asymptote at

$x =$___________

Observations

Graph of $f(x) = log_a(x - h) + k$ is the same as the graph of $y = log_a x$ but is shifted ______ units up/down, and shifted ________units left/right. The vertical asymptote of the graph of $f(x) = log_a(x - h) + k$ is given by $y =$______________.

In general our observations are that the graph of $y = f(x - h) + k$ is the same as the graph of $y = f(x)$ but is shifted ______ units up/down, and shifted ________units left/right. If the gaph of $y = f(x)$ has vertical or horizontal asymptotes, then in the new graph of $y = f(x - h) + k$ the vertical asymptote of the graph is at $x =$_____________ and the horizontal asymptote of the graph is at $y =$______________.

Use your own functions $y = f(x)$ and make observations of what is the graph $y = af(x)$ and also the graph of $y = f(ax)$. Use the space below.

Cover Sheet for Sec 2.1 **Name: ____**

Please write your observations by filling the blanks below.

1. The graph of the function $y = f(x) + k$ is shifted ________________________________ from the graph of the original function $y = f(x)$.
2. The graph of the function $y = f(x) - k$ is shifted ________________________________ from the graph of the original function $y = f(x)$.

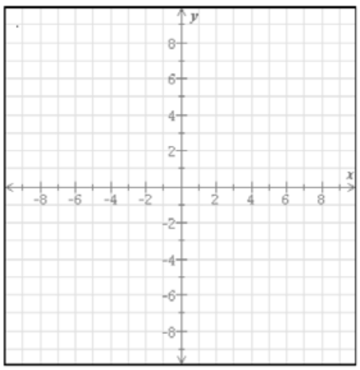
   Statements 1 and 2 can also be written as statements 3 and 4 below.
3. The graph of the function $y - k = f(x)$ is shifted ________________________________ from the graph of the original function $y = f(x)$.
4. The graph of the function $y + k = f(x)$ is shifted ________________________________ from the graph of the original function $y = f(x)$.
5. The graph of the function $y = f(x - h)$ is shifted ________________________________ from the graph of the original function $y = f(x)$.
6. The graph of the function $y = f(x + h)$ is shifted ________________________________ from the graph of the original function $y = f(x)$.
7. The graph of the function $y = af(x)$ is __________________________ from the graph of the original function $y = f(x)$.
8. The graph of the function $y = cf(x)$ is __________________________ from the graph of the original function $y = f(x)$.
9. The graph of the function $y = -f(x)$ is ____ from the graph of the original function.
10. The graph of the function $y = f(-x)$ is ____ from the graph of the original function.
11. For quadratic functions, completing the squares allows you to find the vertex of the parabola. So if we had the function $y - k = a(x - h)^2$, the point (h, k) is referred to as the _______________ of the parabola.
12. For quadratic functions, completing the squares allows you to find the vertex of the parabola. So if we had the function $y + k = a(x + h)^2$, the point $(-h, -k)$ is referred to as the _______________ of the parabola.
13. An even function is symmetric with respect to the _______________.
14. An odd function is symmetric with respect to the _______________________.

Video Log 2.1

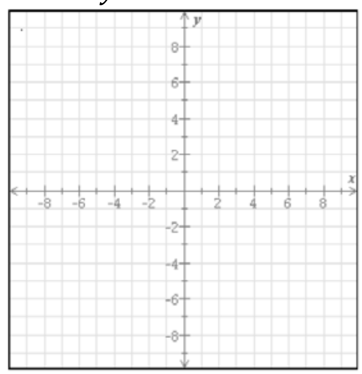
Using the facts from the previous sheet, sketch the graph of the functions below.

Sketch the graph of the functions and relations below. Explain clearly how you decided the graph was the shape you drew. Can you determine the domain and range of the functions and relations that you graphed based on the graphs.

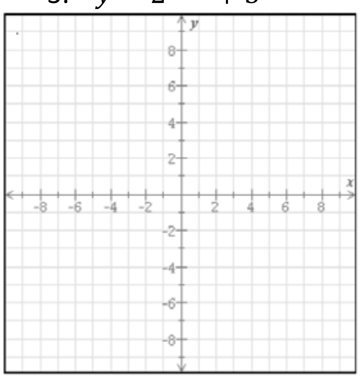
1. $y = 3|x - 5| + 2$



2. $y = x^2 + 3$

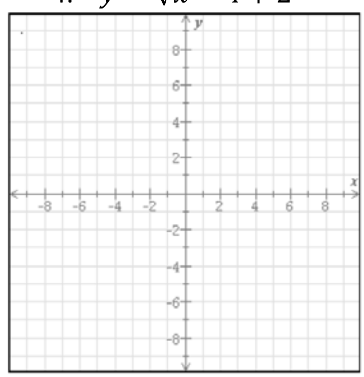


3. $y = 2^{x-1} + 3$

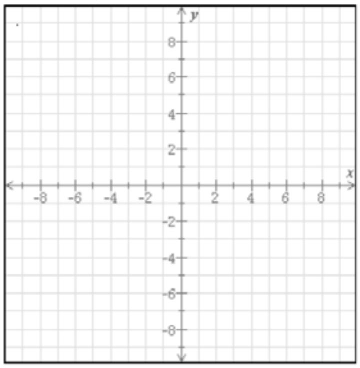


Horizontal Asymptote: ___________

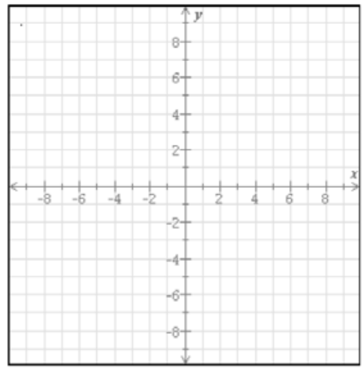
4. $y = \sqrt{x - 4} + 2$



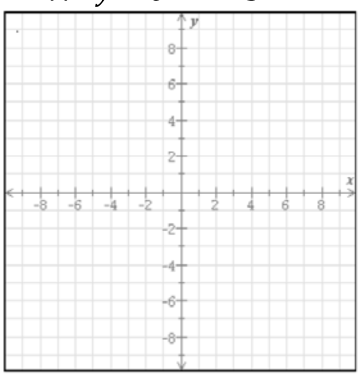
5. $y = -2x^3$



6. $y = -x^4+1$

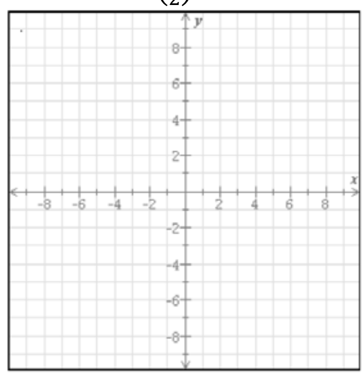


7. $y = e^{x-2} - 3$



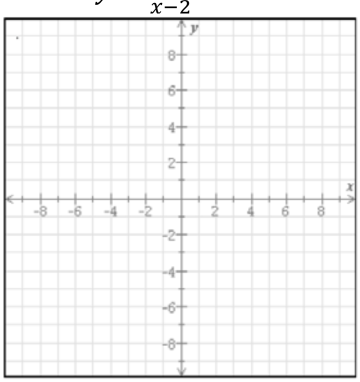
x	y

8. $y = \left(\frac{1}{2}\right)^{x+1} - 3$



x	y

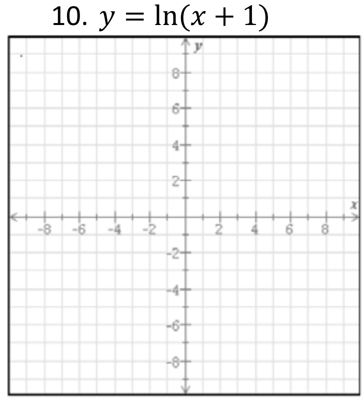
9. $y = \frac{1}{x-2}$



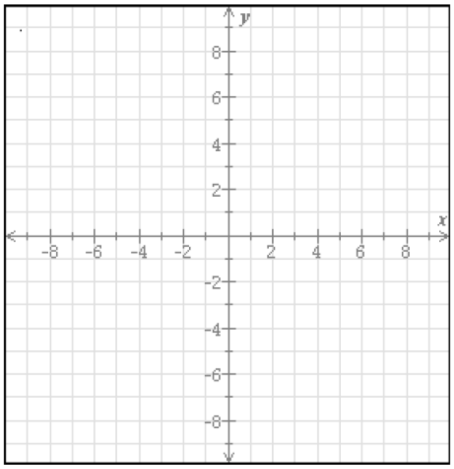
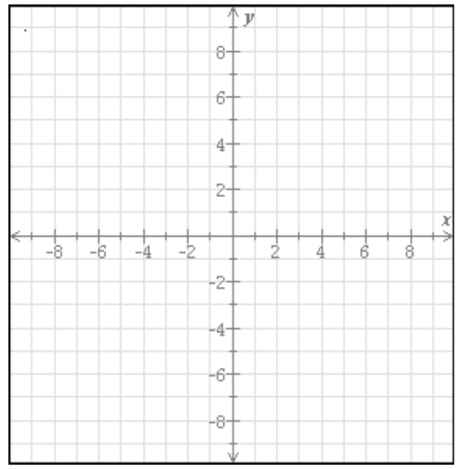
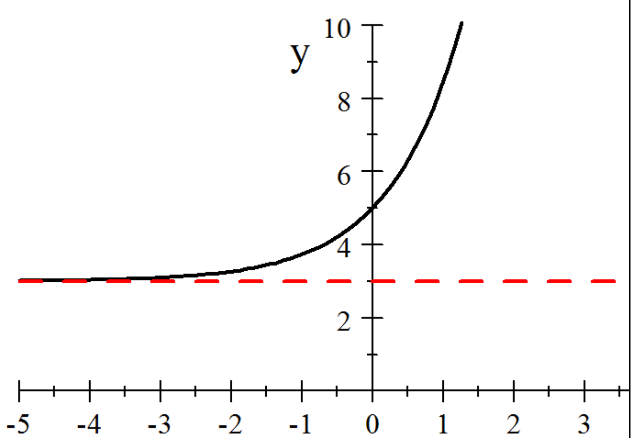
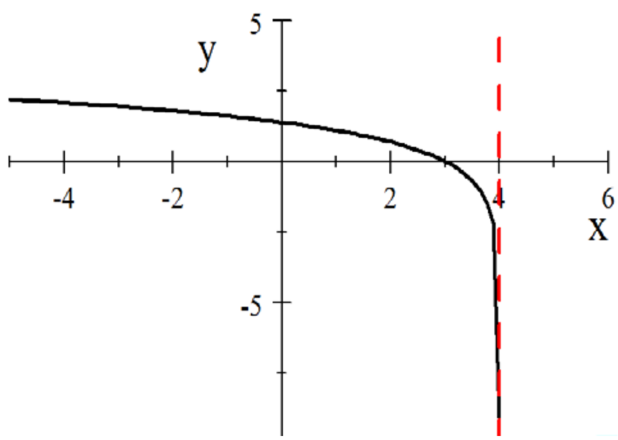
Vertical Asymptote: _______________

Horizontal Asymptote: _______________

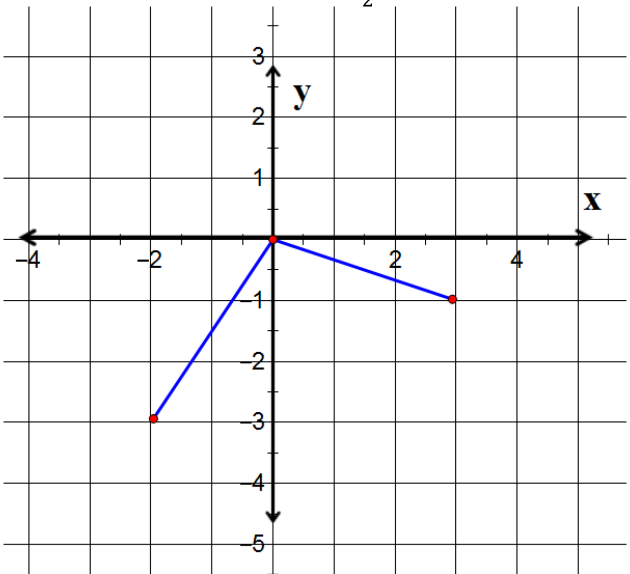
10. $y = \ln(x+1)$



Vertical Asymptote: _______________

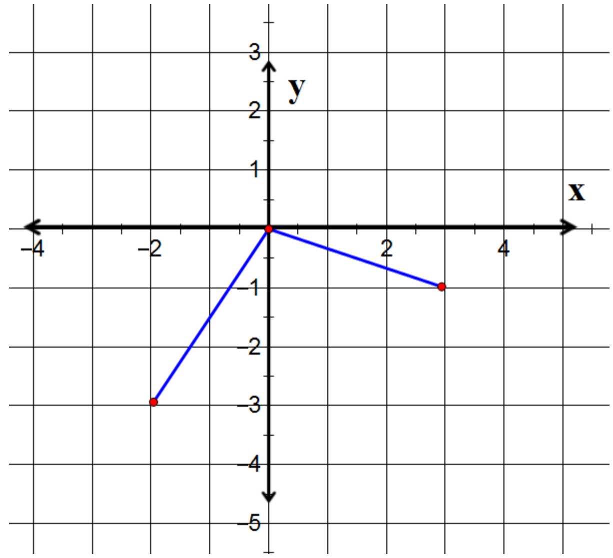
11. $y = \log(x) + 3$  Vertical Asymptote: _____________	12. $y = log_2(x)$  Vertical Asymptote: ______________
13. $y = f(x)$ is the graph below, use that to sketch the graph of $y = f(x - 1) + 2$ 	14. $y = f(x)$ is the graph below, use that to sketch the graph of $y + 1 = -f(x + 2)$ 

15. $y = g(x)$ has the graph below, use that to find the graph of $y = \frac{1}{2}g(x)$.



x	$y = \frac{1}{2}g(x)$

16. $y = g(x)$ has the graph below, use that to find the graph of $y = -g(x)$.



x	$y = -g(x)$