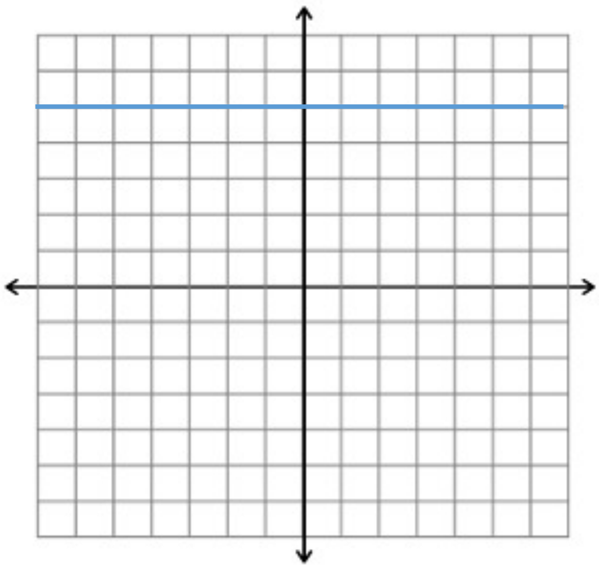
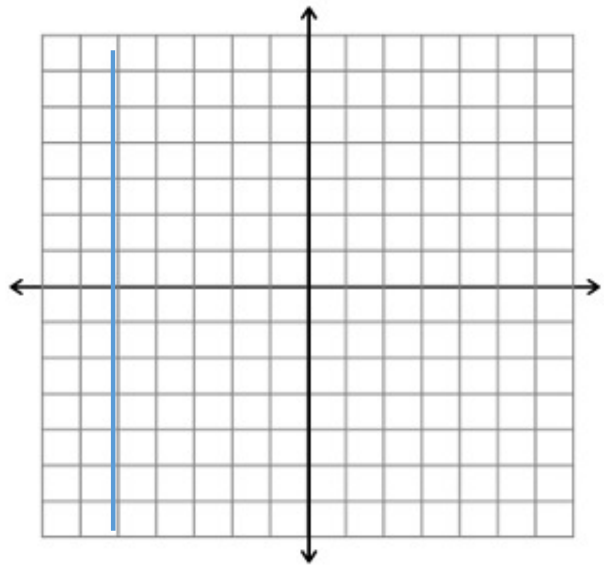
Fall 2016
Solutions to the Math 103/105 Final Exam Review

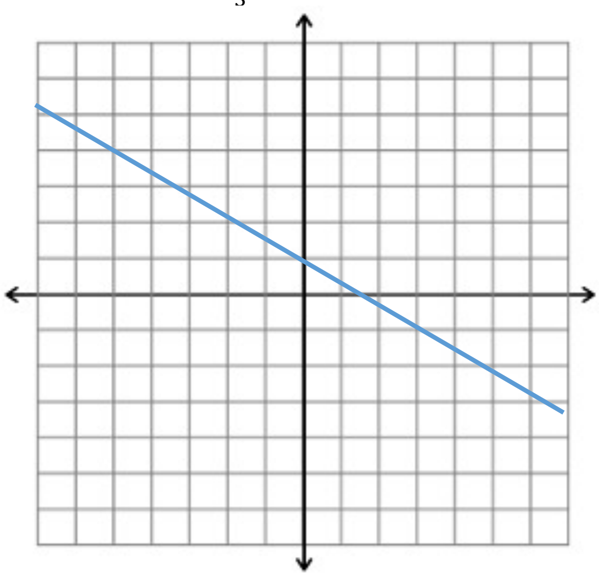
1. $(-3, 5)$ is not a solution because $3(-3) - 4(5) = -9 - 20 = -29 \neq 12$.
   $\left(\frac{5}{3}, -\frac{7}{4}\right)$ is a solution since $3\left(\frac{5}{3}\right) - 4\left(-\frac{7}{4}\right) = 5 + 7 = 12$.
   $\left(-\frac{1}{3}, \frac{3}{4}\right)$ is not a solution because $3\left(-\frac{1}{3}\right) - 4\left(\frac{3}{4}\right) = -1 - 3 = -4 \neq 12$.
2.
   a. $y = 5$



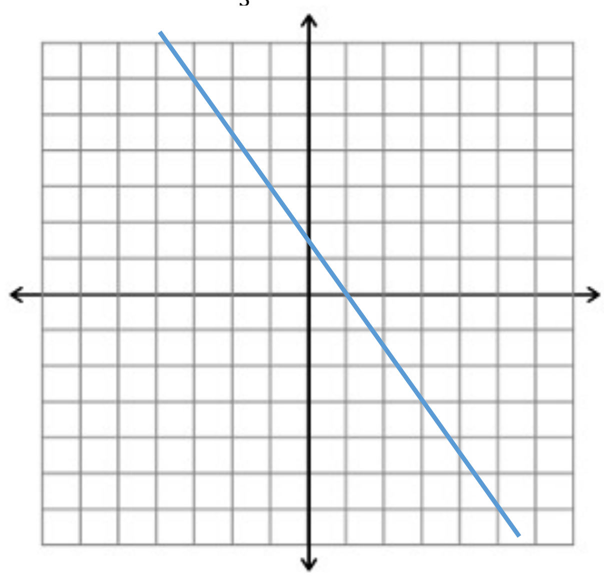
   b. $x = -5$



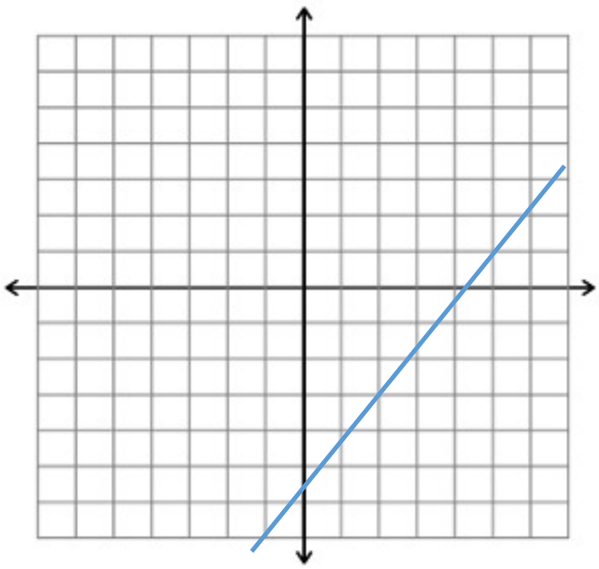
   c. $y = -\frac{2}{3}x + 1$



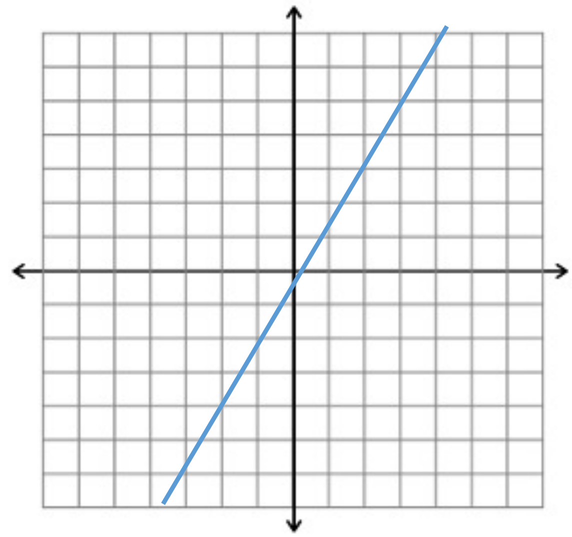
   d. $x = -\frac{2}{3}y + 1$



e. $(2,-3)$, $m = \frac{4}{3}$



f. $(-1,-2)$, $(3,5)$



3.

a. slope of the line $y = \frac{2}{3}x + 4$ is $\frac{2}{3}$. So,

$y - y_1 = m(x - x_1) \quad \rightarrow \quad y - (-1) = \frac{2}{3}(x - 3) \quad \rightarrow \quad y + 1 = \frac{2}{3}x - 2 \quad \rightarrow \quad y = \frac{2}{3}x - 3$

b. slope of the line $y = \frac{2}{3}x + 4$ is $\frac{2}{3}$. So,

$y - y_1 = m(x - x_1) \quad \rightarrow \quad y - (-1) = -\frac{3}{2}(x - 3) \quad \rightarrow \quad y + 1 = -\frac{3}{2}x + \frac{9}{2} \quad \rightarrow \quad y = -\frac{3}{2}x + \frac{7}{2}$

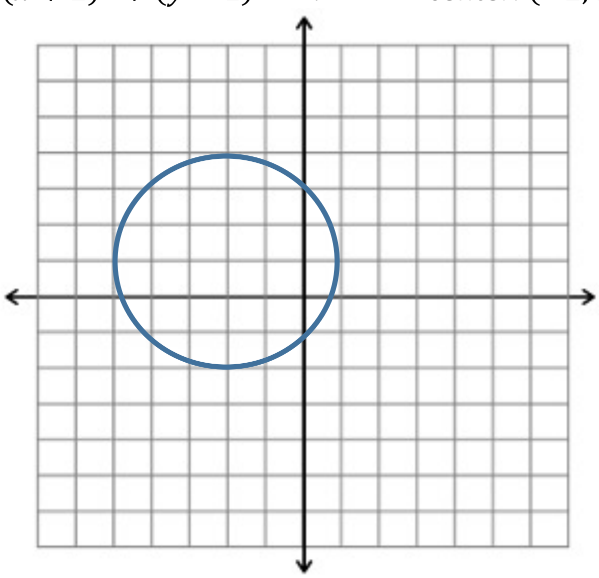
c. slope of the line $x = -3y + 4$ is $-\frac{1}{3}$. So,

$y - y_1 = m(x - x_1) \quad \rightarrow \quad y - 2 = -\frac{1}{3}\big(x - (-1)\big) \quad \rightarrow \quad y - 2 = -\frac{1}{3}x - \frac{1}{3} \quad \rightarrow \quad y = -\frac{1}{3}x + \frac{5}{3}$

d. $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-2}{4-(-1)} = -\frac{4}{5}$

$y - y_1 = m(x - x_1) \quad \rightarrow \quad y - 2 = -\frac{4}{5}\big(x - (-1)\big) \quad \rightarrow \quad y - 2 = -\frac{4}{5}x - \frac{4}{5} \quad \rightarrow \quad y = -\frac{4}{5}x + \frac{6}{5}$

4. $(x + 2)^2 + (y - 1)^2 = 9 \quad \rightarrow$ center: $(-2, 1)$ and $r = 3$



5.

a. $\begin{cases} y = x + 5 \\ 3x + y = 1 \end{cases}$

$3x + (x + 5) = 1 \quad \rightarrow \quad 4x + 5 = 1 \quad \rightarrow \quad 4x = -4 \quad \rightarrow \quad x = -1 \quad \rightarrow \quad y = -1 + 5 = 4$

b. $\begin{cases} x - 2y = 1 \\ x + y = 2 \end{cases} \quad \rightarrow \quad \begin{cases} x - 2y = 1 \\ -x - y = -2 \end{cases}$

$-3y = -1 \quad \rightarrow \quad y = \frac{1}{3} \quad \rightarrow \quad x + \frac{1}{3} = 2 \quad \rightarrow \quad x = 2 - \frac{1}{3} = \frac{5}{3}$

c. $\begin{cases} 3x - 2y = 3 \\ 2x + 3y = 2 \end{cases} \quad \rightarrow \quad \begin{cases} 6x - 4y = 6 \\ -6x - 9y = -6 \end{cases}$

$-13y = 0 \quad \rightarrow \quad y = 0 \quad \rightarrow \quad 3x - 2(0) = 3 \quad \rightarrow \quad 3x = 3 \quad \rightarrow \quad x = 1$

6.

a. $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6.5 - 10}{4 - 1} = -\frac{3.5}{3} = -\frac{7}{6}$

$y - y_1 = m(x - x_1) \quad \rightarrow \quad y - 10 = -\frac{7}{6}(x - 1) \quad \rightarrow \quad y - 10 = -\frac{7}{6}x + \frac{7}{6} \quad \rightarrow \quad y = -\frac{7}{6}x + \frac{67}{6}$

$H = -\frac{7}{6}t + \frac{67}{6}$

b. $\frac{67}{6} = 11.1\overline{6}$ inches

c. doesn't last this long since $H = -\frac{7}{6}(10) + \frac{67}{6} = -\frac{70}{6} + \frac{67}{6} = -\frac{3}{6} = -0.5$

d. $0 = -\frac{7}{6}t + \frac{67}{6} \quad \rightarrow \quad \frac{7}{6}t = \frac{67}{6} \quad \rightarrow \quad 7t = 67 \quad \rightarrow \quad t = \frac{67}{7} \approx 9.57143 \approx$ 9 hours and 34 minutes

7.

a. $A = 8 - 0.25(5) = 8 - 1.25 = 6.75$ liters

b. $A = 8 - 0.25(0) = 8$ liters

c. $0 = 8 - 0.25t \quad \rightarrow \quad 0.25t = 8 \quad \rightarrow \quad t = \frac{8}{0.25} = 32$ minutes

d. t-intercept is 32 which represents the time when the bucket is empty, and A-intercept is 8 which represents the amount of water initially in the bucket.

e. slope is -0.25 which represents the rate in liters per minute in which the bucket is leaking.

9.

a. $\frac{2}{3}(x + 1) - x = 2(x + 3) - 5 \quad \rightarrow \quad \frac{2}{3}x + \frac{2}{3} - x = 2x + 6 - 5 \quad \rightarrow \quad \frac{2}{3}x - x - 2x = 6 - 5 - \frac{2}{3}$

$\frac{2}{3}x - 3x = 1 - \frac{2}{3} \quad \rightarrow \quad -\frac{7}{3}x = \frac{1}{3} \quad \rightarrow \quad -7x = 1 \quad \rightarrow \quad x = -\frac{1}{7}$

b. $\frac{2}{3x - 1} = \frac{4}{x + 1} \quad \rightarrow \quad 2(x + 1) = 4(3x - 1) \quad \rightarrow \quad 2x + 2 = 12x - 4 \quad \rightarrow \quad -10x = -6$

$x = \frac{6}{10} = \frac{3}{5}$

c. $\frac{x}{x + 1} + \frac{2}{2x - 1} = \frac{2x}{2x - 1} \quad \rightarrow \quad \frac{x}{x + 1} = \frac{2x}{2x - 1} - \frac{2}{2x - 1} \quad \rightarrow \quad \frac{x}{x + 1} = \frac{2x - 2}{2x - 1} \quad \rightarrow \quad x(2x - 1) = (x + 1)(2x - 2)$

$2x^2 - x = 2x^2 - 2x + 2x - 2 \quad \rightarrow \quad 2x^2 - x = 2x^2 - 2 \quad \rightarrow \quad -x = -2 \quad \rightarrow \quad x = 2$

d. $\frac{3x}{x^2 + 3x - 4} - \frac{2}{x + 4} = \frac{5}{x - 1} \quad \rightarrow \quad \frac{3}{(x + 4)(x - 1)} - \frac{2(x - 1)}{(x + 4)(x - 1)} = \frac{5(x + 4)}{(x + 4)(x - 1)} \quad \rightarrow \quad 3x - 2(x - 1) = 5(x + 4)$

$3x - 2x + 2 = 5x + 20 \quad \rightarrow \quad x + 2 = 5x + 20 \quad \rightarrow \quad -4x = 18 \quad \rightarrow \quad x = \frac{18}{-4} = -\frac{9}{2}$

e. $\frac{1}{x - 1} + \frac{1}{x + 1} = \frac{-2x + 4}{x^2 - 1} \quad \rightarrow \quad \frac{x + 1}{(x - 1)(x + 1)} + \frac{x - 1}{(x + 1)(x - 1)} = \frac{-2x + 4}{(x + 1)(x - 1)} \quad \rightarrow \quad x + 1 + x - 1 = -2x + 4$

$2x = -2x + 4 \quad \rightarrow \quad 4x = 4 \quad \rightarrow \quad x = 1$

$x = 1$ makes the denominator 0, therefore $x = 1$ is an extraneous solution with no real solution.

f. $3x^2 - 2x - 5 = 0 \quad \rightarrow \quad 3x^2 - 5x + 3x - 5 = 0 \quad \rightarrow \quad x(3x - 5) + 1(3x - 5) = 0$

$(3x - 5)(x + 1) = 0 \quad \rightarrow \quad x = \frac{5}{3}, -1$

g. $5x^2 - 4x + 2 = 0 \quad \rightarrow \quad x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(5)(2)}}{2(5)} = \frac{4 \pm \sqrt{16 - 40}}{10} = \frac{4}{10} \pm \frac{\sqrt{-24}}{10} = \frac{2}{5} \pm \frac{2\sqrt{6}}{10}i = \frac{2}{5} \pm \frac{\sqrt{6}}{5}i$

h. $3(4x - 1) - 2x(x - 1) = 3x^2 - 3x + 3 \quad \rightarrow \quad 12x - 3 - 2x^2 + 2x = 3x^2 - 3x + 3$

$-2x^2 + 14x - 3 - 3x^2 + 3x - 3 = 0 \quad \rightarrow \quad -5x^2 + 17x - 6 = 0 \quad \rightarrow \quad 5x^2 - 17x + 6 = 0$

$5x^2 - 15x - 2x + 6 = 0 \quad \rightarrow \quad 5x(x - 3) - 2(x - 3) = 0 \quad \rightarrow \quad (x - 3)(5x - 2) = 0$

$x = 3, \frac{2}{5}$

i. $3x^2+4x-5=0 \quad\rightarrow\quad x=\frac{-4\pm\sqrt{(4)^2-4(3)(-5)}}{2(3)}=\frac{-4\pm\sqrt{16+60}}{6}=-\frac{4}{6}\pm\frac{\sqrt{76}}{6}=-\frac{2}{3}\pm\frac{2\sqrt{19}}{6}=-\frac{2}{3}\pm\frac{\sqrt{19}}{3}$

j. $x^2-2x=-5 \quad\rightarrow\quad x^2-2x+5=0$

$x=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(5)}}{2(1)}=\frac{2\pm\sqrt{4-20}}{2}=\frac{2}{2}\pm\frac{\sqrt{-16}}{2}=1\pm\frac{4i}{2}=1\pm 2i$

k. $3(x-1)-2x(x+1)=2x+5 \quad\rightarrow\quad 3x-3-2x^2-2x=2x+5 \quad\rightarrow\quad -2x^2+x-3=2x+5$

$2x^2+x+8=0 \quad\rightarrow\quad x=\frac{-1\pm\sqrt{(1)^2-4(2)(8)}}{2(2)}=\frac{-1\pm\sqrt{1-64}}{4}=-\frac{1}{4}\pm\frac{\sqrt{-63}}{4}=-\frac{1}{4}\pm\frac{3\sqrt{7}}{4}i$

l. $\frac{5}{x+1}-\frac{6}{x-3}-7=0 \quad\rightarrow\quad \frac{5(x-3)}{(x+1)(x-3)}-\frac{6(x+1)}{(x+1)(x-3)}-\frac{7(x+1)(x-3)}{(x+1)(x-3)}=0$

$5(x-3)-6(x+1)-7(x+1)(x-3)=0 \quad\rightarrow\quad 5x-15-6x-6-7(x^2-3x+x-3)=0$

$-x-21-7x^2+21x-7x+21=0 \quad\rightarrow\quad -7x^2+13x=0 \quad\rightarrow\quad x(-7x+13)=0$

$x=0\,,\frac{13}{7}$

m. $\sqrt[3]{2x-3}=3 \quad\rightarrow\quad \left(\sqrt[3]{2x-3}\right)^3=(3)^3 \quad\rightarrow\quad 2x-3=27 \quad\rightarrow\quad 2x=24 \quad\rightarrow\quad x=12$

n. $D=RT \quad\rightarrow\quad R=\frac{D}{T}$

o. $ax+by=c \quad\rightarrow\quad by=c-ax \quad\rightarrow\quad y=\frac{c-ax}{b}$

p. $T=\frac{5}{9}(F-32) \quad\rightarrow\quad \frac{9}{5}T=\frac{9}{5}\left(\frac{5}{9}(F-32)\right) \quad\rightarrow\quad \frac{9}{5}T=F-32 \quad\rightarrow\quad F=\frac{9}{5}T+32$

q. $(3-x)^{2/3}=6 \quad\rightarrow\quad \left((3-x)^{2/3}\right)^{3/2}=(6)^{3/2} \quad\rightarrow\quad 3-x=\sqrt{6^3} \quad\rightarrow\quad 3-x=6\sqrt{6}$

$-x=-3+6\sqrt{6} \quad\rightarrow\quad x=3-6\sqrt{6}$

r. $(3x-1)^{3/4}=-8 \quad\rightarrow\quad \sqrt[4]{(3x-1)^3}=-8$ There is no real solution for this equation since no 4th root will be -8. However there is an extraneous solution of $x=\frac{17}{3}$.

s. $\sqrt{2-x}=x+3 \quad\rightarrow\quad \left(\sqrt{2-x}\right)^2=(x+3)^2=(x+3)(x+3) \quad\rightarrow\quad 2-x=x^2+3x+3x+9$

$x^2+7x+7=0 \quad\rightarrow\quad x=\frac{-7\pm\sqrt{(7)^2-4(1)(7)}}{2(1)}=\frac{-7\pm\sqrt{49-28}}{2}=\frac{-7\pm\sqrt{21}}{2}$

However, since $\frac{-7-\sqrt{21}}{2}\approx -5.8$ and $\sqrt{2-x}=x+3=-5.8+3=-2.8$ cannot be negative, $x=\frac{-7-\sqrt{21}}{2}$ is an extraneous solution, and $x=\frac{-7+\sqrt{21}}{2}$ is the real solution.

t. $\sqrt{3x-5}+4=0 \quad\rightarrow\quad \sqrt{3x-5}=-4$ which has no real solution, yet there is an extraneous solution of $x=7$.

u. $|2x-3|=7 \quad\rightarrow\quad 2x-3=7 \quad\rightarrow\quad 2x=10 \quad\rightarrow\quad x=5$

$\rightarrow\quad 2x-3=-7 \quad\rightarrow\quad 2x=-4 \quad\rightarrow\quad x=-2$

v. $|x+5|=-2$ Absolute value of no expression is going to be negative, therefore there is no real solution.

10.

a. $2x-3\le 9 \quad\rightarrow\quad 2x\le 12 \quad\rightarrow\quad x\le 6$ So, in interval notation, the answer is $(-\infty\,,6]$.

b. $\frac{3}{4}-5x>4+2x \quad\rightarrow\quad \frac{3}{4}-4>2x+5x \quad\rightarrow\quad \frac{3}{4}-\frac{16}{4}>7x \quad\rightarrow\quad -\frac{13}{4}>7x \quad\rightarrow\quad -\frac{13}{28}>x$

so, $x<-\frac{13}{28}$ and in interval notation, the answer is $\left(-\infty\,,-\frac{13}{28}\right)$.

c. $-5\le 2x+3<15 \quad\rightarrow\quad -8\le 2x<12 \quad\rightarrow\quad -4\le x<6$ and in interval notation: $[-4\,,6)$

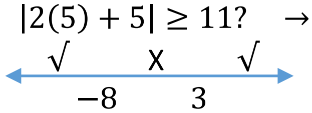
d. $|2x+5|\ge 11 \quad\rightarrow\quad 2x+5=11 \quad\rightarrow\quad 2x=6 \quad\rightarrow\quad x=3$

$\rightarrow\quad 2x+5=-11 \quad\rightarrow\quad 2x=-16 \quad\rightarrow\quad x=-8$

$|2(-10)+5|\ge 11? \quad\rightarrow\quad |-20+5|\ge 11? \quad\rightarrow\quad |-15|\ge 11? \quad\rightarrow\quad 15\ge 11?$ Yes

$|2(0)+5|\ge 11? \quad\rightarrow\quad |5|\ge 11? \quad\rightarrow\quad 5\ge 11?$ No

$|2(5)+5|\ge 11? \quad\rightarrow\quad |10+5|\ge 11? \quad\rightarrow\quad |15|\ge 11? \quad\rightarrow\quad 15\ge 11?$ Yes



So the answer in interval notation is: $(-\infty\,,-8]\cup[3\,,\infty)$

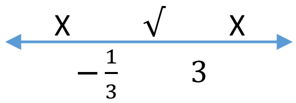
e. $|4-3x|<5 \quad \to \quad 4-3x=5 \quad \to \quad -3x=1 \quad \to \quad x=-\frac{1}{3}$

$\to \quad 4-3x=-5 \quad \to \quad -3x=-9 \quad \to \quad x=3$

$|4-3(-1)|<5? \quad \to \quad |4+3|<5? \quad \to \quad |7|<5? \quad \to \quad 7<5?$ No

$|4-3(0)|<5? \quad \to \quad |4|<5? \quad \to \quad 4<5?$ Yes

$|4-3(4)|<5? \quad \to \quad |4-12|<5? \quad \to \quad |-8|<5? \quad \to \quad 8<5?$ No



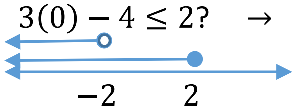
So the answer in internal notation is: $\left(-\frac{1}{3}, 3\right)$

f. $3-x>5 \quad \to \quad 3-x=5 \quad \to \quad -x=2 \quad \to \quad x=-2$

$3-(-3)>5? \quad \to \quad 3+3>5?$ Yes, $3-(0)>5? \quad \to \quad 3>5?$ No, so $x<-2$

$3x-4\leq 2 \quad \to \quad 3x-4=2 \quad \to \quad 3x=6 \quad \to \quad x=2$

$3(0)-4\leq 2? \quad \to \quad -4\leq 2?$ Yes, $\quad 3(3)-4\leq 2? \quad \to \quad 9-4<2?$ No, so $x\leq 2$



Since this is an inequality with “AND” we need the overlapping part. Thus the answer is: $(-\infty, -2)$

11.

a. $2t^3-16=2(t^3-8)=2(t^3-2^3)=2(t-2)(t^2+2t+2^2)=2(t-2)(t^2+2t+4)$

b. $p^3-27=p^3-3^3=(p-3)(p^2+3p+9)$

c. $a^3-27b^3=a^3-(3b)^3=(a-3b)(a^2+3ab+(3b)^2)=(a-3b)(a^2+3ab+9b^2)$

d. $8x^3+125y^3=(2x)^3+(5y)^3=(2x+5y)((2x)^2-(2x)(5y)+(5y)^2)$

$=(2x+5y)(4x^2-10xy+25y^2)$

e. $x^4-y^4=(x^2)^2-(y^2)^2=(x^2+y^2)(x^2-y^2)=(x^2+y^2)(x+y)(x-y)$

f. $x^2-25=x^2-5^2=(x+5)(x-5)$

g. x^2+y^2 this is not factorable (prime)

h. $10x^2-5x+6x-9=10x^2+1x-9=10x^2+10x-9x-9=10x(x+1)-9(x+1)$

$=(x+1)(10x-9)$

i. $2x^2-5x+3=2x^2-2x-3x+3=2x(x-1)-3(x-1)=(x-1)(2x-3)$

j. $x^2+2x-3=(x+3)(x-1)$

k. $8x^3-2x^2=2x^2(4x-1)$

12.

a. $\frac{2-4^2+25\div 5}{|3-5|}=\frac{2-16+5}{|-2|}=\frac{-14+5}{2}=-\frac{9}{2}$

b. $6+4\div 2\times(-3)-2=6+2\times(-3)-2=6+(-6)-2=-2$

c. $2\sqrt{8}+\sqrt[5]{3}+3\sqrt{125}+7\sqrt{2}+6\sqrt[5]{3}-4\sqrt{5}=2\sqrt{4}\sqrt{2}+\sqrt[5]{3}+3\sqrt{25}\sqrt{5}+7\sqrt{2}+6\sqrt[5]{3}-4\sqrt{5}$

$=2(2)\sqrt{2}+\sqrt[5]{3}+3(5)\sqrt{5}+7\sqrt{2}+6\sqrt[5]{3}-4\sqrt{5}$

$=4\sqrt{2}+\sqrt[5]{3}+15\sqrt{5}+7\sqrt{2}+6\sqrt[5]{3}-4\sqrt{5}$

$=11\sqrt{2}+7\sqrt[5]{3}+11\sqrt{5}$

d. $-3^2+4(2-3)=-9+4(-1)=-9-4=-13$

e. $\left(\frac{-2}{3}\right)^{-3}=\left(\frac{3}{-2}\right)^3=\frac{3^3}{(-2)^3}=\frac{27}{-8}=-\frac{27}{8}$

f. $(-2x^{-2}y^{-4})(3x^{-5}y^{-2})=(-2)(3)(x^{-2})(x^{-5})(y^{-4})(y^{-2})=-6x^{-7}y^{-6}=-\frac{6}{x^7y^6}$

g. $\left(\frac{-16a^{-4}b^3}{6a^5b^7}\right)^{-2}=\left(\frac{-8b^3}{3a^5b^7a^4}\right)^{-2}=\left(\frac{-8b^3}{3a^9b^7}\right)^{-2}=\left(\frac{-8}{3a^9b^4}\right)^{-2}=\left(\frac{3a^9b^4}{-8}\right)^2=\frac{9a^{18}b^8}{64}$

h. $\left(\frac{32a^{-4}b^3}{2b^7}\right)^{\frac{1}{2}}=\left(\frac{16a^{-4}}{b^4}\right)^{\frac{1}{2}}=\left(\frac{16}{a^4b^4}\right)^{\frac{1}{2}}=\sqrt{\frac{16}{a^4b^4}}=\frac{4}{a^2b^2}$

i. $\sqrt{\frac{18x^5y^2}{8x^2y^{-3}}}=\sqrt{\frac{9x^3y^2y^3}{4}}=\sqrt{\frac{9x^3y^5}{4}}=\frac{\sqrt{9x^3y^5}}{\sqrt{4}}=\frac{3xy^2\sqrt{xy}}{2}$

j. $\sqrt[3]{-48}=\sqrt[3]{-8}\sqrt[3]{6}=-2\sqrt[3]{6}$

k. $\sqrt{32}=\sqrt{16}\sqrt{2}=4\sqrt{2}$

l. $(3-4i)-(5-2i)=3-4i-5+2i=-2-2i$

m. $\frac{2\sqrt{3}}{\sqrt{5}} = \frac{2\sqrt{3}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{15}}{5}$

n. $\frac{2}{\sqrt[3]{9}} = \frac{2}{\sqrt[3]{9}} \times \frac{\sqrt[3]{3}}{\sqrt[3]{3}} = \frac{2\sqrt[3]{3}}{\sqrt[3]{27}} = \frac{2\sqrt[3]{3}}{3}$

o. $\sqrt{27x^6} - 5\sqrt[3]{x^7y^6} + 2x^2y^3\sqrt[3]{x} - 7x^3\sqrt{3x} = \sqrt{9x^6}\sqrt{3} - 5\sqrt[3]{x^6y^6}\sqrt[3]{x} + 2x^2y^3\sqrt[3]{x} - 7x^3\sqrt{3x}$

$= 3x^3\sqrt{3} - 5x^2y^2\sqrt[3]{x} + 2x^2y^3\sqrt[3]{x} - 7x^3\sqrt{3x}$

p. $\frac{4}{x+2} - \frac{x}{x^2-4} - \frac{2x}{x-2} = \frac{4(x-2)}{(x+2)(x-2)} - \frac{x}{(x+2)(x-2)} - \frac{2x(x+2)}{(x+2)(x-2)} = \frac{4x-8-x-2x^2-4x}{(x+2)(x-2)} = \frac{-2x^2-x-8}{(x+2)(x-2)}$

q. $\left(2\sqrt{3}-4\right)^2 = \left(2\sqrt{3}-4\right)\left(2\sqrt{3}-4\right) = 4\sqrt{9} - 8\sqrt{3} - 8\sqrt{3} + 16 = 4(3) - 16\sqrt{3} + 16$

$= 12 - 16\sqrt{3} + 16 = 28 - 16\sqrt{3}$

r. $\left(2\sqrt{3}-5\right)\left(\sqrt{2}+3\sqrt{3}\right) = 2\sqrt{4} + 6\sqrt{9} - 5\sqrt{2} - 15\sqrt{3} = 2(2) + 6(3) - 5\sqrt{2} - 15\sqrt{3}$

$= 4 + 18 - 5\sqrt{2} - 15\sqrt{3} = 22 - 5\sqrt{2} - 15\sqrt{3}$

s. $\frac{2+\sqrt{3}}{5\sqrt{2}-4} = \frac{(2+\sqrt{3})(5\sqrt{2}+4)}{(5\sqrt{2}-4)(5\sqrt{2}+4)} = \frac{10\sqrt{2}+8+5\sqrt{6}+4\sqrt{3}}{25\sqrt{4}+20\sqrt{2}-20\sqrt{2}-16} = \frac{10\sqrt{2}+8+5\sqrt{6}+4\sqrt{3}}{25(2)-16} = \frac{10\sqrt{2}+8+5\sqrt{6}+4\sqrt{3}}{50-16}$

$= \frac{10\sqrt{2}+8+5\sqrt{6}+4\sqrt{3}}{34}$

t. $\sqrt{3a}(t+p) + \sqrt{3a}(t+p) = 2\sqrt{3a}(t+p)$

u. $(2x-3)^2 = (2x-3)(2x-3) = 4x^2 - 6x - 6x + 9 = 4x^2 - 12x + 9$

v. $(2x^2-1)(3x^2+7x+2) = 6x^4 + 14x^3 + 4x^2 - 3x^2 - 7x - 2 = 6x^4 + 14x^3 + x^2 - 7x - 2$

w. $(2x^3 - 3x^2 + 5x - 1) - (3x^2 + 7x + 2) = 2x^3 - 3x^2 + 5x - 1 - 3x^2 - 7x - 2$

$= -x^3 - 10x^2 + 5x - 3$

x. $(4x^3 - 16x^2 + 21x - 12) \div (2x-3) = 2x^2 - 5x + 3 + \frac{-3}{2x-3}$

$$
\begin{array}{r|l}
 & \;\; 2x^2 - 5x + 3 \\ \hline
2x-3 & \;\; 4x^3 - 16x^2 + 21x - 12 \\
 & -(4x^3 - 6x^2) \\ \hline
 & \quad -10x^2 + 21x \\
 & \quad -(-10x^2 + 15x) \\ \hline
 & \qquad\qquad 6x - 12 \\
 & \qquad\qquad -(6x - 9) \\ \hline
 & \qquad\qquad\quad -3
\end{array}
$$

y. $\frac{9x^2+9x+2}{3x^2-7x-6} \times \frac{2x^2-7x+3}{6x^2-x-1} = \frac{9x^2+6x+3x+2}{3x^2-9x+2x-6} \times \frac{2x^2-6x-1x+3}{6x^2-3x+2x-1} = \frac{3x(3x+2)+1(3x+2)}{3x(x-3)+2(x-3)} \times \frac{2x(x-3)-1(x-3)}{3x(2x-1)+1(2x-1)} =$
$\frac{(x-3)(2x-1)}{(x-3)(3x+2)} \times \frac{(x-3)(2x-1)}{(2x-1)(3x+1)} = \frac{(x-3)(2x-1)}{(3x+2)(3x+1)}$

z. $\frac{9x^2+9x+2}{x^2-9} \div \frac{9x^2+3x-2}{3x^2+8x-3} = \frac{(x-3)(2x-1)}{(x+3)(x-3)} \div \frac{9x^2+6x-3x-2}{3x^2+9x-x-3} = \frac{(x-3)(2x-1)}{(x+3)(x-3)} \div \frac{3x(3x+2)-1(3x+2)}{3x(x+3)-1(x+3)}$

$= \frac{(x-3)(2x-1)}{(x+3)(x-3)} \div \frac{(3x+2)(3x-1)}{(x+3)(3x-1)} = \frac{(x-3)(2x-1)}{(x+3)(x-3)} \times \frac{(x+3)(3x-1)}{(3x+2)(3x-1)} = \frac{(2x-1)(3x-1)}{(3x+2)(3x-1)}$

aa. $\frac{4-\frac{1}{b}}{2-\frac{1}{b}} = \frac{\frac{4b}{b}-\frac{1}{b}}{\frac{2b}{b}-\frac{1}{b}} = \frac{\frac{4b-1}{b}}{\frac{2b-1}{b}} = \frac{4b-1}{b} \times \frac{b}{2b-1} = \frac{4b-1}{2b-1}$

bb. $\frac{\frac{4}{x+1}-1}{2-\frac{1}{x+1}} = \frac{\frac{4}{x+1}-\frac{1(x+1)}{x+1}}{\frac{2(x+1)}{x+1}-\frac{1}{x+1}} = \frac{\frac{4-1(x+1)}{x+1}}{\frac{2(x+1)-1}{x+1}} = \frac{\frac{4-x-1}{x+1}}{\frac{2x+2-1}{x+1}} = \frac{\frac{3-x}{x+1}}{\frac{2x+1}{x+1}} = \frac{3-x}{x+1} \times \frac{x+1}{2x+1} = \frac{3-x}{2x+1}$

13.

a. $\sqrt[2]{9x^4} = 3x^2$

b. $\left(\frac{x^{-2}}{y^{-3}}\right)^{-2} = \frac{x^4}{y^6}$

c. $3x - 4 = yx + 5$ and since $x = 1$, $3(1) - 4 = y(1) + 5 \;\rightarrow\; 3 - 4 = y + 5 \;\rightarrow\; -1 = y + 5$
$y = -6$

d. 6 is the x-intercept and the slope of this line is given by $\frac{3}{4}$

14.

a. $-3^{-2} = -\frac{1}{3^2} = -\frac{1}{9} \neq 9$

b. $x + x = 2x$ NOT x^2

d. $\frac{3x^2+5x}{3x+2} = \frac{x(3x+5)}{3x+2}$ which cannot be simplified. Terms cannot be cancelled and $3x$ is a term in $3x + 5$

e. There is no property of distribution of multiplication over multiplication

f. There is no property of distribution of exponents

g. It should be $x + 7 = 3$, or $x + 7 = -3$, which results to $x = -7 + 3 = -4$ and $x = -7 - 3 = -10$

15.

a. False, because $3\times(2\times5) = 3\times10 = 30$ but $(3\times2)\times(3\times5) = 6\times15 = 90$

b. False, because $(2 + 3)^2 = (5)^2 = 25$ but $(2)^2 + (3)^2 = 4 + 9 = 13$

c. False, because $\sqrt{(3)^2 + (4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$ but $3 + 4 = 7$

d. False, because $\frac{-2}{-3} = \frac{-1\times2}{-1\times3} = \frac{2}{3}$ which is not equal to $-\frac{2}{3}$

e. False, because $2\frac{3}{4} = 2 + \frac{3}{4} = \frac{8}{4} + \frac{3}{4} = \frac{11}{4}$ but $2\times\frac{3}{4} = \frac{6}{4} = \frac{3}{2}$

f. False, because $1 + 1 = 2$ but $(1)^2 = 1$

g. False, because $2 - 3 = -1$ but $3 - 2 = 1$

h. False, because only factors can be cancelled, not terms. $\frac{3}{5} + \frac{2}{3} = \frac{9}{15} + \frac{10}{15} = \frac{19}{15}$ which is greater than $\frac{2}{5}$, not equal to $\frac{2}{5}$

Word Problems:

1.

$$\frac{1}{time\ of\ older\ copy\ machine} + \frac{1}{time\ of\ newer\ copy\ machine} = \frac{1}{time\ of\ both\ copy\ machines}$$

$\frac{1}{2.5} + \frac{1}{t} = \frac{1}{1} \rightarrow \frac{t}{2.5t} + \frac{2.5}{2.5t} = \frac{2.5t}{2.5t} \rightarrow t + 2.5 = 2.5t \rightarrow 2.5 = 1.5t \rightarrow t = \frac{2.5}{1.5} = 1.\overline{6}$ hours

2. $old\ price - 2\ cents = new\ price \rightarrow old\ price = new\ price + 2\ cents = 62 + 2 = 64\ cents$

3. $Adam's\ original\ salary + 5\%\ of\ Adam's\ original\ salary = Adam's\ new\ salary$

   $A + 0.05A = 58000 \rightarrow 1.05A = 58000 \rightarrow A = \frac{58000}{1.05} \approx \$55.238.10$

4. $20\%\ of\ (85 + 90 + 60) + 40\%\ of\ (Final\ Exam) \geq 80 \rightarrow 0.2(85 + 90 + 60) + 0.4E \geq 80$

   $0.2(235) + 0.4E \geq 80 \rightarrow 47 + 0.4E \geq 80 \rightarrow 0.4E \geq 80 - 47 \rightarrow 0.4E \geq 33 \rightarrow E \geq 82.5$

5. Going Against the Wind: $Speed\ of\ Bicyclist - Speed\ of\ Wind = \frac{45}{3} = 15$

   Going with the Wind: $Speed\ of\ Bicyclist + Speed\ of\ Wind = \frac{57}{3} = 19$

   Adding the two equations: $2B = 34 \rightarrow B = 17$ mph speed of the bicyclist

   And so, $17 + W = 19 \rightarrow W = 19 - 17 = 2$ mph speed of wind

6. $2L + 2W = 45$ and $LW = 102$

   $\begin{cases} 2L + 2W = 45 \\ LW = 102 \end{cases} \rightarrow 2W = 45 - 2L \rightarrow w = \frac{45 - 2L}{2} \rightarrow L\left(\frac{(45 - 2L)}{2}\right) = 102$

   $\frac{L(45 - 2L)}{2} = 102 \rightarrow L(45 - 2L) = 204 \rightarrow 45L - 2L^2 = 204 \rightarrow 2L^2 - 45L + 204 = 0$

   $L = \frac{-(-45) \pm \sqrt{(-45)^2 - 4(2)(204)}}{2(2)} = \frac{45 \pm \sqrt{2025 - 1632}}{4} = \frac{45 \pm \sqrt{393}}{4} = \frac{45 \pm 19.82423}{4}$

   $L = 16.2$ inches or $L = 6.3$ inches

   If $L = 16.2$, then $W = \frac{45-2(16.2)}{2} = \frac{45-32.4}{2} = 6.3$ inches, and if $L = 6.3$ inches, $W = 16.2$ inches.

   Thus, the length of the rectangle is 16.2 inches and the width of the rectangle is 6.3 inches.

7.

10 lb or 20%	10 lb or 20%	10 lb or 20%	10 lb or 20%	10 lb or 20%

So, we only received 60% of the order.

8.

Type	Rate	Amount	Rate × Amount
30% Antifreeze	$30\% = 0.3$	10	3
Drain 30% Antifreeze	$30\% = 0.3$	$-x$	$-0.3x$
Pure Antifreeze	$100\% = 1$	$+x$	$+x$
Total: 44% Antifreeze	0.44	10	4.4

$3 - 0.3x + x = 4.4 \quad \rightarrow \quad 3 + 0.7x = 4.4 \quad \rightarrow \quad 0.7x = 1.4 \quad \rightarrow \quad x = 2$ liters

9.

a. $800 million
b. $25
c. $10 or $40
d. $10 to $40