# Exam 1 Review                    Name: ______________________

You are only eligible to take the exam 1 if you have completed Exam 1 Review by the due date. No late work is accepted except under extra ordinary circumstances.

Taking an exam requires synthesis that gives you an overview of the basic principles and techniques you have learned so far, and create a mind-map of how the content is related to each other. Some other things you require are to pull together a mathematical tool box that contains formulas, first steps of different problems, strategies to use to create graphs, functions, and attack problems you are stuck on. This requires careful planning and that is why it is essential to finish this review sheet so you feel prepared.

1. Create a library of functions you have studied so far along with their inverses if they have them. Find the domain and range for the functions and their inverses you listed along with a rough sketch of their graphs.
2. List the basic principles of finding domain, range and evaluating functions that arise from arithmetic of two different functions.
3. What is a composite function? How do you evaluate its domain and range? What is the relationship of composite of two functions that are inverses of each other?
4. Make a list of general principles of creating a function given criteria like in quiz 1 and its review, or homework problems done in class.
5. Make a list of all the properties you have learned so far of the logarithmic functions.
6. How would you identify whether a sequence is arithmetic or geometric? Write a few words or a make a summary statement to help you remember how to find $n^{th}$ terms of arithmetic and geometric sequences.
7. How do you determine if a function is even or odd?
8. Make a generic list of formulas to solve exponential and logarithmic applied problems. It must include compound interest, continuous compound interest, exponential growth and decay, appreciation and depreciation formulas and how they connect to compound and continuous compound interest formulas.

## Review Problems

1. Create a function $y = f(x)$ that satisfies the following criteria.
   a. Is One-to-one
   b. Has domain $(-4,-1] \cup [0,5]$
   c. $f(2) = 9$
   d. For at least one $x$ in the domain of $f$, $f(x) = -2$

Questions about your function-

   i) Do you think the function you came up with is unique? Explain why.
   ii) Find the range of your function.
   iii) What is $f^{-1}(x)$?

2. Create an exponential function so that $f(0) = 200$ and then answer the following questions
   a. What is the inverse of this function?
   b. Find the domain and range of the function and its inverse?
3. Sketch the graphs of the functions below.

a. $f(x) = 2^x$

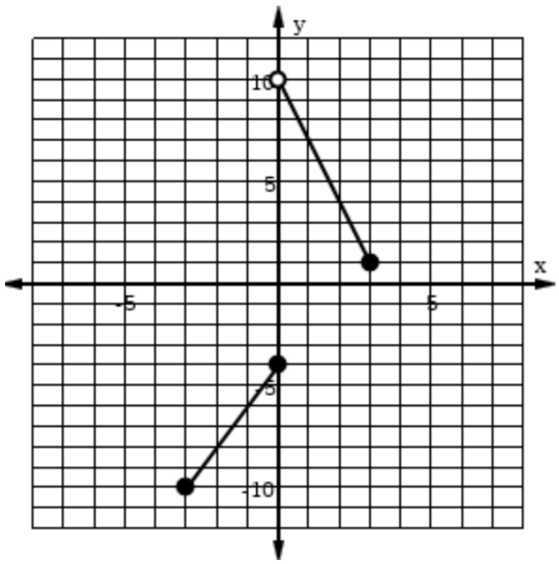
b. $g(x) = log_2 x$

   c. Use the functions in parts a,b to explain the relationship between these two functions.

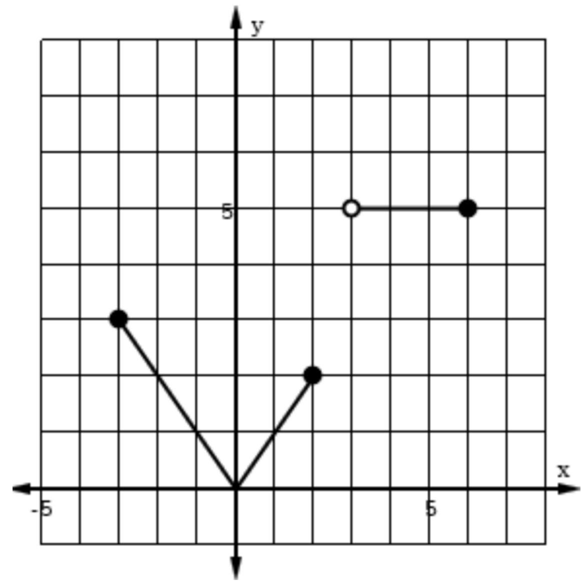
4. What is the relationship of $f(x) = e^x$ and $g(x) = lnx$?

5. The function $f$ is defined as follows: $f(x) = \frac{4x+1}{3x-15}$.
   a. Find $f(4)$. Simplify your answer as much as possible.
   b. Find $f^{-1}(x)$.
   c. Domain and Range of $f(x)$ and $f^{-1}(x)$.
6. Obtain the piece-wise formula for the functions whose graphs are given below. If the functions below are one-to-one, please also find the formulas for their inverse functions and sketch their graphs.

a.




b.




7. A species of bacteria doubles every 20 minutes at room temperature.
   a. Write a function to represent the amount of these bacteria $A(t)$ at room temperature after $t$ hours, if you initially started with 2,000,000 bacteria.
   b. Find the number of bacteria after 2 hours.
   c. Find the number of bacteria at 4 hours.

   Round your answer to the nearest whole number as necessary.

8. Suppose Abby places $\$12,000$ in an account that pays $3.5\%$ interest. Assume that no withdrawals are made from the account. Do not round your answers.
   a. Find the amount in the account at the end of 5 years if the interest was compounded quarterly.
   b. Find the amount in the account at the end of 5 years if the interest was compounded continuously.
   c. Find the amount in the account at the end of 15 years if the interest was compounded quarterly.
   d. Find the amount in the account at the end of 15 years if the interest was compounded continuously.
   e. Which would you prefer compounded quarterly or compounded yearly?

9. Find the difference quotient $\frac{f(x+h)-f(x)}{h}$ where $h \neq 0$ for the function below. Explain what this quotient represents. Simplify your answer as much as possible.

$$f(x) = 5x^2 - 6$$

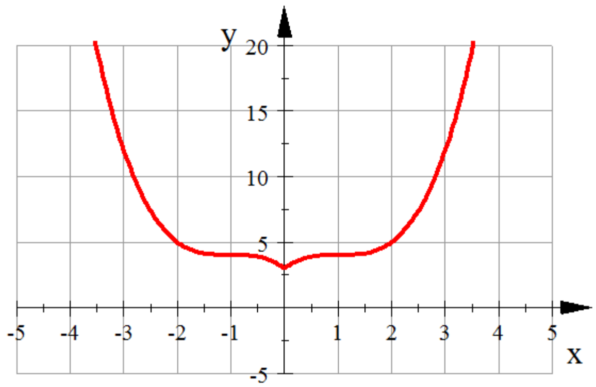
10. Determine if the function below is even, odd or neither.

$$f(x) = 5x^2 + 3$$

11. Determine if the graph of the equation $x^2 + y^2 = 4$ below is symmetric with respect to $x$-axis, $y$-axis, the origin, or neither.

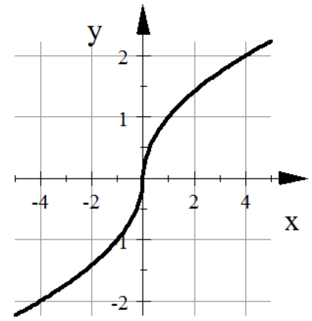
12. For the functions below check all boxes that apply.

a)




| ☐ One-to-one | | |
|---|---|---|
| ☐ E ven | ☐ O dd | ☐ Neit her |

b)




| ☐ One-to-one | | |
|---|---|---|
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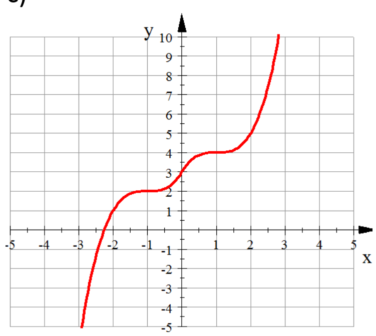
c)




| ☐ One-to-one | | |
|---|---|---|
| ☐ E ven | ☐ O dd | ☐ Neit her |

13. Rewrite the exponential equations in logarithmic form and logarithmic equations in exponential form. If possible simplify your answers.

| Exponential Equation | Logarithmic Equation |
|---|---|
| $e^x = 5$ | |
| $2^{x+1} = 8$ | |
| | $log_2(x) = -1$ |
| | $\log(x+1) = 2$ |
| | $\ln(x+1) = 3$ |
| $5^{1-x} = 3$ | |
| | $log_{\frac{1}{2}}(x) = -3$ |

14.

Fill in the missing values to make the statement a true statement.

$log_5 8 - log_5(\_\_\_\_\_\_) = log_5 4$

$log_2 3 + log_2 5 = log_2(\_\_\_\_\_\_)$

$3log_7 2 = log_7(\_\_\_\_\_)$

$log_5 49 = (\_\_\_\_\_\_)log_5 7$

$\frac{ln5}{ln4} = log_4(\_\_\_\_\_\_)$

Expand the following. Each logarithm in your answer should involve only one variable. Assume that all variables are positive.

$\log(x^3 y^2) =$____________________

$\log_2\left(\frac{x^3 y^2}{\sqrt{z}}\right) =$____________________

$\log\left(\frac{x^3}{\sqrt{z^5 y}}\right) =$____________________

$\ln((4+x)(x-2)) =$____________________

$\ln\left(\frac{x^5 \sqrt[3]{y}}{3z}\right) =$ _______________

Write the following as one term.

$4log_2 x + 2log_2 y =$ ___________

$\frac{1}{3}logx - 2logy + 3logz =$ ____________

Compute the values below exactly

$log_2(8) =$ ___________

$\log(0.000001) =$ ___________

$\ln(e^5) =$ __________

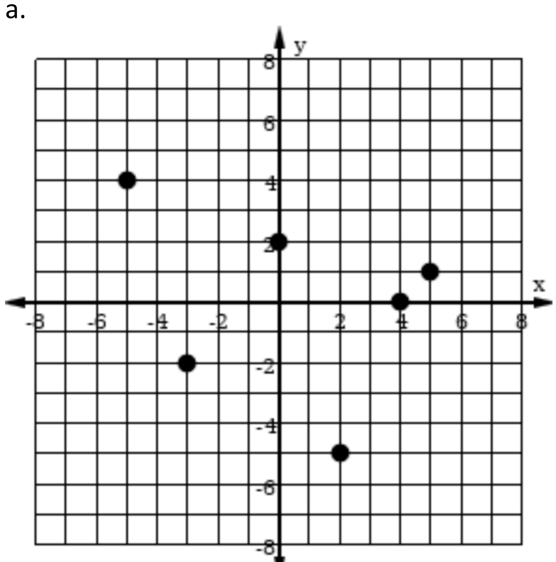
$\ln(\sqrt{e}) =$ ________

$log_5\left(\frac{1}{25}\right) =$ _________

$log_{\frac{1}{3}}(9)$

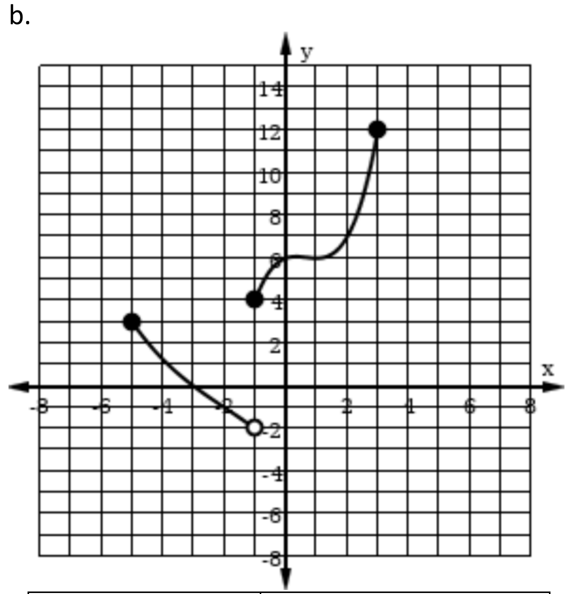
15. Evaluate the following for the given one-to-one functions below.

a.




| | |
|---|---|
| $f(-5)$ | $f^{-1}(-5)$ |
| $f(4)$ | $f^{-1}(4)$ |

b.




| | |
|---|---|
| $f(2)$ | $f^{-1}(0)$ |
| $f(-5)$ | $f^{-1}(-2)$ |

16. Find the symmetries of the functions or relations below and their $x$-intercepts and $y$-intercepts.

A. $4x^2 + 9y^2 = 36$

Symmetries

☐ $x$-axis

☐ $y$-axis

☐ Origin

☐ Neither

Intercepts

$x$-intercept _______________

$y$-intercept _________________

B. $xy^3 = 3$

Symmetries

☐ $x$-axis

☐ $y$-axis

☐ Origin

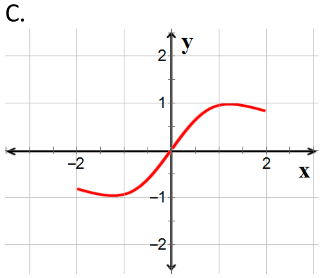
☐ Neither

Intercepts

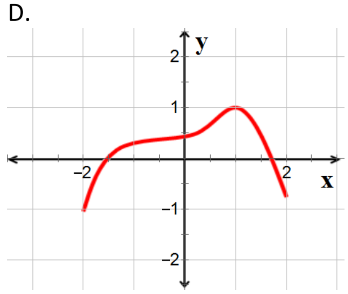
$x$-intercept _______________

$y$-intercept _________________

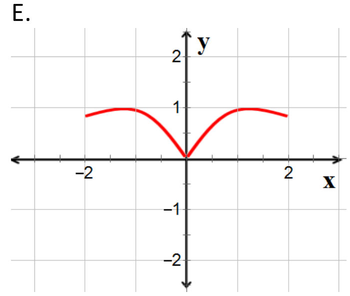
C.




D.




E.




17. Find the first 4 terms of the sequences given below.

| $a_n = n^{th}$ term of the sequence | First term | Second term | Third term | Fourth term | |
|---|---|---|---|---|---|
| $a_n = 5\left(\frac{1}{2}\right)^{2n+1}, n = 0,1,2,\ldots$ | | | | | ☐ Arithmetic<br>☐ Geometric<br>☐ Neither |
| $a_n = 4n + 3, n = 4,5,\ldots$ | | | | | ☐ Arithmetic<br>☐ Geometric<br>☐ Neither |

18. For the sequences below determine if they are arithmetic or geometric. Then find a formula for the $a_n$ and evaluate: $a_5 =$ , and $a_{10} =$ .

| Sequence | Type | $f(n) = a_n = n^{th}$ term | Evaluate these elements. |
|---|---|---|---|
| $\{13,17,21,25, \ldots.\}$ | ☐ Arithmetic<br>☐ Geometric<br>☐ Neither | | $a_5 =$<br><br>$a_{10} =$ |
| $\{7,21,63,189,\ldots\}$ | ☐ Arithmetic<br>☐ Geometric<br>☐ Neither | | $a_5 =$<br><br>$a_{10} =$ |
| $\{1,-1,1,-1, \ldots.\}$ | ☐ Arithmetic<br>☐ Geometric<br>☐ Neither | | $a_5 =$<br><br>$a_{10} =$ |

19. For a given geometric sequence, the $7^{th}$ term, $a_7 = 15$ and the $9^{th}$ term, $a_9 = 135$. Find the $15^{th}$ term $a_{15}$. (Recall that geometric sequences are exponential functions of $(n)$ and the formula for $a_n$ can be written in the form: $a_n = a_0 \cdot r^n$.)

20. Evaluate the following given that $f(x) = \sqrt{x-1}$ and $g(x) = \sqrt{4-x}$

a. $(f+g)(x)$

b. Domain of $f+g$

c. $(f-g)(3)$

d. $\left(\frac{f}{g}\right)(1)$

e. $(f \circ g)(x)$

f. Domain of $f \circ g$

g. $(g \circ f)(0)$