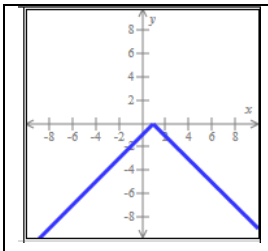
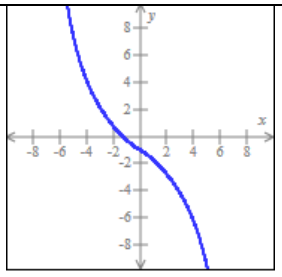
Math 110 Exam 1 Review Answers

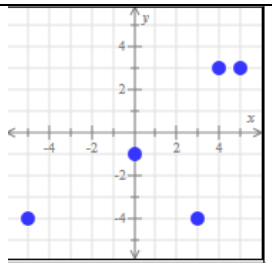
1. Relation 1: It is a function and a one-to-one function. Domain is $[-1\,,\infty)$ and range is $[0\,,\infty)$.
   Relation 2: It is not a function. Domain is $\{-4\,,0\,,1\,,2\}$ and range is $\{-2\,,-1\,,2\,,7\}$.
2. a. $f(4)=-\frac{17}{3}$
   Domain of $f(x)$: $(-\infty\,,5)\cup(5\,,\infty)$
   Range of $f(x)$: $\left(-\infty\,,\frac{4}{3}\right)\cup\left(\frac{4}{3}\,,\infty\right)$
   b. $f^{-1}(x)=\frac{15x+1}{3x-4}$
   Domain of $f^{-1}(x)$: $\left(-\infty\,,\frac{4}{3}\right)\cup\left(\frac{4}{3}\,,\infty\right)$
   Range of $f^{-1}(x)$: $(-\infty\,,5)\cup(5\,,\infty)$
3. a. $f(-3)=-\frac{1}{4}$
   b. $f(5)=2$
4.

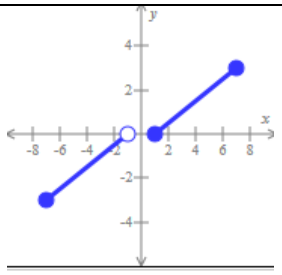
$f(x)=\frac{1}{x-4}$ Domain: $(-\infty\,,4)\cup(4\,,\infty)$ Range: $(-\infty\,,0)\cup(0\,,\infty)$	$g(x)=\sqrt{x-4}$ Domain: $[4\,,\infty)$ Range: $[0\,,\infty)$	$h(x)=\sqrt{4-x}$ Domain: $(-\infty\,,4]$ Range: $[0\,,\infty)$
$r(x)=\sqrt{x-4}+2$ Domain: $[4\,,\infty)$ Range: $[2\,,\infty)$	Domain of T: $\{-3\,,-2\,,0\,,2\}$ Range of T: $\{-3\,,-2\,,2\}$	Domain of h: $[-2\,,3)$ Range of h: $[-4\,,4)$

5. $A(t)=30{,}000(2)^{2t}$
   a. $A(2)=480{,}000$
   b. $A(4)=7{,}680{,}000$
6. a. \$2100
   b. \$2205
7. $\frac{f(x+h)-f(x)}{h}=10x+5h$
8. Even
9. It is symmetric with respect to x-axis, y-axis, and origin.
10. State which of the following functions are one-to-one.


Not One-to-one


One-to-one


Not One-to-one


One-to-one

11. Domain of f: $\{0\,,6\,,7\,,9\}$
    Domain of g: $\{3\,,4\,,5\,,7\,,8\,,9\}$
    Domain of $f\circ g$: $\{3\,,4\,,5\}$
    Domain of $g\circ f$: $\{0\,,6\,,7\,,9\}$
    a. $(f\circ g)(3)=8$
    c. $(f\circ g)(7)=$ undefined
    Range of f: $\{7\,,8\}$
    Range of g: $\{1\,,5\,,6\,,7\,,9\}$
    Range of $f\circ g$: $\{8\}$
    Range of $g\circ f$: $\{1\,,5\}$
    b. $(g\circ f)(9)=1$
    d. $(g\circ f)(0)=5$
12. a. $(g\circ f)(x)=\frac{2x-1}{2x-12}$
    b. $(f\circ g)(6)=17$
    c. Domain of $(g\circ f)(x)$: $(-\infty\,,6)\cup(6\,,\infty)$

13. $f^{-1}(x) = \frac{x+1}{2x-7}$

Domain of f: $\left(-\infty, \frac{7}{2}\right) \cup \left(\frac{7}{2}, \infty\right)$

Range of f: $\left(-\infty, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$

Domain of f^{-1}: $\left(-\infty, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$

Range of f^{-1}: $\left(-\infty, \frac{7}{2}\right) \cup \left(\frac{7}{2}, \infty\right)$

$g^{-1}(x) = \frac{x^2+1}{2}$ for $x \geq \frac{1}{2}$

Domain of g: $\left[\frac{1}{2}, \infty\right)$

Range of g: $[0, \infty)$

Domain of g^{-1}: $[0, \infty)$

Range of g^{-1}: $\left[\frac{1}{2}, \infty\right)$

$h^{-1}(x) = \log_2 x$

Domain of h: $(-\infty, \infty)$

Range of h: $(0, \infty)$

Domain of h^{-1}: $(0, \infty)$

Range of h^{-1}: $(-\infty, \infty)$

$p^{-1}(x) = e^x$

Domain of p: $(0, \infty)$

Range of p: $(-\infty, \infty)$

Domain of p^{-1}: $(-\infty, \infty)$

Range of p^{-1}: $(0, \infty)$

14. $e^x = 5 \quad \rightarrow \quad x = \ln 5$

$2^{x+1} = 8 \quad \rightarrow \quad \log_2 8 = x + 1$

$\log_2(x) = -1 \quad \rightarrow \quad x = 2^{-1}$

$\log(x+1) = 2 \quad \rightarrow \quad x + 1 = 10^2$

$\ln(x+1) = 3 \quad \rightarrow \quad x + 1 = e^3$

$5^{1-x} = 3 \quad \rightarrow \quad 1 - x = \log_5 3$

$\log_{\frac{1}{2}}(x) = -3 \quad \rightarrow \quad x = \left(\frac{1}{2}\right)^{-3}$

15.

A.	B.
$\log_5 8 - \log_5(2) = \log_5 4$ $\log_2 3 + \log_2 5 = \log_2(15)$ $3\log_7 2 = \log_7(8)$ $\log_5 49 = (2)\log_5 7$ $\frac{\ln 5}{\ln 4} = \log_4(5)$	$\log(x^3y^2) = 3\log x + 2\log y$ $\log_2\left(\frac{x^3y^2}{\sqrt{z}}\right) = 3\log_2 x + 2\log_2 y - \frac{1}{2}\log_2 z$ $\log\left(\frac{x^3}{\sqrt{z^5 y}}\right) = 3\log x - \frac{5}{2}\log z - \frac{1}{2}\log y$ $\ln\big((4+x)(x-2)\big) = \ln(4+x) + \ln(x-2)$ $\ln\left(\frac{x^5\sqrt[3]{y}}{3z}\right) = 5\ln x + \frac{1}{3}\ln y - \ln 3 - \ln z$
C.	D.
$4\log_2 x + 2\log_2 y = \log_2(x^4y^2)$ $\frac{1}{3}\log x - 2\log y + 3\log z = \log\left(\frac{z^3\sqrt[3]{x}}{y^2}\right)$	$\log_2 8 = 3$ $\log(0.000001) = -6$ $\ln(e^5) = 5$ $\ln\left(\sqrt{e}\right) = \frac{1}{2}$ $\log_5\left(\frac{1}{25}\right) = -2$

16. $(f+g)(x) = x^2 + 3x + 1$

Domain of $(f+g)$: $(-\infty, \infty)$

$(f+g)(3) = 19$

$(fg)(x) = 3x^3 - x^2 + 6x - 2$

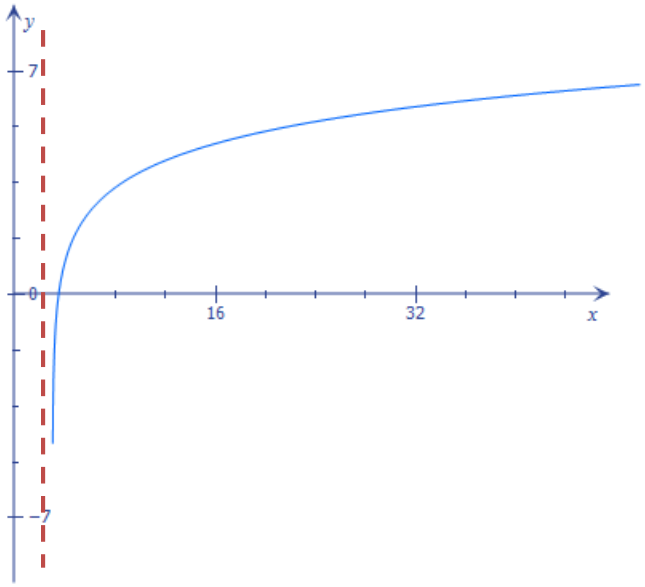
Domain of (fg): $(-\infty, \infty)$

$(fg)(0) = -2$

17. For the rest of the answers, refer to the video log 2.1

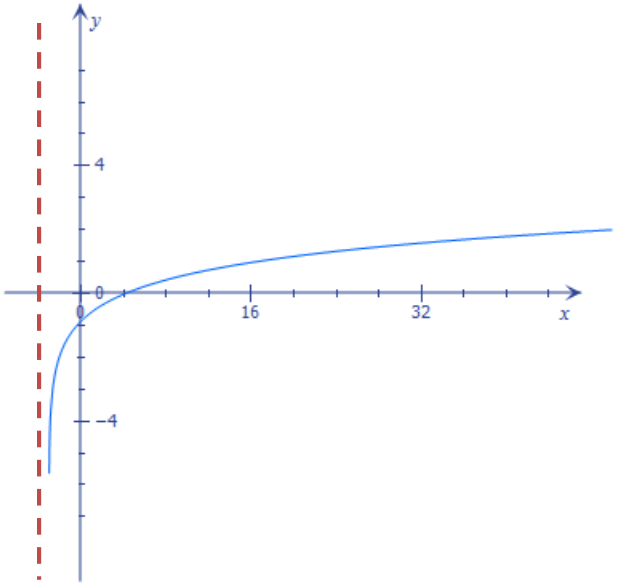
$y = \log_2(x - 3) + 1$

The graph of $y = \log_2 x$ is shifted 3 units to the right and 1 unit up. The vertical asymptote is now at $x = 3$



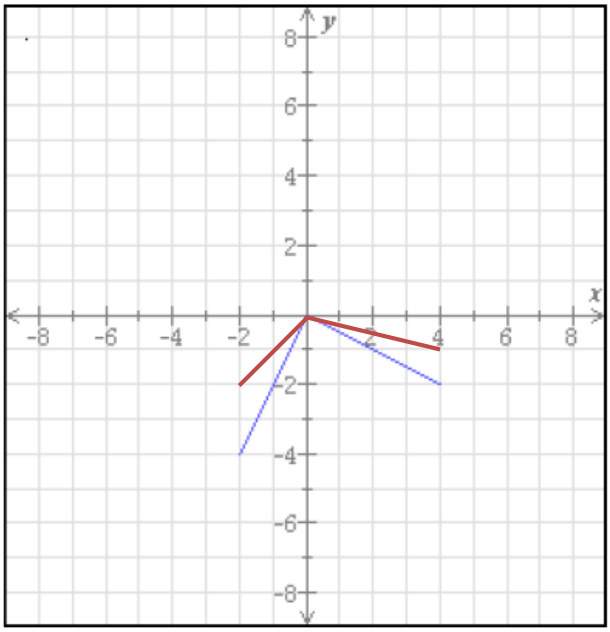
$y = \ln(x + 3) - 2$

The graph of $y = \ln x$ is shifted 3 units to the left and 2 units down. The vertical asymptote is at $x = -3$.



18. $4x^2 + 9y^2 = 36$ is symmetric with respect to x-axis, y-axis, and origin.
x-intercept: $(-3\ ,0)$, $(3\ ,0)$ and y-intercept: $(0\ ,-2)$, $(0\ ,2)$
$xy^3 = 3$ is symmetric with respect to origin. There are no x-intercepts or y-intercepts.

19.



20.

Sequence	Type	$f(n) = a_n = n^{th}$ term	a_n
$\{13,17,21,25,\ldots.\}$	Arithmetic	$a_n = 13 + 4(n-1)$	$a_{20} = 89$
$\{7,21,63,189,\ldots\}$	Geometric	$a_n = 7(3)^{n-1}$	$a_{12} = 1{,}240{,}029$