Exam 1 Review   Name: ______________________________

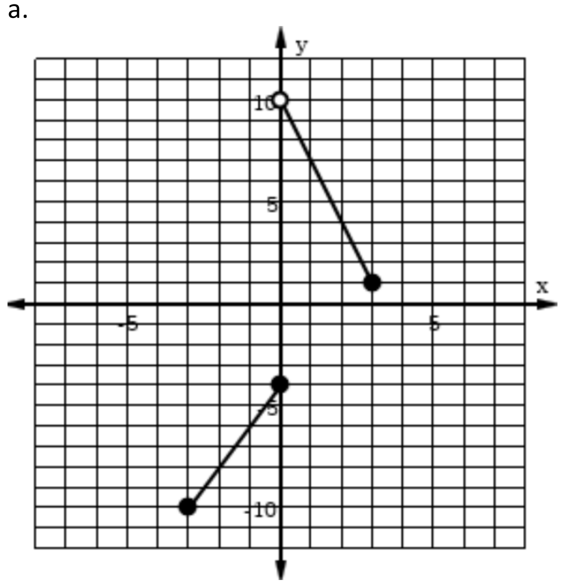
Due Tuesday, October 3, 2017

1. Create a function $y = f(x)$ that satisfies the following criteria.
   a. Is One-to-one
   b. Has domain $(-4,-1] \cup [0,5]$
   c. $f(2) = 9$
   d. For at least one x in the domain of f, $f(x) = -2$

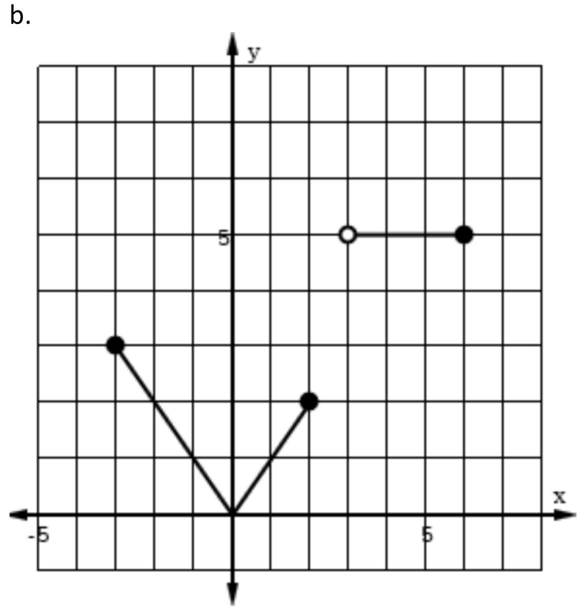
   Questions about your function-

   i) Do you think the function you came up with is unique? Explain why.
   ii) Find the range of your function.
   iii) What is $f^{-1}(x)$?
2. Create an exponential function so that $f(0) = 200$ and then answer the following questions
   a. What is the inverse of this function?
   b. Find the domain and range of the function and its inverse?
3. Sketch the graphs of the functions below.
   a. $f(x) = 2^x$
   b. $g(x) = log_2 x$
   c. Use the functions in parts a,b to explain the relationship between these two functions.
   4. What is the relationship of $f(x) = e^x$ and $g(x) = lnx$?
5. Find the inverse function of $f(x) = \frac{3x-1}{5+x}$. Then find the domain and range of both the function and its inverse.
6. The function f is defined as follows: $f(x) = \frac{4x+!}{3x-15}$.
   a. Find $f(4)$. Simplify your answer as much as possible.
   b. Find $f^{-1}(x)$.
   c. Domain and Range of $f(x)$ and $f^{-1}(x)$.
7. Obtain the piece-wise formula for the functions whose graphs are given below. If the functions below are one-to-one, please also find the formulas for their inverse functions and sketch their graphs.

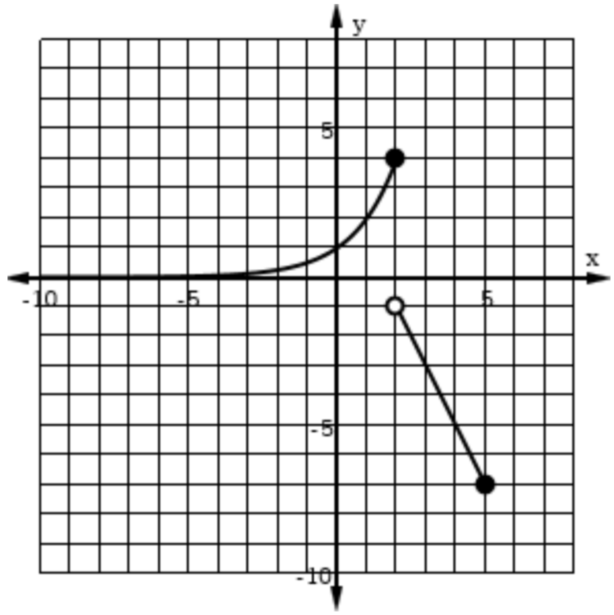
a.



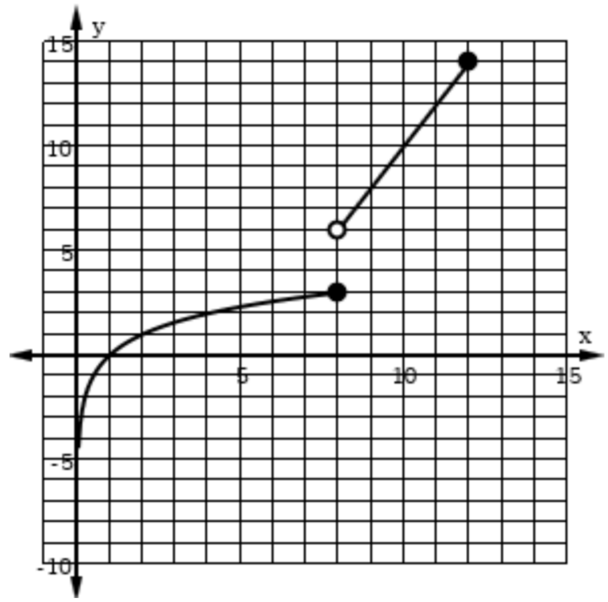
b.



c.



d.



8. A species of bacteria doubles every 30 minutes at room temperature. Write a function to represent the amount of these bacteria $A(t)$ at room temperature after t hours, if you initially started with 30000 bacteria.

a. Find the number of bacteria after 2 hours.

b. Find the number of bacteria at 4 hours.

Round your answer to the nearest whole number as necessary.

9. Suppose Rahul places \$2000 in an account that pays 5% interest compounded each year. Assume that no withdrawals are made from the account. Do not round your answers.
   a. Find the amount in the account at the end of 1 year.
   b. Find the amount in the account at the end of 2 years.

10. Find the difference quotient $\frac{f(x+h)-f(x)}{h}$ where $h \neq 0$ for the function below. Explain what this quotient represents. Simplify your answer as much as possible.

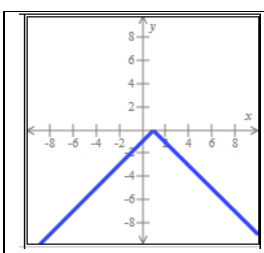
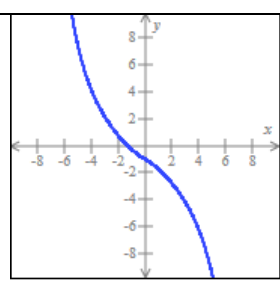
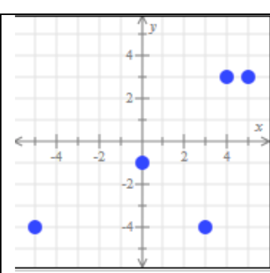
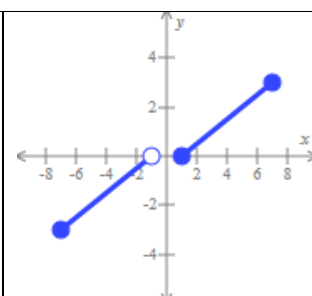
$$f(x) = 5x^2 - 6$$

11. Determine if the function below is even, odd or neither.

$$f(x) = 5x^2 + 3$$

12. Determine if the graph of the equation $x^2 + y^2 = 4$ below is symmetric with respect to x-axis, y-axis, the origin, or neither.

13. State which of the following functions are one-to-one.

 YES NO	 YES NO	 YES NO	 YES NO

14. Rewrite the exponential equations in logarithmic form and logarithmic equations in exponential form. If possible simplify your answers.

Exponential Equation	Logarithmic Equation
$e^x = 5$	
$2^{x+1} = 8$	
	$log_2(x) = -1$
	$\log(x+1) = 2$
	$\ln(x+1) = 3$
$5^{1-x} = 3$	
	$log_{\frac{1}{2}}(x) = -3$

15.

A. Fill in the missing values to make the statement a true statement.

i. $log_5 8 - log_5(\_\_\_\_\_\_) = log_5 4$

ii. $log_2 3 + log_2 5 = log_2(\_\_\_\_\_\_)$

iii. $3log_7 2 = log_7(\_\_\_\_\_)$

iv. $log_5 49 = (\_\_\_\_\_\_)log_5 7$

v. $\frac{ln5}{ln4} = log_4(\_\_\_\_\_\_)$

B. Expand the following. Each logarithm in your answer should involve only one variable. Assume that all variables are positive.

I. $\log(x^3y^2) =$______________________

II. $\log_2\left(\frac{x^3y^2}{\sqrt{z}}\right) =$______________________

III. $\log\left(\frac{x^3}{\sqrt{z^5y}}\right) =$______________________

IV. $\ln((4+x)(x-2)) =$______________________

V. $\ln\left(\frac{x^5\sqrt[3]{y}}{3z}\right) =$ ________________

C. Write the following as one term.

i. $4log_2 x + 2log_2 y =$ ___________

ii. $\frac{1}{3}logx - 2logy + 3logz =$ ____________

D. Compute the values below exactly

i. $log_2(8) =$ ___________

ii. $\log(0.000001) =$ ___________

iii. $\ln(e^5) =$ __________

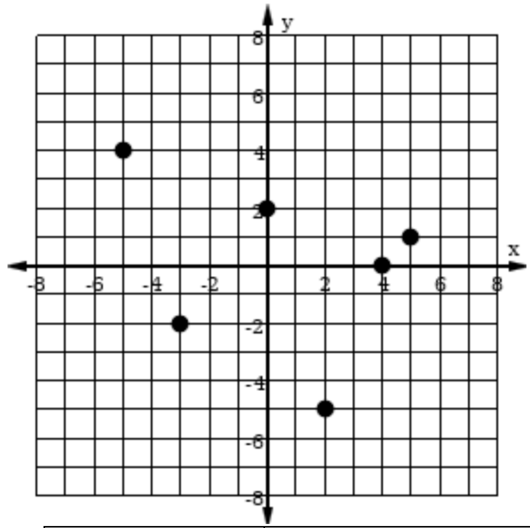
iv. $\ln(\sqrt{e}) =$ ________

v. $log_5\left(\frac{1}{25}\right) =$ _________

vi. $log_{\frac{1}{3}}(9)$

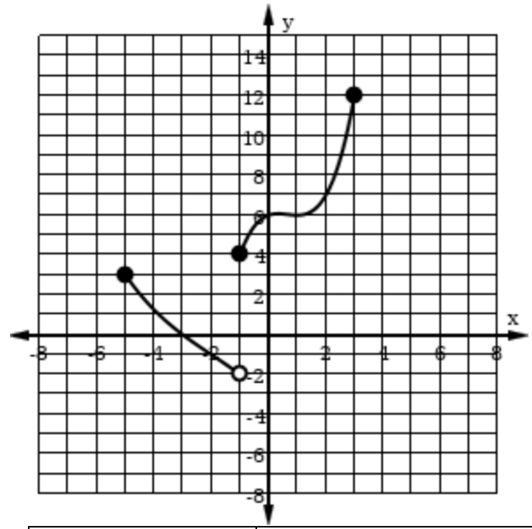
16. Evaluate the following for the given one-to-one functions below.

a.



$f(-5)$	$f^{-1}(-5)$
$f(4)$	$f^{-1}(4)$

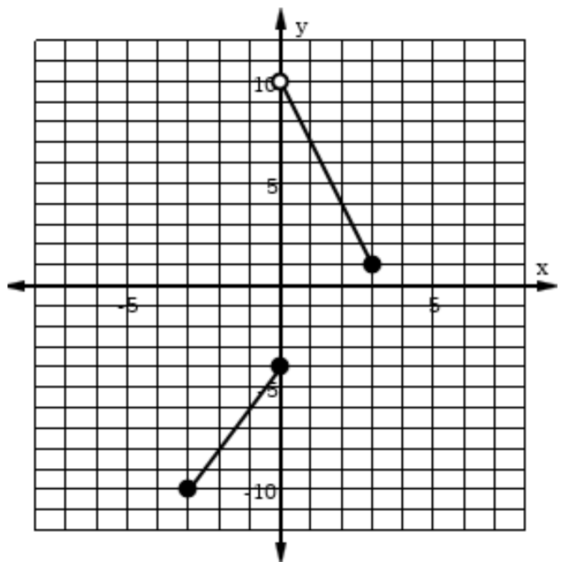
b.



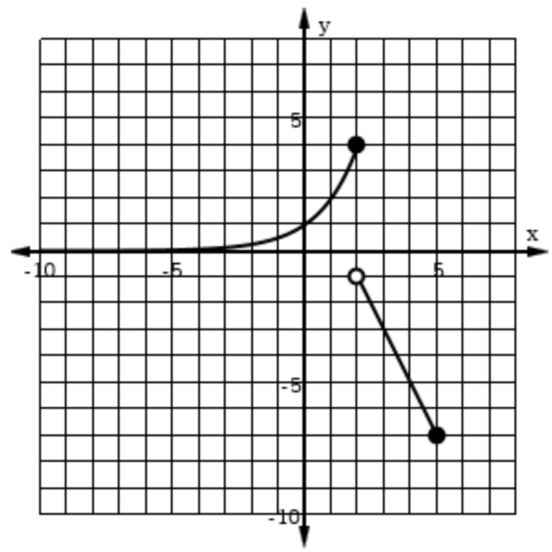
$f(2)$	$f^{-1}(0)$
$f(-5)$	$f^{-1}(-2)$

17. Obtain the piece-wise formula for the functions whose graphs are given below. If the functions below are one-to-one, please also find the formulas for their inverse functions and sketch their graphs.

a.



b.



18. Find the symmetries of the functions or relations below and their x-intercepts and y-intercepts.

A. $4x^2 + 9y^2 = 36$

Symmetries

- ☐ x-axis
- ☐ y-axis
- ☐ Origin
- ☐ Neither

Intercepts

x-intercept ________________

y-intercept __________________

B. $xy^3 = 3$

Symmetries

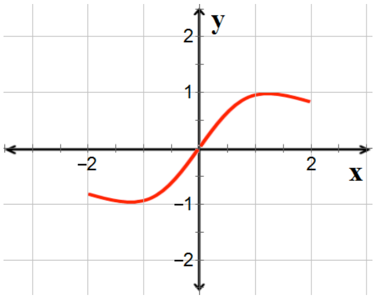
- ☐ x-axis
- ☐ y-axis
- ☐ Origin
- ☐ Neither

Intercepts

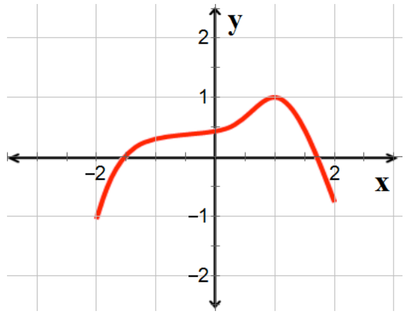
x-intercept ________________

y-intercept __________________

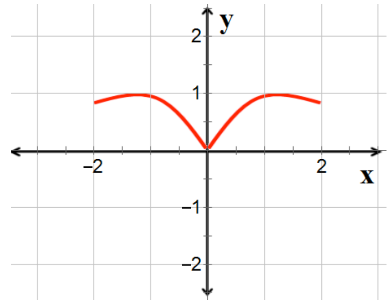
C.



D.



E.



19. Find the first 4 terms of the sequences given below.

$a_n = n^{th}$ term of the sequence	First term	Second term	Third term	Fourth term	
$a_n = 5\left(\frac{1}{2}\right)^{2n+1}, n = 0,1,2,\ldots$					☐ Arithmetic ☐ Geometric ☐ Neither
$a_n = 4n + 3, n = 4,5,\ldots$					☐ Arithmetic ☐ Geometric ☐ Neither
$a_n = \frac{2n-1}{n+2}, n = 1,2,3,\ldots$					☐ Arithmetic ☐ Geometric ☐ Neither

20. For the sequences below determine if they are arithmetic or geometric. Then find a formula for the a_n and evaluate: $a_5 =$, and $a_{10} =$.

Sequence	Type	$f(n) = a_n = n^{th}$ term	Evaluate these elements.
$\{13,17,21,25,\ldots.\}$	☐ Arithmetic ☐ Geometric ☐ Neither		$a_5 =$ $a_{10} =$
$\{7,21,63,189,\ldots\}$	☐ Arithmetic ☐ Geometric ☐ Neither		$a_5 =$ $a_{10} =$
$\{1,-1,1,-1,\ldots.\}$	☐ Arithmetic ☐ Geometric ☐ Neither		$a_5 =$ $a_{10} =$

21. For a given geometric sequence, the 7^{th} term, $a_7 = 15$ and the 9^{th} term, $a_9 = 135$. Find the 15^{th} term a_{15}. (Recall that geometric sequences are exponential functions of (n) and the formula for a_n can be written in the form: $a_n = a_0 \cdot r^n$.)